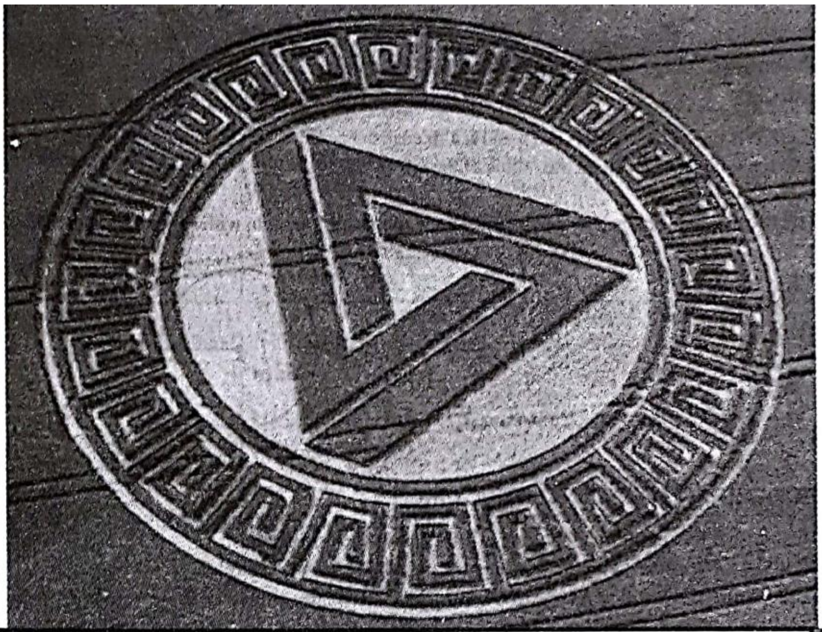
chapter 10



CIRCLES

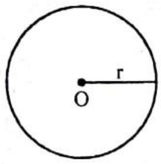
Introduction

In our daily life we come across many objects which are round (Circular) in shape like coins, discs, tyres, clock etc. In earlier classes, we have already studied about circles and some other properties related to the circle. In this chapter, you will study more about the circle and its related properties. You will study tangent to a circle, common tangent to circles, direct and indirect common tangents, important facts and theorems related to it.

CIRCLE :

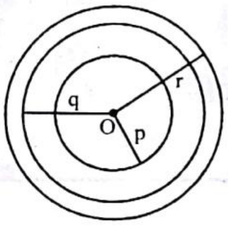
A circle is the set of all points in a plane at a fixed distance from a fixed point in the plane. The fixed point is called the center of the circle. The fixed distance is called the radius of the circle.

Usually a circle is named by specify its center and radius. A circle with centre 'O' and radius 'r' is named as circle (O, r).



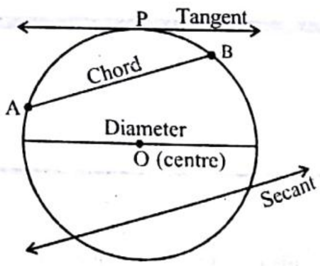
CONGRUENT AND CONCENTRIC CIRCLES :

We say that two circles are congruent if they have the same radius.
If two or more circles with the same center are called concentric circles.
In the figure, circles $(O, p), (O, q), (O, r)$ are three concentric circles.



TERMS RELATED TO A CIRCLE :

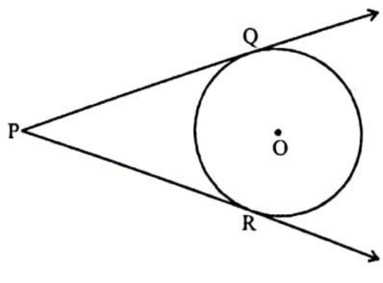
- A segment that connects two distinct points on a circle is called a **chord**.
- A chord that passes through the center of a circle is called a **diameter**.
- A line that intersects with the circle at exactly two points is called **a secant** (line).



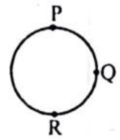
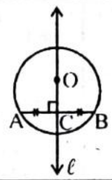
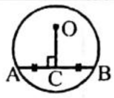
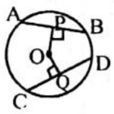
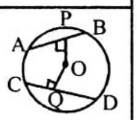
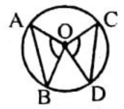
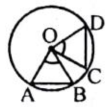
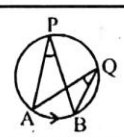
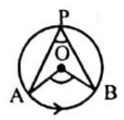
- **A tangent** to a circle is a straight line which touches the circle at only one point. The point where the tangent touches the circle is called the point of contact of the tangent to the circle. In the figure, P is the point of contact of the tangent to the circle.

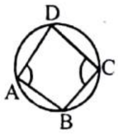
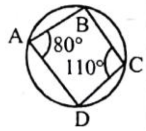
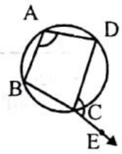
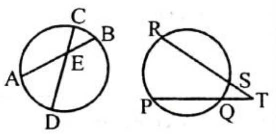
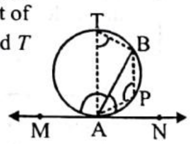
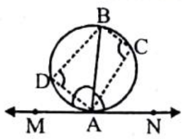
NUMBER OF TANGENTS FROM AN EXTERIOR POINT TO A CIRCLE :

Only two tangents can be drawn from an exterior point to a circle. In the figure, PQ and PR are the two tangents from an exterior point P to a circle.

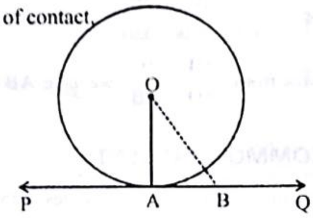


IMPORTANT POINTS TO BE REMEMBER

1.	One and only one circle passes through three non-collinear points.	In the figure, one and only one circle passes thought three non-collinear points P, Q and R. 
2.	Perpendicular bisector of a chord of a circle passes through the centre of the circle.	In the figure, line ℓ is the perpendicular bisector of chord AB. Hence the line ℓ passes through the centre O of the circle. 
3.	Perpendicular drawn from the centre to any chord, bisects the chord.	In the figure, OC is the perpendicular drawn from centre O to the chord AB. Hence OC bisect AB at C i.e. $AC = CB$ 
4.	Two equal chords of a circle are equidistant from the centre of the circle.	In the figure, AB and CD are two equal chords. Hence distance of chords AB and CD from centre O are equal i.e. $OP = OQ$ 
5.	If two chords of a circle are equidistant from the centre of a circle, then the two chords are equal.	In the figure, perpendicular distances OP and OQ from centre O of two chords AB and CD are equal. Hence $AB = CD$ 
6.	Equal chords of a circle subtend equal angles at the centre of the circle.	In the figure, AB and CD are two equal chords. Hence $\angle AOB = \angle COD$ 
7.	If two chords of a circle subtend equal angle at the centre, then the two chords are equal.	In the figure, $\angle AOB = \angle COD$. Hence $AB = CD$ 
8	Angles subtended by an arc of a circle at different points on the rest of the circle are equal.	In the figure, $\angle APB$ and $\angle AQB$ are the angle subtended by an arc AB of the circle at two different points on the rest of the circle. Hence $\angle APB = \angle AQB$ 
9.	Angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining part of the circle.	In the figure, $\angle AOB$ is subtended by an arc AB at the centre O and $\angle APB$. is the angle subtended by the arc AB at any pont P on the remaining part of the circle at two different points on the rest of the circle. Hence $\angle AOB = 2\angle APB$ 

10.	Opposite angles of a cyclic quadrilateral are supplementary.	In the figure, $ABCD$ is a cyclic quadrilateral. Hence $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$ 
11.	If any pair of opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	In the figure, $\angle A + \angle C = 180°$ Hence quadrilateral, $ABCD$ is cyclic. 
12.	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	In the figure, $\angle DCE$ is an exterior angle of cyclic quadrilateral $ABCD$. Interior opposite angle of $\angle DCE$ is $\angle A$. Hence $\angle DCE = \angle A$ 
13.	If two chords of a circle intersects (internally or externally) each other at a point. Then product of the lengths of their segments are equal.	In figure (i), two chords AB and CD of the circle intersects each other at point E internally. $\therefore AE \times EB = CE \times ED$ In figure (ii), two chords PQ and RS intersects each other at point T externally. Hence $PT \times TQ = RT \times TS$  (i) (ii)
14.	If a line touches the circle at a point and if a chord is drawn from the point of contact then the angle formed between the chord and the tangent is equal to the angle subtended by the chord in the alternate segments.	In the figure, a line MN touches a circle at a point A, AB is the chord through the point of contact of the line ℓ with the circle. P and T are any two points on the circle situated opposite sides of chord AB. So $\angle BAN$ & $\angle ATB$ and $\angle BAM$ & $\angle APB$ are two pairs of angles in the alternate segments. Hence $\angle BAN = \angle ATB$, $\angle BAM = \angle APB$ 
15.	A line drawn through the end point of a chord of a circle such that any pair of angle formed between the line and the chord is equal to the angle subtended by the chord in the alternate segment. Then, the line is tangent to the circle at that point.	In the figure $\angle$BAN $= \angle ADB$ Hence MN is tangent to the circle at point A of the circle. 

THEOREM 1 :



Statement : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given : PQ is a tangent of a circle at a point A and O is the centre of the circle.

To prove : $OA \perp PQ$

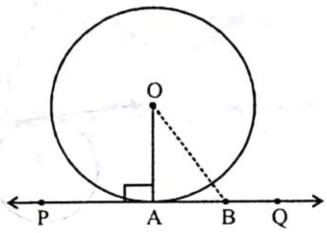
Construction : Take any point B on PQ. Then join OB.

Proof : Point B is outside the circle because any point on tangent other than point of contact is always outside the circle. Thus, the distance of centre from any point other than the point of contact on tangent is greater than its distance from point of contact. Therefore, $OB > OA$

Hence, out of all the line segments joining the centre O to the points on the tangent, OA is the least.

Since perpendicular from a point O on the tangent PQ is least of all line segments joining the centre to the points on the tangent. Hence $OA \perp PQ$.

THEOREM 2 (CONVERSE OF THEOREM 1) :



Statement : A straight line drawn at a point on the circle and perpendicular to the radius through that point is tangent to the circle.

Given : In Fig., O is the centre of the circle and OA is a radius.

$OA \perp PQ$

To prove : Straight line PQ is tangent to circle at point A.

Construction : Take any point B on line PQ and join QB.

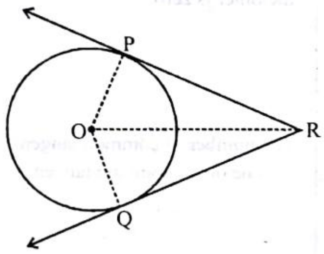
Proof : Given that radius OA is perpendicular on PQ, then among all the line segments joining O to any point on PQ, OA is shortest. $\Rightarrow OB > OA$

Hence, point B lie outside the circle. Similarly all points other than A on PQ will lie out side the circle. So line PQ intersects the circle at only one point hence PQ is a tangent to the circle.

THEOREM 3 :

Statement : The lengths of two tangents drawn from an external point to a circle are equal.

Given : O is the centre of the circle. RP and RQ are two tangents drawn from external point R to the circle.



To prove : $RP = RQ$

Construction : Join OP, OQ and OR.

Proof : We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\angle OPR = \angle OQR = 90°$ .... (1)

In ΔOPR and ΔOQR,

$\angle OPR = \angle OQR = 90°$ [By (1)]

$OR = OR$ (Common sides)

$OP = OQ$ (Radii of the same circle)

So, from right angle-hypotenuse-side congruency

$\Delta OPR \cong \Delta OQR$

Since corresponding sides of two congruent triangles are equal.

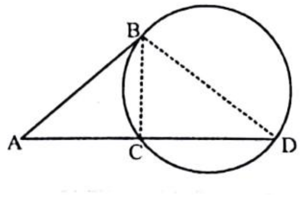
$\therefore$ $RP = RQ$

Corollary : Centre of the circle lies on the bisector of the angle between the two tangent drawn from an exterior point.

THEOREM 4 :

Statement : If AB is a tangent to a circle at B and ACD is a secant then $AB^2 = AC \times AD$.

Given : AB is a tangent to a circle at B and ACD is a chord.



To prove : $AB^2 = AC \times AD$

Construction: Join BC and BD.

Proof: In Δs ABC and ADB we have

$\angle A = \angle A$ (common)

$\angle ABC = \angle ADB$ [$\because$ angles in the alt. segments are equal]

$\therefore$ $\Delta ABC \sim \Delta ADB$ [AA criterian]

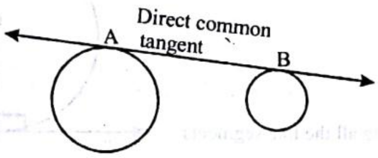
∴ their corresponding sides must be proportional.

i.e., $\frac{AB}{AD} = \frac{AC}{AB} = \frac{BC}{DB}$

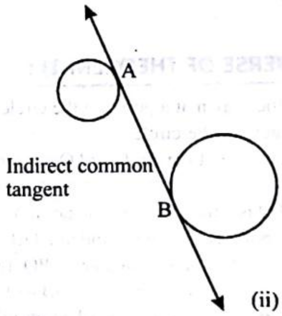
Now from $\frac{AB}{AD} = \frac{AC}{AB}$, we have $AB \times AB = AC \times AD$ or $AB^2 = AC \times AD$

COMMON TANGENTS:

If a line is tangent to two circles in the same plane, then the line is called a common tangent to the circles.
It the two circles are on the same side of the tangent, then the common tangent is called Direct common tangent.
If the two circles are on the opposite side of the tangent, then the common tangent is called Indirect (or transverse common tangent).



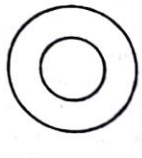
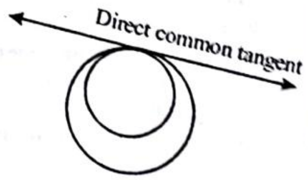
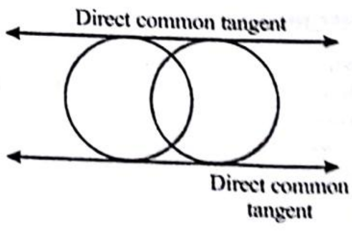
(i)



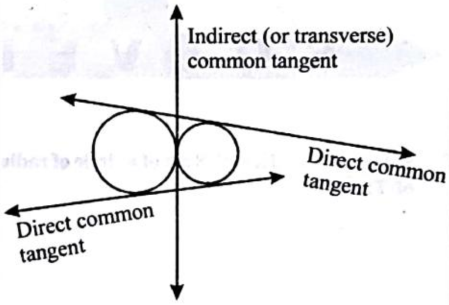
(ii)

In the figure (i) AB is the direct common tangent and in figure (ii) AB is the Indirect (or transverse) common tangent.

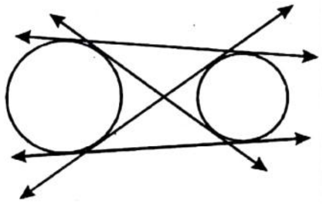
IMPORTANT POINTS TO BE REMEMBER

1.	The number of common tangents to the circles one lying inside the other is zero.	
2.	The number of common tangents to two circles touching internally is one i.e. one direct common tangent.	
3.	The number of common tangents to two intersecting circles is two, i.e., two direct common tangents.	

4. The number of common tangents to two circles touching externally is, three, i.e., two direct common tangents and one indirect or transverse common tangent.



5. The number of common tangents to non-intersecting circles is four, i.e. 2 direct common tangents and 2 indirect (or transverse) common tangents.

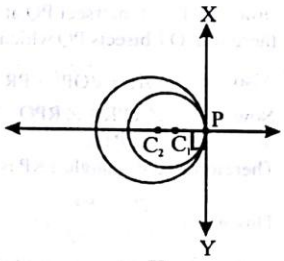


6. When two circles touch each other internally or externally, then the line joining the centres is perpendicular to the tangent drawn at the point of contact of the two circles.

Case (i)

Two circles with centres C_1 and C_2 touch each other internally at P. C_1C_2P is the line drawn through the centres and XY is the common tangent drawn at P which is common tangent to both the circles.

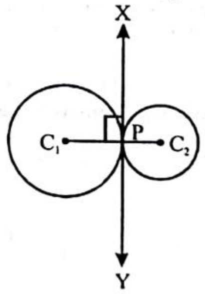
$\therefore$ C_1C_2 is perpendicular to XY



Case (ii)

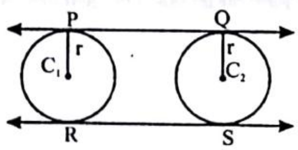
The given two circles with centres C_1 and C_2 touch each other externally at P.

C_1PC_2 is the line joining the centres of the circles and XY is the common tangent to the two circles drawn at P.



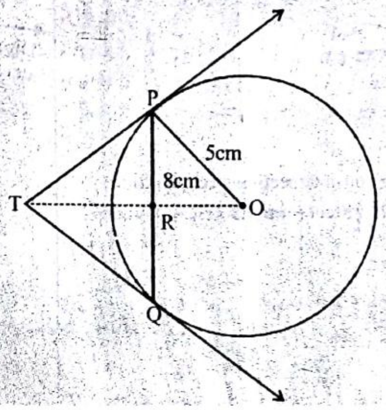
7. The direct common tangents to two circles of equal radii are parallel to each other.

Let two circles of equal radii 'r' have centres C_1 and C_2 and PQ and RS be the direct common tangents drawn to the circles. Then PQ is parallel to RS.



MISCELLANEOUS SOLVED EXAMPLES

1. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig.). Find the length of TP.



Sol. Join OT. Let it intersect PQ at the point R. Then Δ TPQ is isosceles and TO is the angle bisector of $\angle$ PTQ. So, OT $\perp$ PQ and therefore, OT bisects PQ which gives PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}\text{cm} = 3\text{ cm}.$

Now, $\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$

So, $\angle RPO = \angle PTR$

Therefore, right triangle TRP is similar to the right triangle PRO by AA similarity.

This gives $\dfrac{TP}{PO} = \dfrac{RP}{RO}$ i.e., $\dfrac{TP}{5} = \dfrac{4}{3}$ or $TP = \dfrac{20}{3}$cm.

TP can also be found by using the Pythagoras theorem, as follows :

Let $TP = x$ and $TR = y$.

Then $x^2 = y^2 + 16$ (Taking right Δ PRT) ...... (1)

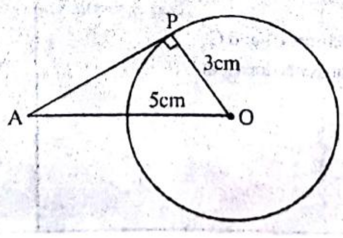
$x^2 + 5^2 = (y+3)^2$ (Taking right Δ OPT) ...... (2)

Subtracting (1) from (2), we get

$25 = 6y - 7$ or $y = \dfrac{32}{6} = \dfrac{16}{3}$

Therefore, $x^2 = \left(\dfrac{16}{3}\right)^2 + 16 = \dfrac{16}{9}(16+9) = \dfrac{16 \times 25}{9}$ [From (1)] or $x = \dfrac{20}{3}$

2. Find the length of tangent drawn from a point whose distance from the centre of a circle is 5 cm, and the radius of the circle is 3cm.



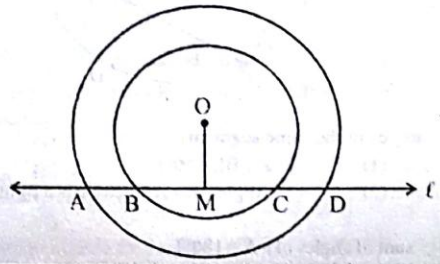
Sol. In figure, O is the centre of the circle and OA = 5 cm. and OP = 3 cm.

$\because$ The tangent is perpendicular to the radius through the point of contact hence in right angled triangle OPA (by Baudhayan theorem.)

$$OA^2 = OP^2 + PA^2$$
$$PA^2 = OA^2 - OP^2 = (5)^2 - (3)^2 = 25 - 9$$
$$PA^2 = 16 \text{ or } PA = 4 \text{ cm.}$$

$\because$ The length of tangent is 4 cm.

3. Two concentric circles with centre O have A, B, C, D as the points of intersection with the line ℓ as shown in the figure. If AD = 12 cm and BC = 8 cm, find the length of AB, CD, AC and BD.



Sol. Since OM ⊥ BC, a chord of the circle. $\therefore$ it bisects BC.

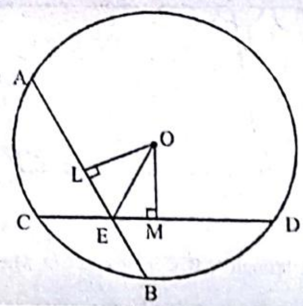
$\therefore \quad BM = CM = \frac{1}{2}(BC) = \frac{1}{2}(8) = 4$ cm.

Since OM ⊥ AD, a chord of the circle. $\therefore$ it bisects AD.

$\therefore \quad AM = MD = \frac{1}{2} AD = \frac{1}{2}(12) = 6$ cm.

Now, $AB = AM - BM = 6 - 4 = 2$ cm.
$CD = MD - MC = 6 - 4 = 2$ cm
$AC = AM + MC = 6 + 4 = 10$ cm
$BD = BM + MD = 4 + 6 = 10$ cm.

4. Chords AB and CD of a circle with centre O, intersect at a point E. If OE bisects ∠AED, then Prove that AB = DC.



Sol. Draw OL ⊥ AB and OM ⊥ CD.

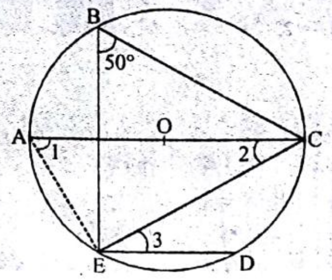
In Δ OLE and Δ OME

	∠OLE = ∠OME	(90° each)
	∠LEO = ∠MEO	(given)
and	OE = OE	(common)
$\therefore$	Δ OLE ≅ Δ OME	(By AAS congruence of the triangle)
⇒	OL = OM	

Thus, chords AB and CD are equidistant from centre. But we know that only equal chords are equidistant from centre.

So, AB = DC

5. In the given figure, the chord ED is parallel to the diameter AC. Find $\angle CED$.



Sol. $\angle CBE = \angle CAE$ ($\because$ angles in the same segment)

$\angle CAE = \angle 1 = 50°$ ....(1) ($\because \angle CBE = 50°$)

$\angle AEC = 90°$ ....(2) ($\because$ Angles in a semi circle is a right angle)

Now in ΔAEC,

$\angle 1 + \angle AEC + \angle 2 = 180°$ [$\because$ sum of angles of a $\Delta = 180°$]

$\therefore$ $50° + 90° + \angle 2 = 180°$

$\Rightarrow$ $\angle 2 = 40°$ ....(3)

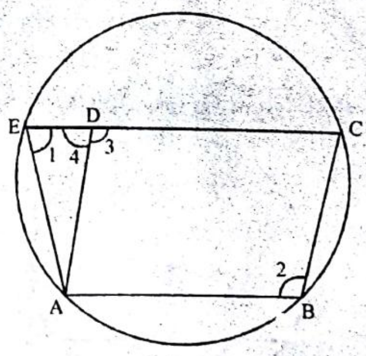
Also, $ED \parallel AC$ (Given)

$\therefore$ $\angle 2 = \angle 3$ (Alternate angles)

$\therefore$ $40° = \angle 3$ i.e., $\angle 3 = 40°$

Hence $\angle CED = 40°$

6. ABCD is a parallelogram. The circle through A, B and C intersects CD when produced at E. Prove that AD = AE.



Sol. Given : ABCD is a parallelogram. The circle through A, B, C intersects CD, when produced at E.

To prove : AE = AD.

Proof : Since ABCE is a cyclic quadrilateral

$\therefore$ $\angle 1 + \angle 2 = 180°$ (opposite angles of a cyclic quadrilateral are supplementary) ....(1)

Also, $\angle 3 + \angle 4 = 180°$ (linear pair) ....(2)

From (1) and (2), we get

$\angle 1 + \angle 2 = \angle 3 + \angle 4$ ....(3)

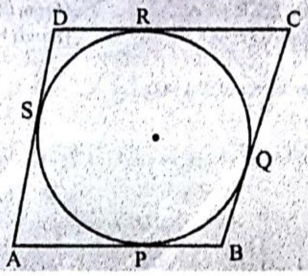
But $\angle 2 = \angle 3$ (opposite angles of a parallelogram) ....(4)

$\therefore$ From (3) and (4), we get $\angle 1 = \angle 4$

Now in ΔADE, since $\angle 1 = \angle 4$

AD = AE (sides opp. to equal angles are equal)

7. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.



Sol. Given : Sides AB, BC, CD and DA of a ||gm ABCD touch a circle at P, Q, R and S respectively.

To prove : Parallelogram ABCD is a rhombus.

Proof :

$AP = AS$ .... (1)

$BP = BQ$ .... (2)

$CR = CQ$ .... (3)

$DR = DS$ .... (4)

[Tangents drawn from an external point to a circle are equal]

Adding (1), (2), (3) and (4), we get

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow AB + AB = AD + AD$ [In a ||gm ABCD, opp. sides are equal]

$\Rightarrow 2AB = 2AD$

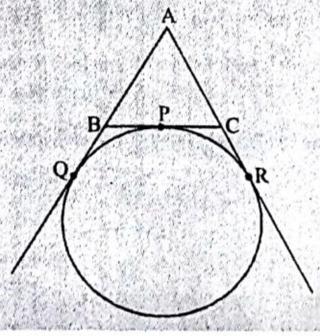
or $AB = CD$

But $AB = CD$ and $AD = BC$ [Opposite sides of a ||gm

$\therefore AB = BC = CD = DA$

Hence, ||gm ABCD is a rhombus.

8. A circle touches the side BC of a Δ ABC at P and touches AB and AC when produced at Q and R respectively shown in figure, show that $AQ = \frac{1}{2}$ (Perimeter of Δ ABC)



Sol. Given : A circle is touching side BC of Δ ABC at P and touching AB and AC when produced at Q and R respectively.

To prove : $AQ = \frac{1}{2}$ (Perimeter of Δ ABC)

Proof :

$AQ = AR$ .... (1)

$BQ = BP$ .... (2)

$CP = CR$ .... (3)

[Tangents drawn from an external point to a circle are equal]

Now, Perimeter of $\Delta ABC = AB + BC + CA$

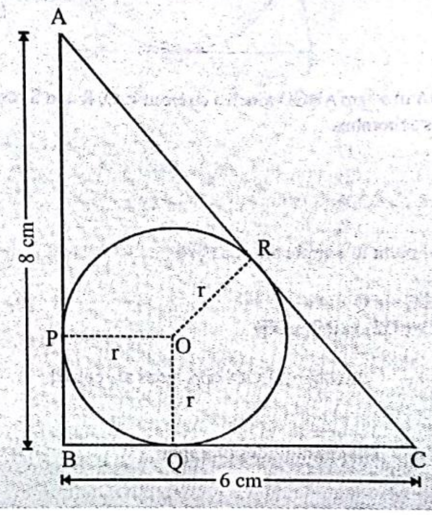
$= AB + BP + PC + CA$

$= (AB + BQ) + (CR + CA)$ [From (2) and (3)]

$= AQ + AR = AQ + AQ$ [From (1)

$\Rightarrow \quad AQ = \frac{1}{2}$ (Perimeter of Δ ABC)

9. In the given figure, ABC is a right angled triangle, right angle at B such that BC = 6 cm. and AB = 8 cm. Find the radius of its incircle.



Sol. Since lengths of two tangents drawn from an external point to a circle are equal, therefore,

$AR = AP = AB - BP \Rightarrow AP = AR = (8 - r)$ cm.

and $CR = CQ = CB - BQ = (6 - r)$ cm

$\therefore \quad AC = AR + CR = (8 - r + 6 - r)$ cm $= (14 - 2r)$ cm.

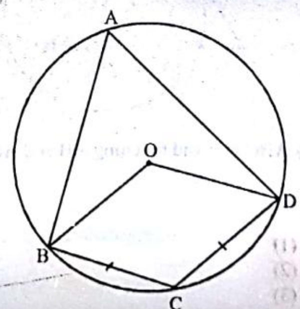
Now, $AC^2 = AB^2 + BC^2$

($\because \Delta$ ABC is a right angled triangle)

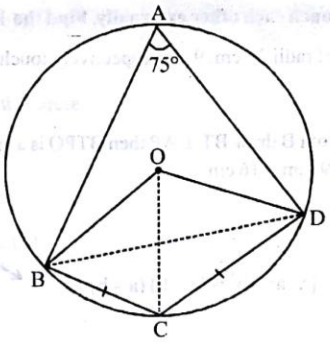
$\Rightarrow \quad (14 - 2r)^2 = 8^2 + 6^2$

$\Rightarrow \quad r = 2$ cm.

10. In the given figure O is the centre of the circle, $\angle BAD = 75°$ and chord BC = chord CD. Find (i) $\angle BOD$, (ii) $\angle OBD$, (iii) $\angle BCD$.



Sol. Join BD and OC.



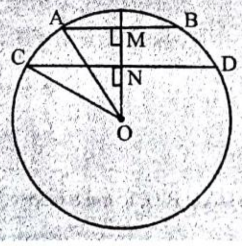
(i) We know that $\angle BOD = 2\angle BAD$
[Angle at the centre is double the angle in the remaining circumference made by the same arc]
$\therefore$ $\angle BOD = 2\times 75° = 150°$

(ii) In ΔOBD, $OB = OD$ (Radii of the same circle)
$\angle OBD = \angle ODB$ [Angles opposite to equal sides are equal]

$$= \frac{180° - 150°}{2} = \frac{30°}{2} = 15°$$

(iii) $\angle BCD = 180° - 75° = 105°$ [Opposite angles of a cyclic quadrilateral are supplementary]

11. AB and CD are two chords of a circle such that AB = 6cm, CD = 12 cm and AB || CD. If the distance between AB and CD is 3cm, find the radius of the circle.



Sol. Let AB and CD be two parallel chords of a circle with centre O and radius r cm.
AB = 6 cm, CD = 12 cm.
Let $OM \perp AB$ and $ON \perp CD$. Since AB || CD, therefore, M, N and O are collinear.
Given that distance between AB and CD = 3cm = MN
$OA = r$ cm, $OC = r$ cm.
Let $ON = x$ cm then $OM = (x + 3)$ cm

$$AM = \frac{1}{2}AB = \frac{1}{2}\times 6\text{ cm} = 3\text{ cm}$$

and $CN = \frac{1}{2}CD = \frac{1}{2}\times 12\text{ cm} = 6\text{ cm}$

In r.t. ΔOAM, $OA^2 = AM^2 + OM^2$ and in rt. ΔOCN, $OC^2 = CN^2 + ON^2$
$\Rightarrow$ $r^2 = (3)^2 + (x+3)^2$ .... (i)
and $r^2 = (6)^2 + x^2$ .... (ii)
From (i) and (ii), $(3)^2 + (x+3)^2 = (6)^2 + x^2$
$\Rightarrow$ $9 + x^2 + 6x + 9 = 36 + x^2$
$\Rightarrow$ $6x = 36 - 9 - 9 = 18$
$\Rightarrow$ $x = 3$ Thus, from (ii) $r = \sqrt{45}$
or $r = 3\sqrt{5}$ cm $= 3\times 2.236$ cm $= 6.708$ cm $= 6.71$ cm.
Hence, radius of the circle is 6.71 cm.

12. Two circles of radii 25 cm and 9 cm touch each other externally. Find the length of the direct common tangent.

Sol. Two circles with centres A and B and of radii 25 cm, 9 cm respectively touch each other at point C.

$BC = 9$ cm, $CA = 25$ cm

$BA = BC + CA = 9\text{ cm} + 25\text{ cm} = 34$ cm

Let PQ be a direct common tangent. From B draw $BT \perp AP$ then BTPQ is a rectangle.

$\therefore \quad AT = AP - TP = AP - BQ = (25 - 9)\text{ cm} = 16$ cm

Now from right ΔABT, we have

$AB^2 = AT^2 + BT^2$

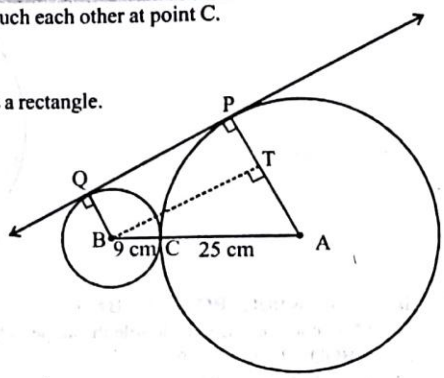
$\Rightarrow \quad BT^2 = AB^2 - AT^2 = (34)^2 - (16)^2$

$BT^2 = (34 + 16)(34 - 16) \qquad [\because a^2 - b^2 = (a + b)(a - b)]$

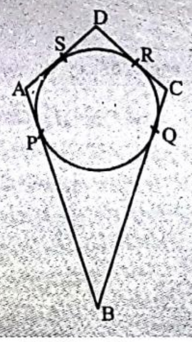
$= 50 \times 18 = 900$

$\Rightarrow \quad BT = +\sqrt{900} = 30$ cm

$\therefore \quad$ Length of direct common tangent $= PQ = BT = 30$ cm.



13. Let ABCD be a quadrilateral with an incircle. The sum of the opposite sides are equal i.e., AB + CD = AD + BC.



Sol. Let the incircle touch the sides AB, BC, CD, DA at P, Q, R and S respectively,

Now, $AP = AS$

$AP = BQ$

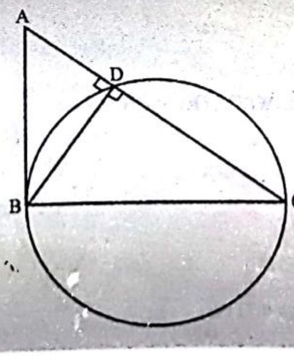
$CR = CQ$

$DR = DS$

So $\quad AB + CD = AP + BP + CR + DR = AS + BQ + CQ + DS = AS + DS + BQ + CQ = AD + BC$

Hence $\quad AB + CD = AD + BC$.

14. In a right triangle ABC, the perpendicular BD on the hypotenuse AC is drawn. Prove that
(i) $AC \times AD = AB^2$ (ii) $AC \times CD = BC^2$



Sol. We draw a circle with BC as diameter. Since $\angle BDC = 90°$.

$\therefore$ The circle on BC as diameter will pass through D.

$\therefore$ BC is a diameter and $AB \perp BC$

Also, AB is a tangent to the circle at B

Since AB is a tangent and ADC is a secant to the circle.

$\therefore$ $AC \times AD = AB^2$

This proves (i)

Again $AC \times CD = AC \times (AC - AD)$

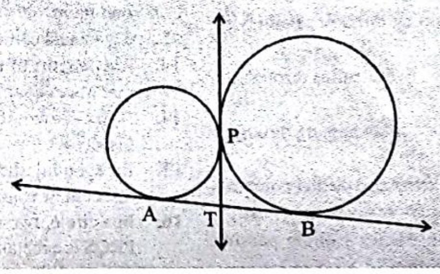
$= AC^2 - AC \times AD$

$= AC^2 - AB^2$ [Using (i)]

$= BC^2$ [ΔABC is a right triangle]

Hence, $AC \times CD = BC^2$. This proves (ii)

15. Two circles touch externally at P and a common tangent touches them at A and B. Prove that
(i) the common tangent at P bisects AB
(ii) AB subtends a right angle at P



Sol. Let PT be the common tangent at any point P. Since the tangent to a circle from an external point are equal.

$\therefore$ $TA = TP, TB = TP \Rightarrow TA = TB$

i.e., PT bisects AB at T

$TA = TP$ gives $\angle TAP = \angle TPA$ (from ΔPAT)

$TB = TP$ gives $\angle TBP = \angle TPB$ (from ΔPBT)

$\therefore$ $\angle TAP + \angle TBP = \angle TPA + \angle TPB = \angle APB$

$\Rightarrow$ $\angle TAP + \angle TBP + \angle APB = 2\angle APB = 180°$ [sum of $\angle$ s of a $\Delta = 180°$]

$\Rightarrow$ $\angle APB = 90°$

16. Two circles of radii 5 cm and 3 cm and centres A and B touch internally. If the perpendicular bisector of segment AB meets the bigger circle in P and Q. then find the length of PQ.

Sol. When two circles touch each other internally, then

Distance between their centres = Difference of their radii

$\Rightarrow AB = (5 - 3)\text{ cm} = 2\text{ cm}$.

Now PQ is the $\perp$ bisector of segment AB.

$\therefore AM = MB = 1\text{ cm}$

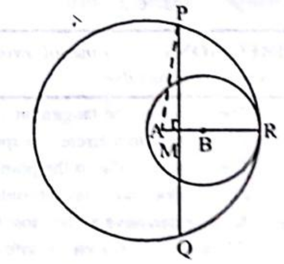
From rt angled. ΔAMP, we have

$AP^2 = AM^2 + MP^2 \Rightarrow 5^2 = 1^2 + MP^2$

$\Rightarrow MP^2 = 25 - 1 = 24\text{ cm} \Rightarrow MP = +\sqrt{24}\text{ cm} = 2\sqrt{6}\text{ cm}$

Now $AM \perp PQ \Rightarrow PM = MQ$

$\therefore PQ = 2\,PM = 2\times 2\sqrt{6}\text{ cm} \Rightarrow PQ = 4\sqrt{6}\text{ cm}$.



1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. A tangent to a circle touches it at .... point (s).
2. A line intersecting a circle in two points is called a ....
3. A circle can have .... parallel tangents at the most.
4. The common point of a tangent to a circle and the circle is called ....
5. There is no tangent to a circle passing through a point lying .... the circle.
6. The tangent to a circle is .... to the radius through the point of contact.
7. There are exactly two tangents to a circle passing through a point lying .... the circle.
8. The lengths of the two tangents from an external point to a circle are ....
9. There is one and only one tangent to a circle passing through a point lying .... the circle.
10. The line containing the radius through the point of contact is called the .... to the circle at the point.
11. The tangent at any point of a circle is .... to the radius through the point of contact.
12. The tangents drawn at the ends of a diameter of a circle are.................. .
13. The .... lies on the bisector of the angle between the two tangents.
14. If two circles intersect at two distinct points, then the number of common tangents is ....
15. From a point P which is at a distance of 13 cm from the centre of the circle of radius 5 cm, a tangent is drawn to the circle. The length of the tangent is ....

T/F True / False :

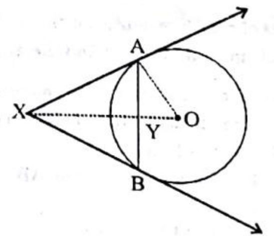
DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. There is only one tangent at a point of the circle.
2. The tangent to a circle is a special case of the secant.
3. The perpendicular at the point of contact to the tangent to a circle does not pass through the centre.
4. A circle can have at the most two parallel tangents.
5. If P is a point on a circle with centre C, then the line drawn through P and perpendicular to CP is the tangent to the circle at the point P.
6. The parallelogram circumscribing a circle is a rhombus.
7. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
8. There cannot be more than two tangents to a circle parallel to a given secant.
9. Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
10. The centre of the circle lies on the bisector of the angle between the two tangents.
11. A tangent to a circle is a line that intersects the circle in only one point.
12. Two equal chords of a circle are always parallel.
13. The lengths of tangents drawn from an external point to a circle are equal.
14. A line drawn from the centre of a circle to a chord always bisects it.
15. Line joining the centers of two intersecting circles always bisect their common chord.
16. In a circle, two chords PQ and RS bisect each other. Then PRQS is a rectangle.

MTF Match the Following :

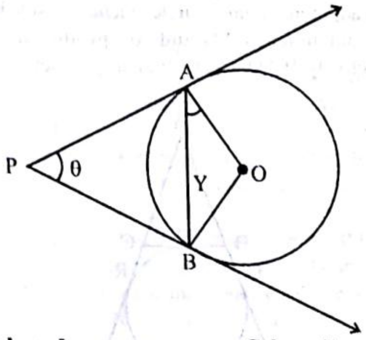
DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D....) in column I have to be matched with statements (p, q, r, s....) in column II.*

1. If AB is a chord of length 6 cm. of a circle of radius 5cm, the tangents at A and B intersect at a point X (figure), then match the column.



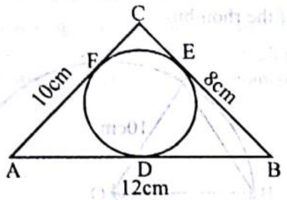
Column I	Column II
(A) AY	(p) 4 cm.
(B) OY	(q) 3.75 cm.
(C) XA	(r) 5 cm.
(D) OA	(s) 3 cm.

2. If two tangents PA and PB are drawn to a circle with centre O from an external point P (figure), then match the column.



Column I	Column II
(A) $\angle PAB$	(p) $90°$
(B) $\angle OAP$	(q) $\theta/2$
(C) $\angle OAB$	(r) $90-\frac{\theta}{2}$
(D) $\angle AOB$	(s) $180°-\theta$

3. For a circle which is inscribed in a Δ ABC having sides 8cm, 10cm and 12 cm (figure), then match the column.

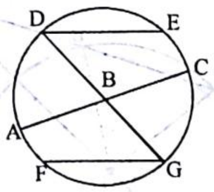


Column I	Column II
(A) AD	(p) 1
(B) BE	(q) 7
(C) CF	(r) 5
(D) AD/AF	(s) 3

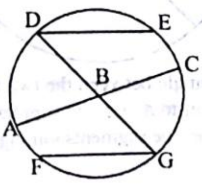
VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

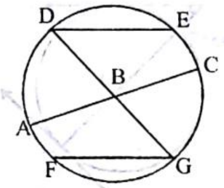
1. Name the centre of this circle.



2. Name two chords of the given circle that are not diameters.

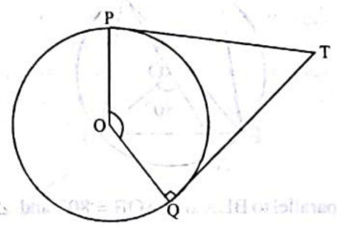


3. If DG is 5 inches long, then how long is DB ?



4. Two circles touch externally at a point P. From a point T, on the tangent at P, tangents TQ and TR are drawn to the circles with points of contact Q and R respectively. Prove that TQ = TR.

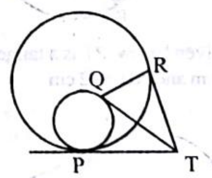
5. In figure, if TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.



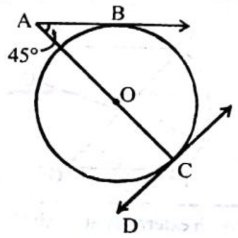
6. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the Length of PQ.

7. In the following figure, PT = 4 cm, TQ = 6 cm and RT = 3 cm, then find the value of TS

8. The accompanying diagram shows two circles which touch internally at a point P from T on a common tangent PT, tangent segments TQ and TR are drawn to both circles. Which type of triangle is TQR ?

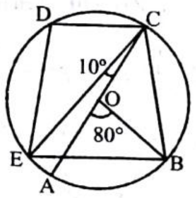


9. The diagram shows a circle with centre O. Line AB is tangent to the circle at point B and line DC is tangent to the circle at point C.



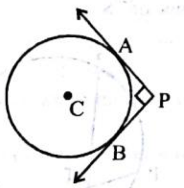
If the radius of the circle is 2 cm, what is the measure of AC ?

10. The diagram shows a circle where AC is the diameter and O is the centre.

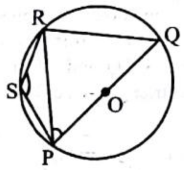


If CD is parallel to BE and $\angle AOB = 80°$ and $\angle ACE = 10°$, what is the measure of $\angle BEC$?

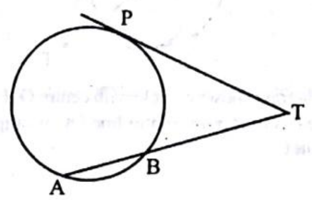
11. In the adjoining figure, PA and PB are tangents from P to a circle with centre C. If the radius of the circle is 4 cm and $PA \perp PB$, then find the length of each tangent.



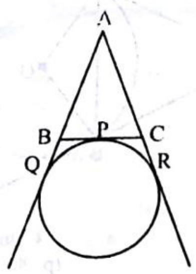
12. In the given figure, POQ is a diameter and PQRS is a cyclic quadrilateral. If $\angle PSR = 150°$, find $\angle RPQ$



13. In the figure given below, PT is a tangent to the circle. Find PT if AT = 16 cm and AB = 12 cm.



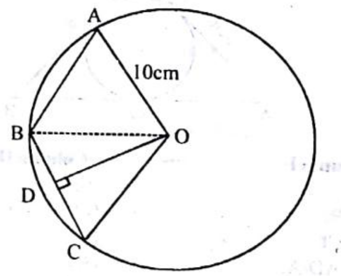
14. In the adjoining figure, a circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of ΔABC.



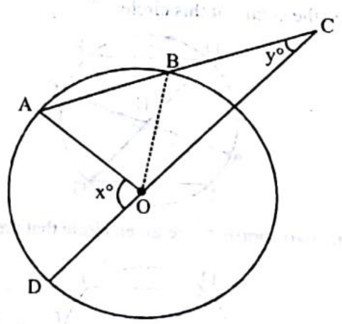
SAQ Short Answer Questions :

DIRECTIONS : *Give answer in two to three sentences.*

1. OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O. If the radius of the circle is 10 cm. Find the area of the rhombus.



2. In the given figure, AB is the chord of a circle with centre O. AB is produced to C such that BC = OB.CO is joined and produced to meet the circle in D. If $\angle ACD = y°$ and $\angle AOD = x°$, prove that $x° = 3y°$.



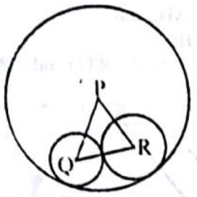
3. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

4. Suppose that we have two distinct circles (O, r) and (P, q). If r = 25 inches, q = 50 inches and the length of the segment OP is 60 inches, is it possible that circles (O, r) and (P, q) are tangent (i.e. they have exactly one point in common) ? Why?

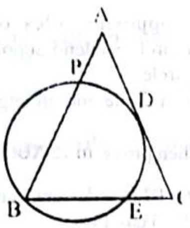
5. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

6. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. Find the length of the common chord.

7. In the given figure, three circles with centres P, Q and R are drawn such that the circles with centres Q and R touches each other externally and they touch the circle with centre P internally. If PQ = 10 cm, PR = 8 cm and QR = 12 cm, then find the diameter of the largest circle.

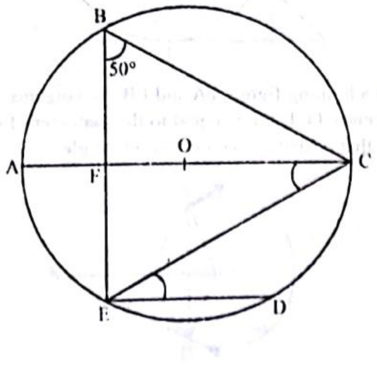


8. The accompanying diagram shows isosceles triangle ABC in which AB = AC. A circle passing through B intersects AB at P, BC at E and AC at D. D is the mid point of AC, then find ratio of AP to AB.



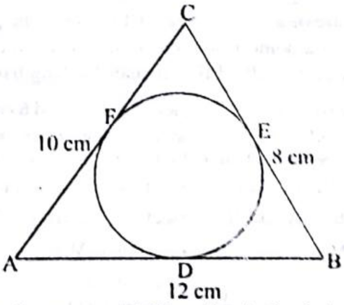
9. AB is a line segment of length 48 cm. C is its middle point. On AB, AC, CB semicircles are described. Determine the radius of the circle inscribed in the space enclosed by three semicircles.

10. In the given figure, the chord ED is parallel to the diameter AC. Find $\angle$CED.



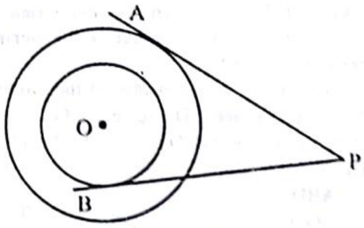
11. ΔABC is right angled at B such that BC = 6 cm and AB = 8 cm. Find the radius of its incircle.

12. A circle is inscribed in a Δ ABC having sides 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF.



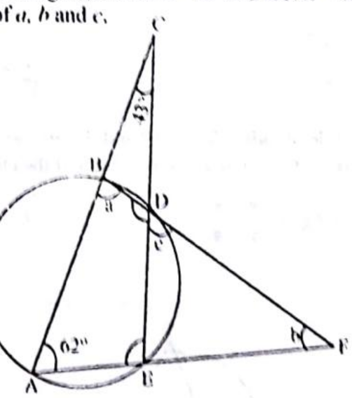
13. AB is a diameter and AC is a chord of a circle such that $\angle$ BAC = 30°. If then tangent at C intersects AB produced in D, prove that BC = BD.

14. In the adjoining figure, two concentric circles with centre O are of radii 5cm and 3cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, then find BP.



15. The radius of the circumcircle of a right triangle is 15 cm, and the radius of its inscribed circle is 6 cm. Find the sides of the triangle.

16. In the given figure, if $\angle$ACE = 43° and $\angle$CAF = 62°, find the values of a, b and c.



17. The radius of a circle is 13 cm and the length of one of its chord is 10 cm. What is the distance of the chord from the centre.

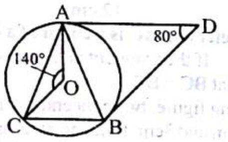
LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. The centre of a circle of radius 13 units is the point (3, 6). P (7, 9) is a point inside the circle. APB is a chord of the circle such that AP = PB. Calculate the length of AB.

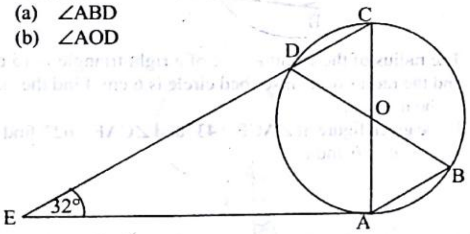
2. Two chords *AB* and *CD* of lengths 8 cm and 6 cm of a circle are parallel and are on the same side of its centre. If the distance between them is 1 cm. Find the radius of the circle.

3. In given figure, O is centre of the circumcircle of ΔABC. Tangents at A and B intersect at D. Given $\angle ADB = 80°$ and $\angle AOC = 140°$, calculate the $\angle ACB$.



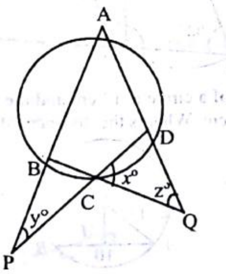
4. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral PQSR?

5. In the diagram, O is the centre of the circle ABCD. The straight lines AC and BD intersect at O.
The tangent at A meets CD produced at E and $\angle$ AED = 32°.
Calculate
(a) $\angle$ABD
(b) $\angle$AOD

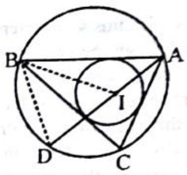


6. In a circle of radius 5 cm. *AB* and *AC* are two chords such that *AB* = *AC* = 6 cm. Find the length of the chord *BC*.

7. In given fig., if $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then calculate the values of *x*, *y* and *z*.

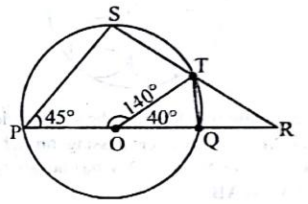


8. In fig., I is the incentre of ΔABC. AI when produced meets the circumcircle of ΔABC in Δ. If $\angle BAC = 66°$ and $\angle ACB = 80°$, calculate
(i) $\angle$DBC (ii) $\angle$IBC
(iii) $\angle$BID



9. In a right triangle ABC, the perpendicular BD on the hypotenuse AC is drawn. Prove that
(i) $AC \times AD = AB^2$ and
(ii) $AC \times CD = BC^2$

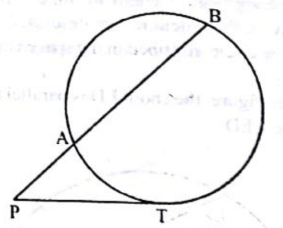
10. In the given figure, find $\angle$RTQ and $\angle$RQT



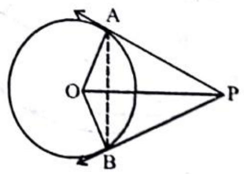
11. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

12. Suppose that O is a circle and an angle ABC is an angle inscribed in O. Then prove $m \angle ABC = \frac{1}{2} m \angle AOC$

13. In the given figure, PT is a tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT.



14. In the adjoining figure, PA and PB are tangents to a circle with centre O. If OP is equal to the diameter of the circle, prove that ΔABP is an equilateral triangle.



2 EXERCISE

MCQ Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. Two chords AB and CD of a circle intersect at E such that AE = 2.4 cm., BE = 3.2 cm and CE = 1.6 cm. The length of DE is –

(a) 1.6 cm. (b) 3.2 cm.
(c) 4.8 cm. (d) 6.4 cm.

2. ACB is a tangent to a circle at C. CD and CE are chords such that $\angle ACE > \angle ACD$. If $\angle ACD = \angle BCE = 50°$, then –

(a) CE = CD
(b) ED is not parallel to AB
(c) ED passes through the centre of the circle
(d) ΔCDE is a right-angled triangle

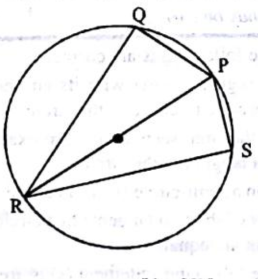
3. The locus of the middle points of equal chords of a circle with centre at O is –

(a) a straight line
(b) a circle with centre different from O
(c) a circle with centre at O
(d) a circle intersecting the given circle at the end of the chord.

4. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is –

(a) 3r (b) 6r
(c) 9r (d) 12r

5. In the figure if $\angle QPR = 67°$ and $\angle SPR = 72°$ and RP is a diameter of the circle, then $\angle QRS$ is equal to -



(a) 18° (b) 23°
(c) 41° (d) 67°

6. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and AB = 24 cm, then the distance O_1O_2 is equal to –

(a) 44 cm (b) 51 cm
(c) 40.5 cm (d) 45 cm

7. AB and CD are two chords of a circle intersecting at the point P outside the circle. If PA = 12 cm, CD = 7 cm and PC = 15 cm, then AB is equal to –

(a) 15.5 cm (b) 4 cm
(c) 8 cm (d) 10 cm

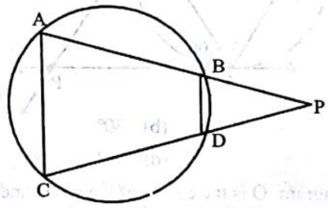
8. If tangents QR, PR, PQ and drawn respectively at A, B, C to the circle circumscribing an acute-angled ΔABC so as the form another ΔPQR, then the $\angle RPQ$ is equal to –

(a) $\angle BAC$ (b) $180° - \angle BAC$
(c) $\frac{1}{2}(180° - \angle BAC)$ (d) $180° - 2\angle BAC$

9. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length of PQ is

(a) 12 cm (b) 13 cm
(c) 8.5 cm (d) $\sqrt{119}$ cm

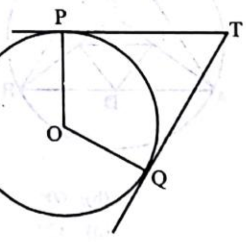
10. In the figure below (not to scale), AB = CD and $\overline{AB}$ and $\overline{CD}$ are produced to meet at the point *p*.



If $\angle BAC = 70°$, then $\angle P$ is

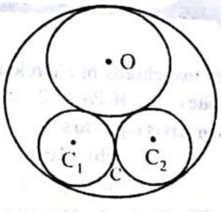
(a) 30° (b) 40°
(c) 45° (d) 50°

11. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110°$, then $\angle PTQ$ is



(a) 60° (b) 70°
(c) 80° (d) 90°

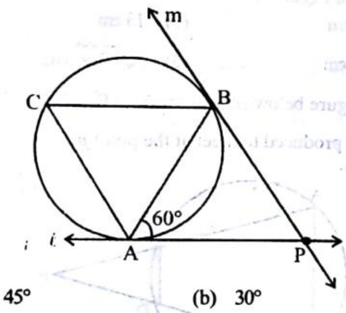
12. Two circles of unit radius touch each other and each of them touches internally a circle of radius 2 units, as shown in the following figure. The radius of the circle which touches all the three circles



(a) 5 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) None of these

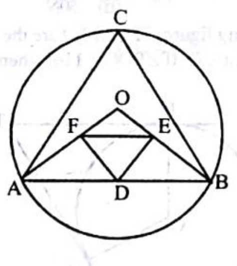
13. In the diagram below, if ℓ and m are two tangents and AB is a chord making an angle of 60° with the tangent ℓ, then the angle between ℓ and m is



(a) 45° (b) 30°

(c) 60° (d) 90°

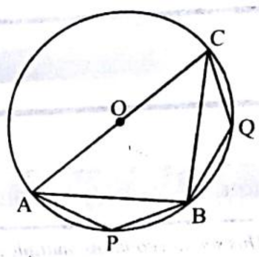
14. In the diagram, O is the centre of the circle and D, E and F are mid points of AB, BO and OA respectively. If $\angle DEF = 30°$, then $\angle ACB$ is



(a) 30° (b) 60°

(c) 90° (d) 120°

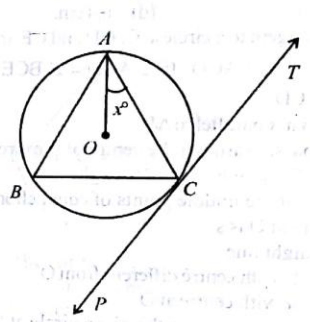
15. In the below diagram, O is the centre of the circle, AC is the diameter and if $\angle APB = 120°$, then $\angle BQC$ is



(a) 30° (b) 150°

(c) 90° (d) 120°

16. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumcentre of ΔABC. If $\angle ACP = 118°$, then the measure of $\angle x$. is



(a) 28° (b) 32°

(c) 42° (d) 38°

MTOC More than One Correct

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d). Out of which ONE OR MORE may be correct.*

1. Which of the following is/are correct.

(a) A line segment lying with its end points lying on a circle is called a chord of the circle.

(b) A line that intersects a circle at exactly one point is called a tangent to the circle.

(c) Angle in a semi-circle is a right angle.

(d) Lengths of the two tangents to a circle from an external point are equal.

2. Which of the following statement (s) is/are true?

(a) Two chords of a circle equidistant from the centre are equal

(b) Equal chords in a circle subtend equal angles at the centre

(c) Angle in a semicircle is a right angle

(d) None of the above.

3. Which of the following is a cyclic quadrilateral?
   (a) Rhombus (b) Rectangle
   (c) Parallelogram (d) Trapezium
4. Which of the following is/are not correct.
   (a) A secant is a line that intersects a circle in four distinct points.
   (b) In a circle, the perpendicular from the centre to a chord bisects the chord.
   (c) The point common to a circle and its tangent is called the point of contact.
   (d) Adjacent angles of a cyclic quadrilateral are supplementary.

PBQ Passage Based Questions :

DIRECTIONS : *Study the given passage(s) and answer the questions given below each passage.*

The lengths of two parallel chords of a circle are 6 cm and 8cm. The smaller chord is at a distance of 4 cm from the centre.

1. The radius of the circle is.
   (a) 10 cm (b) 5 cm
   (c) 3 cm (d) none of these
2. The distance of the other chord from the centre is.
   (a) 3 cm (b) 6 cm
   (c) 4 cm (d) none of these

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

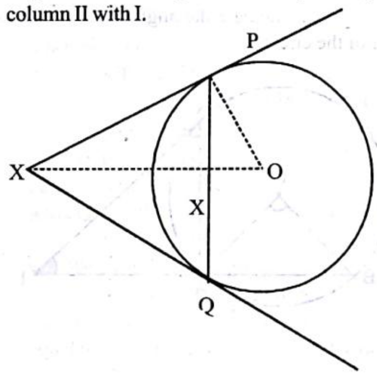
(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion is correct** but **Reason is incorrect**.
(d) If **Assertion is incorrect** but **Reason is correct**.

1. **Assertion:** If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.
   Reason: $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$
2. **Assertion:** If in a cyclic quadrilateral, one angle is 40°, then the opposite angle is 140°
   Reason: Sum of opposite angles in a cyclic quadrilateral is equal to 360°
3. **Assertion:** If length of a tangent from an external point to a circle is 8cm, then length of the other tangent from the same point is 8 cm.
   Reason: length of the tangents drawn from an external point to a circle are equal.

MMQ Multiple Matching Questions :

DIRECTIONS : *Each question has statements (A, B, C, D....) given in Column I and statements (p, q, r, s....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1. If PQ is a chord of length 8 cm of a circle of radius 5 cm, the tangents at P and Q intersect at point X, then match the column II with I.

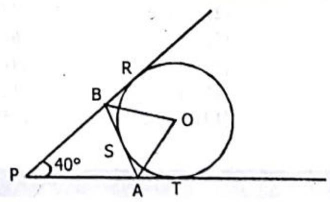


Column I	Column II
(A) XP	(p) 5 cm
(B) OY	(q) 0.5 m
(C) XQ	(r) $\frac{20}{3}$ cm
(D) OP	(s) 3 cm
	(t) 0.067 m

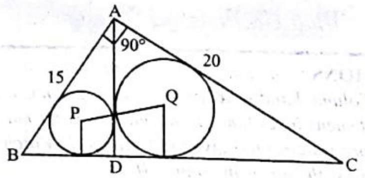
HOTS HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

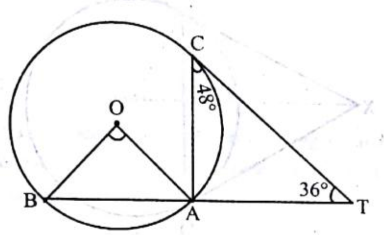
1. The radii of two concentric circles are 13 cm and 8 cm. PQ is a diameter of the bigger circle. QR is a tangent to the smaller circle touching it at R. Find the length PR.
2. As shown in the figure, triangle PAB is formed by three tangents to circle with centre O and $\angle APB = 40°$. Find $\angle AOB$.



3. In the figure, ΔACB is a right angled triangle. CD is the altitude. Circles are inscribed within the triangles ACD, BCD. P and Q are the centres of the circles. Find the distance PQ.

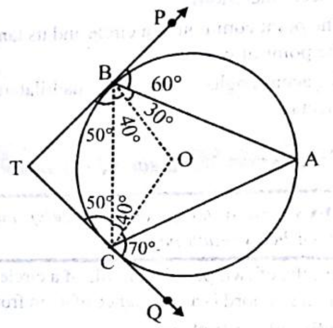


4. A, B and C are three points on a circle. The tangent at C meets BA produced at T. Given that $\angle ATC = 36°$ and that the $\angle ACT = 48°$, calculate the angle subtended by AB at the centre of the circle.



5. In a circle C (O, r), OP is equal to diameter of the circle. PA and PB are two tangents. Prove that Δ ABP is an equilateral triangle.

6. In the given fig. TBP and TCQ are tangents to the circle whose centre is O. Also, $\angle PBA = 60°$ and $\angle ACQ = 70°$. Determine $\angle BAC$ and $\angle BTC$.



SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

(1) One	(2) Secant	(3) Two
(4) Point of contact	(5) inside	
(6) perpendicular	(7) outside	(8) equal
(9) on	(10) normal	
(11) Perpendicular	(12) Parallel	(13) centre
(14) 2	(15) 12 cm	

TRUE / FALSE

(1) True	(2) True	(3) False
(4) True	(5) True	(6) True
(7) True	(8) True	(9) True
(10) True	(11) True	(12) False
(13) True	(14) False	(15) True
(16) True		

MATCH THE FOLLOWING :

(1) (A) → s; (B) → p; (C) → q; (D) → r

(2) (A) → r; (B) → p; (C) → q; (D) → s

(3) (A) → q; (B) → r; (C) → s; (D) → p

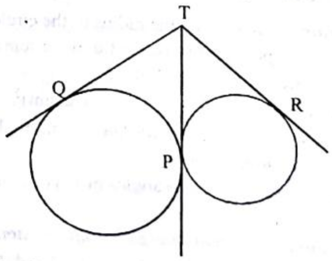
VERY SHORT ANSWER QUESTIONS :

(1) Point B represents the centre.

(2) DE and FG

(3) The diameter of a circle is twice as long as the radius.
So, DB = 5 inches ÷ 2 = 2.5 inches

(4) In figure, two circles touch each other at P. PT is the tangent (common) to the circles. TQ and TR are also tangents at Q and R respectively.



$\therefore$ TQ = TP and TP = TR

(Tangents from an external point to the circle are equal)

Comparing two equations, we get TQ = TR

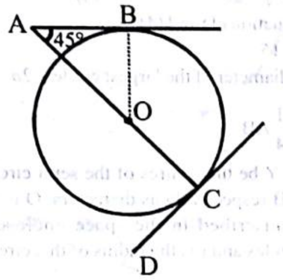
(5) $\angle PTQ = 70°$

(6) $PQ^2 = 119$.

(7) 8 cm.

(8) TP and TQ are tangents from point T on smaller circle.
So, TQ = TP and TP and TR are tangents from point T on bigger circle, so, TP = TR
$\Rightarrow TQ = TR$
Hence, TQR is isosceles triangle.

(9) Join OB.
$\angle OBA = 90°$ [Since radius is perpendicular to tangent]
By Pythagorus theorem $AB^2 + OB^2 = AO^2$.
In ΔABO, $\angle ABO = 90°$ and $\angle BAO = 45° \Rightarrow \angle BOA = 45°$.
So, OBA is an isosceles right angled triangle
$\therefore OB = AB = 2$ cm



$\Rightarrow AO^2 = 2^2 + 2^2 \Rightarrow AO = 2\sqrt{2}$ cm and $AC = AO + OC$
$= 2\sqrt{2} + 2$.

(10) $\angle AOB + \angle BOC = 180° \Rightarrow \angle BOC = 180° - 80° = 100°$
$\angle BEC = \frac{1}{2}\angle BOC = 50°$

(11) Join CA and CB, $CA \perp AP$, $CB \perp BP$. Check that CAPB is a square. AP = CA = 4cm.

(12) $\angle RPQ = 60°$

(13) Given AT = 16 cm and AB = 12 cm.
We know that
$PT^2 = TA \times TB$
$\Rightarrow PT^2 = 16 \times 4 \Rightarrow PT = \sqrt{64} = 8$ cm.
Hence, PT is of 8 cm.

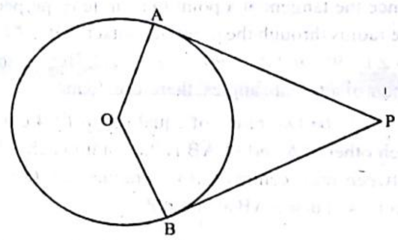
(14) 10 cm.
[**Hint.** $AQ = \frac{1}{2}$ (perimeter of ΔABC)
$\Rightarrow$ perimeter of $\Delta ABC = 2AQ = (2 \times 5)$ cm = 10 cm.]

SHORT ANSWER QUESTIONS :

(1) Since OABC is a rhombus
$\therefore$ OA = AB = BC = OC = 10 cm
Now, $OD \perp BC \Rightarrow CD = \frac{1}{2}BC = \frac{1}{2}(10) = 5$ cm.
$\therefore$ By pythagoras theorem,
$OC^2 = OD^2 + DC^2$
$\Rightarrow OD^2 = OC^2 - DC^2 = (10)^2 - (5)^2 = 100 - 25 = 75$
$\Rightarrow OD = \sqrt{75} = 5\sqrt{3}$
$\therefore$ area $(\Delta ABC) = \frac{1}{2} BC \times OD = \frac{1}{2}(10) \times 5\sqrt{3}$
$= 25\sqrt{3}$ sq. cm.
So, Area of rohmbus = 2 (Area of Δ OAB)
$= 2(25\sqrt{3}) = 50\sqrt{3}$ sq.cm.

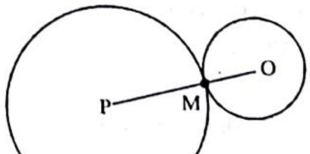
(2) Since BC = OB (Given)
$\therefore \angle OCB = \angle BOC = y°$
($\because$ angles opposite to equal sides are equal)
$\angle OBA = \angle BOC + \angle OCB = y° + y° = 2y°$
($\because$ exterior angles of a Δ is equal to the sum of the opposite interior angles)
Also, OA = OB ($\because$ Radii of the same circle)
$\angle OAB = \angle OBA = 2y°$ ($\because$ Angles opposite to equal sides of a triangle are equal)
$\angle AOD = \angle OAC + \angle OCA$
$\Rightarrow x° = 2y° + y° \Rightarrow x° = 3y°$ ($\because$ exterior angle of a Δ is equal to the sum of the opposite interior angles)

(3)



Proof : Since OA and OB are radii of circle and PA and PB are tangents, $OA \perp AP$ and $OB \perp BP$
$\therefore \angle OAP = \angle OBP = 90°$
Now in quadrilateral APBO, we have
$\angle A + \angle P + \angle B + \angle O = 360°$
$\Rightarrow 90° + \angle P + 90° + \angle O = 360°$
$\therefore \angle P + \angle O = 180°$
$\Rightarrow \angle APB + \angle AOB = 180°$

(4) Draw the sketch that we want to be a true one: circles (O, r) and (P, q) are tangent.



We have the following system of equations:

$OP = PM + MO = 60$

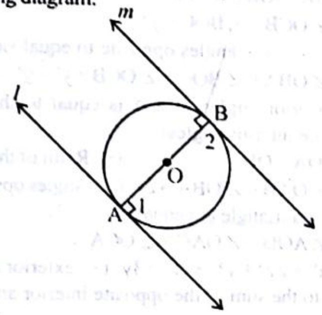
$PM = q = 50$

$MO = r = 25$

$PM + MO = 25 + 50 = 75$

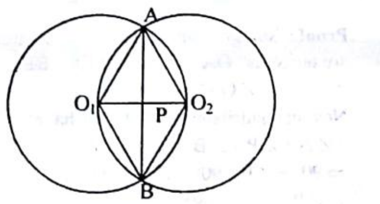
Due to the obvious contradiction between the first and the last equations (O, r) and (P, q) cannot be tangent.

(5) Let l and m be the tangents drawn at the end points A and B of a diameter AB of a circle with centre O. We need to show that l and m are parallel. Mark the angles as shown in the adjoining diagram.



Since the tangent at a point to a circle is perpendicular to the radius through the point of contact, $AB \perp l$ and $BA \perp m$ $\Rightarrow \angle 1 = 90°$ and $\angle 2 = 90° \Rightarrow \angle 1 = \angle 2$. But $\angle 1$ and $\angle 2$ form a pair of alternate angles, therefore, $l \parallel m$.

(6) Let there be two circle of equal radius (= 4 cm), intersect each other at A and B. AB is the common chord. Distance between their centres O_1O_2 = radius. So, $O_1O_2 = 4$ cm. O_1O_2 will bisect AB at 90° at P.



So, $AP = PB$ and $O_1P = O_2P = \frac{1}{2}O_1O_2 = \frac{4}{2} = 2$

In right angle ΔAPO_2

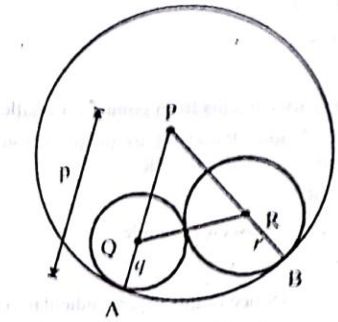
$O_2A^2 = AP^2 + O_2P^2$

$4^2 = AP^2 + 2^2$

$AP^2 = 4^2 - 2^2 = 12$

$\Rightarrow AP = 2\sqrt{3}$, $AB = 2AP = 4\sqrt{3}$

(7) Let radius of the circle with centre P, Q and R are respectively p, q and r.



Now, $PQ = p - q = 10$ ...(1)

$PR = p - r = 8$ ...(2)

and $QR = q + r = 12$ ...(3)

On adding equation (2) and (3),

$p + q = 20$ ...(4)

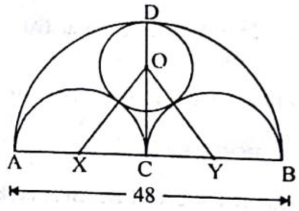
From equation (1) and (4),

$p = 15$

Hence, diameter of the largest circle $= 2p = 30$ cm

(8) $AP = \frac{1}{4}AB$

(9) Let X, Y be the centres of the semi circles described on AC, CB respectively as diameters. O is the centre of the circle inscribed in the space enclosed by the three semicircles and r is the radius of this circle.



$AX = XC = 12$ cm

$OC \perp AB$ and $OC = CD - OD$

$\Rightarrow OC = 24 - r$.

$OX = OY = 12 + R$

ΔXCO is right angled at C.

So, $OX^2 = XC^2 + OC^2$

$\Rightarrow (12 + r)^2 = 12^2 + (24 - r)^2$

$\Rightarrow 144 + 24r + r^2 = 12^2 + 576 + r^2 - 48r$

$\Rightarrow 72r = 576 \Rightarrow r = 8$

$\therefore$ Radius of circle is 8 cm.

(10) Join AE, AB and CD. $\angle CBE = \angle CAE$

($\because$ Angles on the same segment)

$\angle CAE = \angle 1 = 50°$ $(\angle CBE = 50°)$ ...(1)

$\angle AEC = 90°$ ...(2)

($\because$ Angles in a semi circle is a right angle.)

Now in ΔAEC,

$\angle 1 + \angle AEC + \angle 2 = 180°$

[$\because$ Sum of angles of a $\Delta = 180°$]

$\therefore \quad 50° + 90° + \angle 2 = 180°$

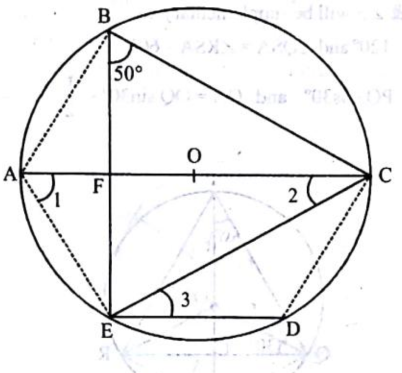
$\Rightarrow \quad \angle 2 = 40° \quad \ldots(3)$

Also, ED $\parallel$ AC (Given)

$\therefore \quad \angle 2 = \angle 3$ (Alternate angles)

$\therefore \quad 40° = \angle 3$ i.e., $\angle 3 = 40°$

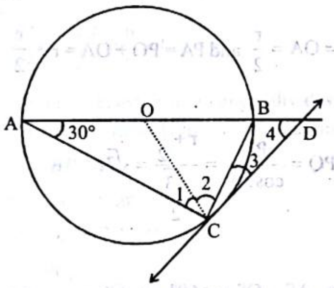
Hence, $\angle$CED $= 40°$.



(11) r = 2 cm.

(12) AD = 7 cm, BE = 5 cm, CF = 3 cm

(13) Join OC



OC $\perp$ CD $\therefore \angle 2 + \angle 3 = 90°$

OC = OA $\therefore \angle 1 = 30°$

Now, $\angle 1 + \angle 2 = 90°$ [angle is semi circle]

$\therefore \quad \angle 2 = 90° - 30° = 60° \Rightarrow \angle 3 = 30°$

In ΔACD, $\angle ACD + \angle CAD + \angle 4 = 180°$

$\Rightarrow \quad \angle 4 = 30°$

In ΔBCD, $\angle 3 = \angle 4 \quad \therefore$ BC = BD.

(14) $4\sqrt{10}$ cm;

Hint. Join OA, OB and OP.

OA $\perp$ AP, OB $\perp$ BP.

In ΔOAP, $\angle OAP = 90°$, $OP^2 = OA^2 + AP^2 = 5^2 + 12^2 = 169$

$\Rightarrow \quad$ OP = 13 cm.

In ΔOBP, $\angle OBP = 90°$, $BP^2 = OP^2 - OB^2 = 13^2 - 3^2 = 160$

$\Rightarrow \quad BP = 4\sqrt{10}$ cm

(15) The sides are 18, 24, 30.

(16) **Given:** $\angle ACE = 43°$ and $\angle CAF = 62°$.

In ΔAEC, $\angle AEC + \angle ACE + \angle CAE = 180°$

$\Rightarrow \quad \angle AEC + 43° + 62° = 180°$

$\Rightarrow \quad \angle AEC = 180° - (62° + 43°)$

$= 180° - 105° = 75°$.

We know that opposite angles of a cyclic quadrilateral are supplementary.

Here, AEDB is a cyclic quadrilateral.

$\therefore \quad a = \angle ABF = 180° - 75° = 105°$

And $\quad \angle BAE + \angle BDE = 180°$

$\Rightarrow \quad \angle BDE = 180° - 62° = 118°$

$\Rightarrow \quad c = \angle EDF = 180° - \angle BDE$ (straight angle)

$= 180° - 118° = 62°$

In ΔABF, $\quad b = \angle BFA = 180° - (62° + 105°)$

$= 180° - 167° = 13°$.

(17) Let the centre of the circle be 'O' and AB is the given chord.

$AB = 10$ cm, $OA = 13$ cm

$OM \perp AB$, OM = distance of chord AB from centre.

As $OM \perp AB$, So M is the mid point of AB.

$\therefore \quad AM = MB = 5$ cm

Now in ΔAOM

$AO^2 = AM^2 + OM^2$

$OM^2 = AO^2 - AM^2$

$OM^2 = 13^2 - 5^2 = 169 - 25 = 144$

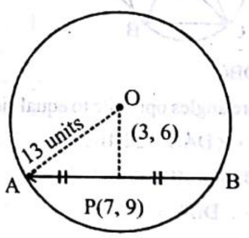
LONG ANSWER QUESTIONS :

(1) Given: The centre of the circle is O (3, 6). and P (7, 9) is a point on the chord AB of the circle such that AP = PB, i.e., P is midpoint of the chord.

Join OP and OA.

Radius of the circle OA = 13 units.

By distance formula:



$OP = \sqrt{(3-7)^2 + (6-9)^2}$

$\Rightarrow \quad OP = \sqrt{(-4)^2 + (-3)^2}$

$\Rightarrow \quad OP = \sqrt{16 + 9}$

$\Rightarrow \quad OP = \sqrt{25} = 5$ units.

We know that a straight line drawn from the centre of a circle to a chord is perpendicular to the chord.

Thus, $\angle OPA = 90°$

(2) Let AB and CD are two parallel chords.

Draw $\perp OM$ from centre O to AB and extend it to N.

As $AB \parallel CD$, $ON \perp CD$

Now $AM = \frac{1}{2}AB \Rightarrow AM = \frac{1}{2}\times 8 = 4$ cm

$CN = \frac{1}{2}CD = \frac{1}{2}\times 6$, $CN = 3$ cm

Given that $MN = 1$ cm

Let radius $OA = OC = x$ cm and $OM = y$ cm

From ΔOAM, $OA^2 = AM^2 + OM^2$

$y^2 = 4^2 + x^2$ ....(1)

From ΔOCN $OC^2 = CN^2 + ON^2$

$y^2 = 3^2 + (x+1)^2$ ....(2)

From (1) and (2)

$4^2 + x^2 = 3^2 + (x+1)^2$

$(x+1)^2 - x^2 = 4^2 - 3^2$

$(x+1+x)(x+1-x) = 16-9$

$\Rightarrow 2x+1 = 7 \Rightarrow 2x = 6 \Rightarrow x = 3$

$y^2 = 4^2 + 3^2$ From (1)

$y^2 = 25$

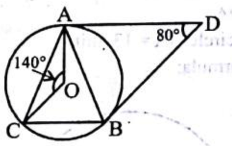
$\Rightarrow y = \sqrt{25} \Rightarrow y = 5$ cm

$\therefore$ Radius = 5 cm

(3) AD and BD are tangents from an external point D to a circle with centre O.

$\Rightarrow AD = BD$

In ΔADB, we have $AD = BD$ [from above]



$\angle DAB = \angle DBC$

[Since these are angles opposite to equal sides of a triangle]

$180° = \angle ADB + \angle DAB + \angle DBA$

$\Rightarrow 80° + \angle DAB + \angle DBA = 180°$.

So, $\angle DAB + \angle DBA = 100°$

$\Rightarrow \angle DAB = \angle DBA = 50°$

$OA \perp AD$ [$\because$ The radius of a circle is perpendicular to the tangent through the point of contact]

$\Rightarrow \angle OAD = 90°$

$\angle OAB = \angle OAD - \angle DAB = 90° - 50° = 40°$

In ΔOAC, we have

$OA = OC$ [Radii of the same circle]

$\therefore \angle OAC = \angle OCA$ [$\angle$s opp. to equal sides]

But $\angle AOC = 140°$

$\Rightarrow \angle OAC + \angle OCA = 180° - \angle AOC = 180° - 140° = 40°$

$\Rightarrow \angle OAC = \angle OCA = 20°$

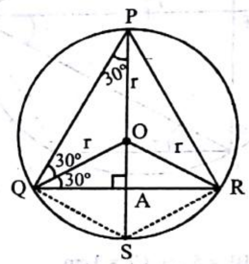
Now, $\angle CAB = \angle OAC + \angle OAB = 20° + 40° = 60°$.

(4) As PQR is an equilateral triangle, hence PS will be perpendicular to QR and will divide it into 2 equal parts.

$\angle P$ & $\angle S$ will be supplementary so

$\angle S = 120°$ and $\angle QSA = \angle RSA = 60°$.

$PA = PQ\cos 30°$ and $OA = OQ\sin 30° = \frac{r}{2}$



$\Rightarrow AS = OA = \frac{r}{2}$ and $PA = PO + OA = r + \frac{r}{2}$

Hence, $PQ = \frac{PA}{\cos 30°} = \frac{r+\frac{r}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}r$

In ΔQAS, $AS = QS\cos 60° \Rightarrow QS = \frac{r/2}{1/2} = r$

In Δs QAS and RAS,

Since $AQ = AR$, AS is common and $\angle QAS = \angle RAS = 90°$

So, $QS = RS$.

$\therefore$ Perimeter of $PQSR = 2(PQ + QS) = 2(\sqrt{3}+1)r$

(5) (a) Since A, B, C, D are four points on circle having centre O and AC and BD intersect at O.

So, $OA = OB = OC = OD$. Since EA is tangent at A, $OA \perp EA$.

In Δ AOB, $OA = OB \Rightarrow \angle BAO = \angle ABO$. So, it is isosceles.

Similarly, ΔDOC is also isosceles

$\angle ABD = \angle CAB = \angle BDC = \angle ACD$.

From right angle ΔEAC

$\angle ACE = 90 - 32 = 58°$

$\angle ACE$ is $\angle ACD = \angle ABD$.

So, $\angle ABD = 58°$

(b) $\angle AOB = \angle DOC$ and $\angle AOB + \angle OAB + \angle OBA = 180°$

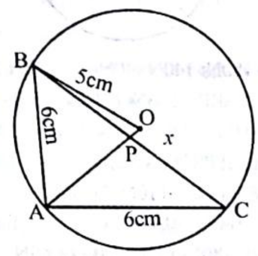
$\Rightarrow \angle AOB + 2 \times 58 = 180 \Rightarrow \angle AOB = 180 - 116$

$\Rightarrow \angle AOB = 64° = \angle DOC$

$\angle AOD = 180 - \angle DOC = 180 - 64$

So, $\angle AOD = 116°$.

(6) Here AB and AC are two equal chords of a circle with centre O.



$\therefore$ Centre O lies on the angle bisector of $\angle BAC$.

Also $\frac{BP}{PC} = \frac{AB}{AC} = \frac{6}{6} = 1$

[$\because$ Internal bisector of an angle divides the opposite sides in the ratio of the sides containing the angle]

$\Rightarrow \quad BP = PC$

Now if $\quad OP = x$ cm

then $AP = OA - OP = (5 - x)$ cm

Now in right ΔOPB

$PB^2 = 5^2 - x^2$ ....(1)

Also in right ΔAPB, $PB^2 = AB^2 - AP^2$

$= 6^2 - (5 - x)^2$ ....(2)

From (1) and (2), we get

$5^2 - x^2 = 6^2 - (5 - x)^2$

$\Rightarrow \quad 25 - x^2 = 36 - (25 - 10x + x^2)$

$\Rightarrow \quad 25 - x^2 = 11 + 10x - x^2$

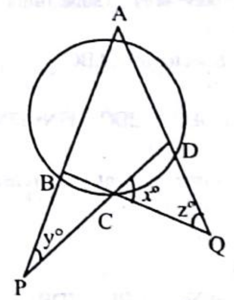
$\Rightarrow \quad 10x = 14 \Rightarrow x = \frac{14}{10} = 1.4$

Now $PB^2 = 5^2 - x^2 = 25 - (1.4)^2 = 25 - 1.96 = 23.04$

$\Rightarrow \quad PB = \sqrt{23.04} = 4.8$

Hence $BC = 2PB = 2 \times 4.8 = 9.6$ cm.

(7) In the figure, $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k$ (say)



$\Rightarrow \quad x = 3k, y = 4k, z = 5k$

In cyclic quad. ABCD, the side BC is produced to Q

$\therefore \quad \angle A = \angle DCQ = x$

[$\because$ Ext. $\angle$ of a cyclic quad. is equal to its int. opp. $\angle$]

Also, in ΔCDQ, the side QD is produced to A

$\angle ADP = x + z$

[$\because$ Ext. $\angle$ of a Δ is equal to the sum of its two interior opposite $\angle$s]

Now, in ΔADP

$\angle A + \angle ADP + \angle APD = 180°$

[$\because$ sum of $\angle$s of a $\Delta = 180°$]

$\therefore x + (x + z) + y = 180°$

$\Rightarrow \quad 2x + y + z = 180°$

$\Rightarrow \quad 2(3k) + 4k + 5k = 180°$

$\Rightarrow \quad 6k + 4k + 5k = 180°$

$\Rightarrow \quad 15k = 180°; k = 180° \div 15 = 12°$

$x = 3k = 3 \times 12 = 36°$

$y = 4k = 4 \times 12 = 48°$

and $z = 5k = 5 \times 12 = 60°$

(8) Join DB and IB

$\angle BAC = 60°$

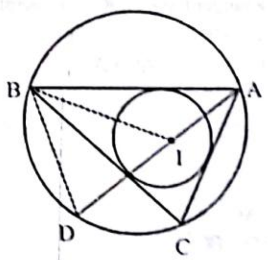
AI is the bisector of $\angle BAC$

$\therefore \quad \angle DAC = 33°$

Now $\angle DBC = \angle DAC$

[Angles in the same segment of a circle are equal]

$\Rightarrow \quad \angle DBC = 33° \quad [\because \angle DAC = 33°]$



Again $\angle BAC = 66°, \angle ACB = 80°$

$\therefore \quad \angle ABC = 180° - (\angle BAC + \angle ACB)$

$= 180° - (66° + 80°) = 180° - 146°$

$\Rightarrow \quad \angle ABC = 34°$

$\therefore$ BI is the bisector of $\angle ABC$

$\therefore \quad \angle IBC = 17°$

Now $\angle DBI = \angle DBC + \angle IBC = 33° + 17° = 50°$

Again $\angle ADB = \angle ACB$

[Angles in the same segment of a circle are equal]

But $\angle ACB = 80°$ (Given)

$\therefore \quad \angle ADB = 80°$

Now, in ΔIBD,

$\angle BID = 180° - (\angle DBI + \angle IDB)$

$= 180° - (50° + \angle ADB)$

$= 180° - (50° + 80°) = 180° - 130° = 50°$

Hence (i) $\angle DBC = 33°$

(ii) $\angle IBC = 17°$

(iii) $\angle BID = 50°$

(9) **Given :** A ΔABC right $\angle$d at AC is the hypotenuse $BD \perp AC$.

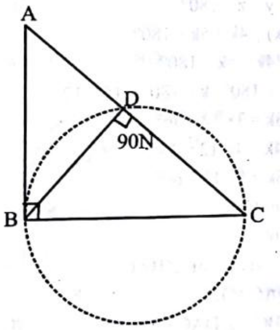
To prove. (i) $AC \times AD = AB^2$

(ii) $AD \times CD = BC^2$.

Proof :

(i) Draw circle circumscribing ΔBDC.

BC is the diameter because $\angle BDC = 90°$



In ΔABC, $\angle ABC = 90°$

$\therefore \quad AB \perp BC$

$\therefore$ AB is a tangent and ADC is a secant

So, $AC \times AD = AB^2$ .... (1)

Also, $AC \times CD = AC \times (AC - AD)$

$= AC^2 - AC \times AD$

$= AC^2 - AB^2$ [From (1), $AC \times AD = AB^2$]

[By pythagoras theorem]

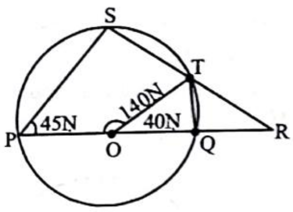
$= BC^2$

Hence, $AC \times AD = AB^2$

and $AC \times CD = BC^2$

(10) $\angle PST = \frac{1}{2}$ (Reflex $\angle POT$) [Angle at the circumference is $\frac{1}{2}$ of angle at the center]

$\Rightarrow \angle PST = \frac{1}{2}(360° - 140°) = 110°$



$\angle TOQ = 180° - 140° = 40°$ [Linear pair]

In ΔSPR, $\angle SRP = 180° - (110° + 45°) = 25°$

$\Rightarrow$ In ΔOTR $\angle OTR = 180° - (40° + 25°) = 115°$

Quadrilateral PSTQ is a cyclic quadrilateral

$\therefore \angle TQP = 180° - 110° = 70°$

$\Rightarrow RQT = 180° - 70° = 110°$ [linear pair]

$\therefore \angle RTQ = 180° - (25° + 110°) = 45°$

(11) Given a quadrilateral ABCD which circumscribes a circle with centre O.

We need to prove that

$\angle AOB + \angle COD = 180°$ and $\angle BOC + \angle AOD = 180°$

Let the sides AB, BC, CD and DA of the quadrilateral ABCD touch the circle at the points P, Q, R and S respectively. Mark the angles at O as shown in the adjoining figure.

In ΔOAP and ΔOAS,

$OA = OA, OP = OS$ (radii)

and $AP = AS$ (lengths of tangents)

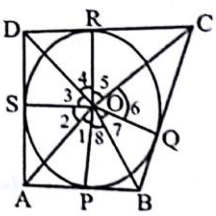
$\therefore \quad \Delta OAP \cong \Delta OAS$ (by SSS rule of congruency)

$\therefore \quad \angle 1 = \angle 2$ .... (i)

Similarly, $\angle 3 = \angle 4$ .... (ii)

$\angle 5 = \angle 6$ .... (iii)

and $\angle 7 = \angle 8$ .... (iv)



But $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

(sum of angles at a point)

$\Rightarrow\ 2(\angle 1+\angle 4+\angle 5+\angle 8)=360°$

[using (i), (ii), (iii) and (iv)]

$\Rightarrow\ (\angle 1+\angle 8)+(\angle 4+\angle 5)=180°$

$\Rightarrow\ \angle AOB+\angle COD=180°$.

Similarly, $\angle BOC+\angle AOD=180°$

(12) We need to show that an inscribed angle is equal to the half of the corresponding central angle. In order to do this we have to consider all possible cases that may happen for an inscribed angle.

Case 1: One of the angle sides passes through O.

Let's denote the degree measure of ABC as x. Then we need to show that the degree measure of AOC is 2x.

Indeed, the triangle AOB is an isosceles one, therefore the degree measure of BAO is x. From the theorem regarding the sum of angles in a triangle it follows that

$m\angle BOA=180°-2x$

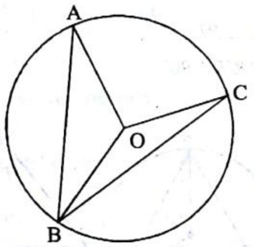
But angles BOA and AOC are supplementary angles, so

$m\angle AOC=180°(180°-2x)=2x$

Case 2: O is between angle sides.

Let's denote the degree measure of ABC as x again. And let's denote the degree measure of OBC as y. Since the triangle BOC is an isosceles one and from the theorem regarding the sum of angles in a triangle it follows that

$m\angle BOC=180°-2y$



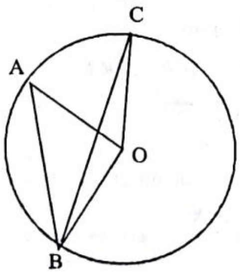
By the same reasoning we have that

$m\angle AOB=180°-2(x-y)$

$360°=m\angle AOC+m\angle COB+m\angle BOA$

$m\angle AOC=360°-(180°-2y)-(180°-2(x-y))=2x$

Case 3: Both sides of the angle are on the same side from O.



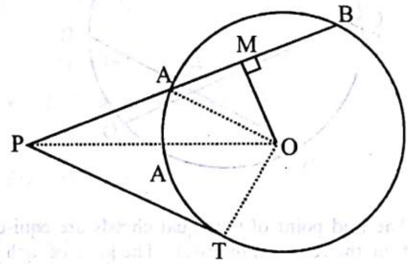
If we denote the degree measure of ABC as x and that of CBO as y then the degree measure of ABO is x+y. Following the same pattern of reasoning as the one for the previous two cases we have that

$m\angle COB=180°-2y$, $m\angle AOB=180°-2(x+y)$

$m\angle AOC=m\angle COB-m\angle AOB$

$=180°-2y(180°-2(x+y)=2x$

(13) Joint OT, OP and OA. Draw $OM\perp AB$. $OT\perp PT$



$\therefore\ OP^2=PT^2+OT^2$ .... (1)

Also, $OP^2=PM^2+OM^2$ .... (2)

From (1) and (2), $PT^2+OT^2=PM^2+OM^2$

$PT^2+OT^2=(PA+AM)^2+OM^2$

$PT^2+OT^2=PA^2+AM^2+2PA.AM+OM^2$

$PT^2+OT^2=PA^2+2PA.AM+OA^2$

$PT^2=PA^2+2PA.AM$ [$\because$ OT = OA]

$PT^2=PA^2+PA.AB$ [$\because$ OM bisects AB]

$PT^2=(4)^2+4\times 5$

$PT^2=36$

$PT=6$ cm.

(14) Let OP meet the circle at Q. Join AQ. As OP is equal to the diameter of the circle and OQ is radius, so OQ = QP i.e. Q is mid-point of OP. Since PA is tangent to the circle at A and OA is its radius, $OA\perp AP$ i.e. $\angle OAP=90°$

In right triangle OAP, Q is mid-point of hypotenuse,

$\therefore\ AQ=OQ=QP$

Also OA = OQ (radii of same circle)

$\Rightarrow\ OA=OQ=AQ\Rightarrow\Delta OAQ$ is equilateral

$\Rightarrow\ \angle AOQ=60°$

$\Rightarrow\ \angle AOP=60°$.

In ΔOAP, $\angle OPA+\angle AOP+\angle OAP=180°$

$\Rightarrow\ \angle OPA+60°+90°=180°\Rightarrow\angle OPA=30°$

$\Rightarrow\ \angle APB=60°$ ($\therefore$ OP is bisector of $\angle APB$)

Also $PA=PB\Rightarrow\angle PAB=\angle PBA$.

In ΔPAB, $\angle PAB+\angle PBA+\angle APB=180°$

$\Rightarrow\ 2\angle PAB+60°=180°\Rightarrow\angle PAB=60°$

$\Rightarrow$ triangle ABP is equilateral.

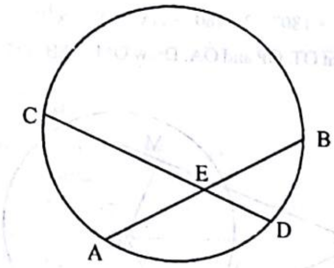
Exercise 2

MULTIPLE CHOICE QUESTIONS :

(1) (c) Apply the rule, $AE \times EB = CE \times ED$

$\Rightarrow$ $2.4 \times 3.2 = 1.6 \times ED$

$\therefore$ $ED = 4.8$ cm.



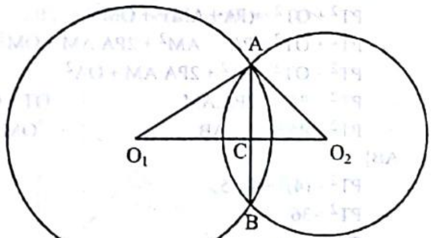
(2) (a)

(3) (c) The mid point of the equal chords are equi-distant from the centre of the circle. The locus of such points is a circle with same centre O.

(4) (b) Side of the regular hexagon inscribed in a circle of radius r is also r, the perimeter is 6r.

(5) (c)

(6) (b)



C is the mid-point of AB so that AC = 12

$AO_1 = 37$ and $AO_2 = 20$

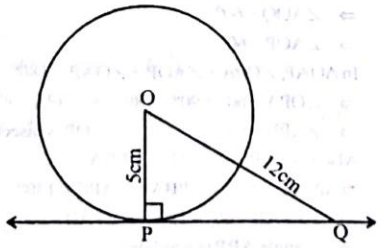
$\therefore$ $CO_1 = \sqrt{37^2 - 12^2} = 35$,

$CO_2 = \sqrt{20^2 - 12^2} = 16$

$\therefore$ $O_1O_2 = 35 + 16 = 51$

(7) (a) **(8)** (a)

(9) (d)



[**Hint.** $OP \perp PQ$ because tangent is perpendicular to radius through that point. In ΔOPQ, $\angle OPQ = 90°$ By Pythagoras theorem,

$PQ^2 = OQ^2 - OP^2 = 12^2 - 5^2 = 119$.]

(10) (b) Exterior angle of a cyclic quadrilateral is equal to its interior opposite angle.

$\angle BAC = \angle DCA$ and proceed.

(11) (b) [**Hint.** $OP \perp PT$, $OQ \perp QT$.

In quad. OPTQ, $\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360°$

$\Rightarrow 110° + 90° + \angle PTQ + 90° = 360° \Rightarrow \angle PTQ = 70°$]

(12) (c)

(13) (c) Tangents drawn to a circle from an external point are equal.

(14) (b) (i) ADEF is a parallelogram.

(ii) $\angle FAD = 30°$ and

$\angle OAD = \angle OBA$

(angles opposite to equal sides)

(15) (b) (i) APBC is a cyclic quadrilateral.

(ii) $\angle ABC$ is an angle in a semi circle.

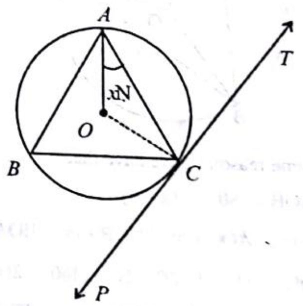
(iii) ABQC is a cyclic quadrilateral.

(16) (a) Join OC.

OC is the radius and PT is the tangent to circle at point C

$\therefore$ $OC \perp PT$

$\therefore$ $\angle OCP = 90°$



given $\angle ACP = 118°$

$\therefore$ $\angle ACO = \angle ACP - \angle OCP$

$= 118° - 90°$

$\angle ACO = 28°$

Since O is the circumcentre, thus $OA = OC$ (radius)

$\therefore$ $\angle OAC = \angle OCA$

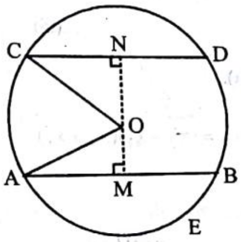
$\therefore$ $x = 28°$

MORE THAN ONE CORRECT :

(1) (a, b, c, d) **(2)** (a, b, c) **(3)** (a, b, c) **(4)** (a, d)

PASSAGE BASED QUESTIONS :

(1) (b)



$AB = 6$ cm, $CD = 8$ cm. $OM = 4$ cm

$AM = \frac{1}{2} \times (AB) = \frac{1}{2} \times (6) = 3$ cm

In ΔOAM,
By Pythagorus theorem,
$OA^2 = OM^2 + AM^2$

$OA = \sqrt{16+9} = 5$ cm.

$\therefore$ radius $= OA = 5$ cm.

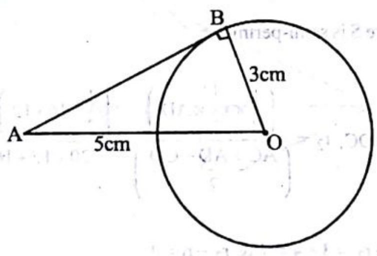
(2) (a) $CN = \frac{1}{2} \times CD = \frac{1}{2} \times 8 = 4$ cm.

In ΔCON,
By Pythagorus theorem,
$(OC)^2 = (ON)^2 + (CN)^2$
$(ON)^2 = (OC)^2 - (CN)^2$

$ON = \sqrt{(5)^2 - (4)^2} = 3$ cm

ASSERTION & REASON :

(1) (a)



$(OA)^2 = (AB)^2 + (OB)^2$

$(AB) = \sqrt{25-9} = 4$ cm.

Both Assertion and Reason are correct. Also, Reason is the correct explanation of the Assertion.

(2) (c) angle $+ 40° = 180°$
angle $= 180° - 40° = 140°$.

(3) (a)

MULTIPLE MATCHING QUESTIONS :

(1) (A) $\to$ r, t (B) $\to$ s (C) $\to$ r, t (D) $\to$ p

$PY = \frac{1}{2} \times PQ = \frac{1}{2}(8) = 4$ cm.

Now, By Pythagorus theorem,
$(OP)^2 = (OY)^2 + (PY)^2$

$OY = \sqrt{(5)^2 - (4)^3} = 3$ cm.

Now, ΔPXY and ΔPOY are similar

$\therefore \frac{PX}{PO} = \frac{PY}{OY} = \frac{4}{3}$

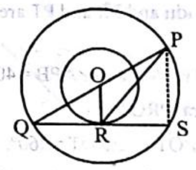
$PX = \frac{4 \times 5}{3} = \frac{20}{3}$

$PX = XQ = \frac{20}{3} cm.$

$OP =$ radius $= 5$ cm.

HOTS SUBJECTIVE QUESTIONS :

(1) Let the line QR intersects the bigger circle at S. Join PS.
O is the mid-point of PQ.
[$\because$ PQ is a diameter of the bigger circle]



QR is a tangent to the smaller circle and OR is a radius through the point of contact R.

$\therefore$ $OR \perp QR \Rightarrow OR \perp QS$

Since OR is $\perp$ to a chord QS of the bigger circle

$\therefore$ QR = RS

[$\because$ perpendicular from the centre to a chord bisects the chord]

$\Rightarrow$ R is the mid-point of QS.

$\therefore$ In ΔQSP, O is the mid-point of PQ and R is the mid-point of QS.

$\therefore$ $OR = \frac{1}{2} PS$

[Since segment joining the mid-points of any two sides of a triangle is half of the third side]

$\Rightarrow$ $PS = 2OR = 2 \times 8$ cm $= 16$ cm.

From right ΔOQR, $OR^2 + QR^2 = OQ^2$

$\Rightarrow \quad QR = \sqrt{OQ^2 - OR^2} = \sqrt{(13)^2 - (8)^2}$

$= \sqrt{169 - 64} = \sqrt{105}$

$\therefore \quad RS = QR = \sqrt{105}$

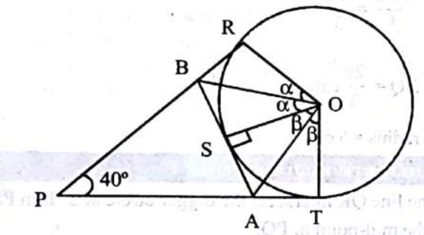
$PR^2 = RS^2 + PS^2 = 16^2 + \left(\sqrt{105}\right)^2$

$\Rightarrow \quad PR^2 = 256 + 105 = 361 \Rightarrow PR = 19$

(2) AS and AT are two tangents from A, so, AT = AS. Join O and S. Since AB is tangent at S and OS is radius $\Rightarrow OS \perp AB$ and $OS = OT$

BS and BR are two tangents from B. So, BS = BR and since OS and OR are radius of the same circle,

$\Rightarrow OS = OR$



Therefore, OA and OB bisect angles SOT and SOR

OR and OT are radii and PR and PT are tangent at R and T respectively.

So, $\angle ORP = \angle OTP = 90°$ and $\angle APB = 40° = \angle TPR$

So, in quadrilateral PROT

$\angle ORP + \angle TPR + \angle OTP + \angle ROT = 360°$.

So, $90° + 40° + 90° + \angle ROT = 360° \Rightarrow \angle ROT = 140°$

Let $\angle SOR = 2\alpha$ and $\angle SOT = 2\beta$ and OB and OA are respective bisectors, so, $\angle ROB = \angle BOS = \alpha$ and $\angle SOA = \angle AOT = \beta$.

$\angle AOB = \angle BOS + \angle SOA = \alpha + \beta$.

$\angle ROT = \angle SOR + \angle SOT = 2\alpha + 2\beta = 2(\alpha + \beta) = 140°$

$\Rightarrow \alpha + \beta = 70° = \angle AOB$. So, $\angle AOB = 70°$.

(3) In right angled ΔBAC,

$BC^2 = 15^2 + 20^2$ or $BC = 25$

Let $BD = x_1$ and $DC = x_2$

In right angled ΔADB,

$AB^2 = x_1^2 + AD^2$

or $15^2 = x_1^2 - AD^2 = 225$ ....(i)

In right angled ΔADC,

$AC^2 = DC^2 + AD^2$

$\Rightarrow \quad 20^2 = x_2^2 + AD^2$ ....(ii)

$x_1 + x_2 = 25$ ....(iii)

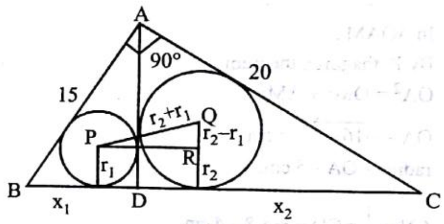
Subtracting (i) from (ii),

$\Rightarrow 400 - 225 = x_2^2 - x_1^2 = (x_2 - x_1)(x_2 + x_1)$ or

$175 = (x_2 - x_1) \times 25$

$\Rightarrow x_2 - x_1 = 7$ ....(iv)

Adding, (iii) & (iv), we get, $x_2 = 16$ & $x_1 = 9$



Using x_1, x_2 in (ii), we get, $400 = 256 + AD^2 \Rightarrow AD = 12$

In ΔADB, using the formula for inradius

$$r_1 = \frac{\text{Area}}{S} = \frac{\left(\frac{1}{2} \times x_1 \times AD\right)}{\left(\frac{AB + BD + AD}{2}\right)} = \frac{\left(\frac{1}{2} \times 9 \times 12\right) \times 2}{15 + 12 + 9} = 3$$

Where S is semi-perimeter

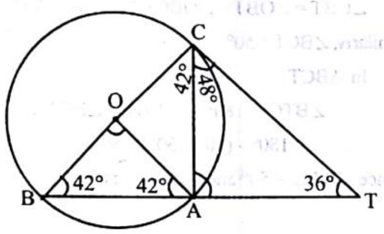
$$\text{In } \Delta ADC,\ r_2 = \frac{\left(\frac{1}{2} \times x_2 \times AD\right)}{\left(\frac{AC + AD + CD}{2}\right)} = \frac{\left(\frac{1}{2} \times 16 \times 12\right) \times 2}{20 + 12 + 16} = 4$$

$\therefore r_1 + r_2 = 3 + 4 = 7$ & $r_2 - r_1 = 1$

In ΔPQR, $PQ^2 = QR^2 + PR^2$

$\Rightarrow PQ = \sqrt{(r_1 + r_2)^2 + (r_2 - r_1)^2} = \sqrt{7^2 + 1^2} = \sqrt{50}$

(4) Let O be the centre of the circle. Join OC.



Since angle between the radius and the tangent is 90°, i.e.

$\angle OCT = 90°$

$\therefore \quad \angle OCA = \angle OCT - \angle ACT$

$= 90° - 48° = 42°$

and $OA = OC = OB$ [Radii of the same circle]

$\Rightarrow \quad \angle OAC = \angle OCA = 42°$

[Angles opposite to equal sides are equal]

In ΔACT, exterior $\angle CAB = \angle ACT + \angle CTA$

$\Rightarrow \quad 48° + 36° = 84°$

$\therefore \quad \angle OAB = \angle CAB - \angle OAC$

$\Rightarrow \quad \angle OAB = 84° - 42° = 42°$

$\Rightarrow \quad \angle OBA = \angle OAB = 42°$

In isosceles ΔAOB,

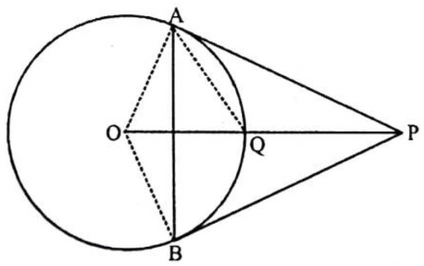
$\angle OBA + \angle OAB + \angle AOB = 180°$

$\Rightarrow \quad 42° + 42° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 180° - 84° = 96°$.

Hence, the angle subtended by AB at centre is 96°.

(5) Let OP meet the circle at Q (fig.).



We are given : $OP = \text{diameter} = 2r$

$\Rightarrow OP = OQ + OP \Rightarrow 2r = r + QP$

$\Rightarrow QP = r$

$\therefore$ Q is the mid-point of OP.

Now AP is tangent at A and OA is radius $\Rightarrow OA \perp AP$

$\therefore \Delta OAP$ is right triangle at $A = 90°$ and Q is mid-point of hypotenuse OP.

$\therefore \quad AQ = OQ = OA$

$\therefore \quad \Delta OAQ$ is equilateral

$\Rightarrow \quad \angle AOQ = 60°$

$\therefore \quad \angle APO = 180° - 90° - 60° = 30°$

$\therefore \quad \angle APB = 2\angle APO = 2 \times 30 = 60°$

Also; $PA + PB$

$\Rightarrow \quad \angle PAB = \angle PBA$

Now in ΔAPB, we have

$\angle APB + \angle PAB + \angle PBA = 180°$

$\Rightarrow \quad 60° + 2\angle PAB = 180° \Rightarrow \angle PAB = 60°$

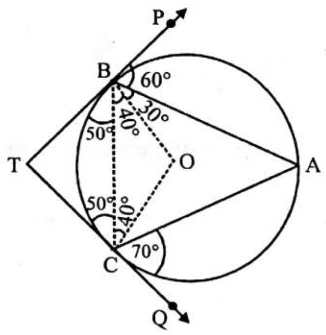
$\therefore \quad \angle PAB = \angle PBA = \angle APB = 60°$

$\therefore \quad \Delta APB$ is equilateral

(6) Join BC, BO and CO.

$\angle OBP = 90°$

[$\because$ Tangent to a circle is $\perp$ to the radius through the point of contact]



$\therefore \quad \angle OBA = 90° - 60° = 30°$.

Now $\angle ACQ = \angle CBA$

[$\angle$s in the alternate segment]

$\therefore \quad \angle CBA = 70°$ [$\because \angle ACQ = 70°$]

$\therefore \quad \angle OBC = \angle CBA - \angle OBA = 70° - 30° = 40°$

In ΔOBC,

$OB = OC$

$\therefore \quad \angle OBC = \angle OCB$

$\Rightarrow \quad \angle OCB = 40°$ [$\because \angle OBC = 40°$]

$\therefore \quad \angle BOC = 180° - (40° + 40°) = 100°$

Again $\angle BAC = \frac{1}{2}\angle BOC$

[$\therefore$ Angle subtended by an arc at the centre is double the angle subtended by the same arc at the circumference]

$\Rightarrow \quad \angle BAC = \frac{1}{2} \times 100° = 50°$

Again $\angle OBT = 90°$

[$\because$ Tangent to a circle is $\perp$ to the radius through the point of contact]

$\therefore \quad \angle CBT = \angle OBT - \angle OBC = 90° - 40° = 50°$

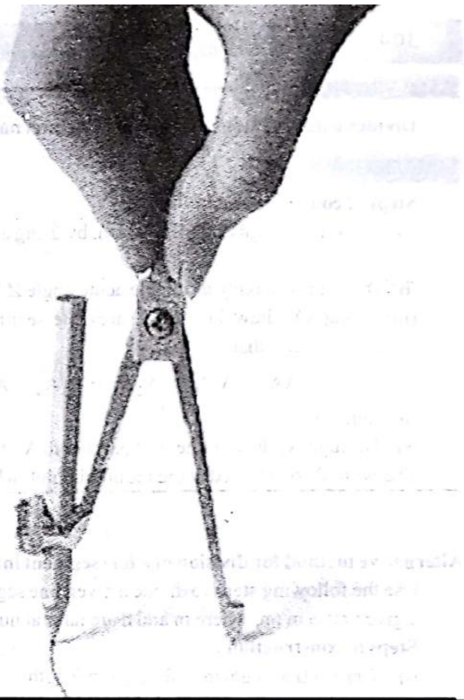
Similarly, $\angle BCT = 50°$

$\therefore$ In ΔBCT

$$\angle BTC = 180° - (\angle CBT + \angle BCT)$$
$$= 180° - (50° + 50°) = 80°$$

Hence $\angle BAC = 50°$ and $\angle BTC = 80°$

chapter 11



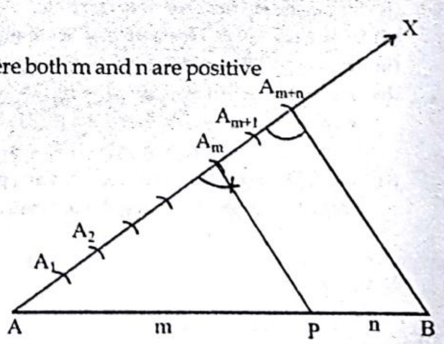
CONSTRUCTIONS

Introduction

One of the aims of the studying Geometry is to acquire the skill of drawing figures accurately. You have learnt how to construct geometrical figures namely triangles, quadrilateral and circles with the help of ruler and compasses. You have constructed angles of 30°, 60°, 90°, 120° and 45°. You have also drawn perpendicular bisector of a line segment and bisector of an angle.

DIVISION OF A LINE SEGMENT :

In order to divide a line segment internally in a given ratio m : n, where both m and n are positive integers, we follow the following steps :



Steps of construction :

(i) Draw a line segment AB of given length by using a ruler

(ii) Draw any ray AX making a suitable acute angle with AB.

(iii) Along AX draw (m + n) arcs intersecting the rays AX at $A_1, A_2, ..., A_m, A_{m+1}, ..., A_{m+n}$ such that $AA_1 = A_1A_2 = ... = A_{m+n-1}A_{m+n}$.

(iv) Join BA_{m+n}

(v) Through the point A_m draw a line parallel to $A_{m+n}B$ by making $\angle AA_mP = \angle AA_{m+n}B$.

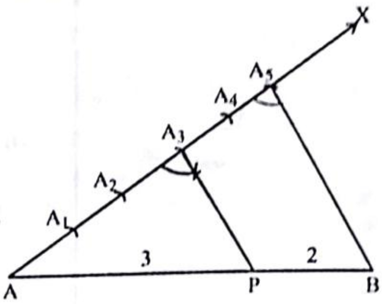
Suppose this line meets AB at point P.

The point P so obtained is the required point which divides AB internally in the ratio m : n.

ILLUSTRATION 11.1

Divides a line segment of length 12 cm internally in the ratio 3 : 2.

SOLUTION:



Steps of construction :

(i) Draw a line segment AB = 12 cm. by using a ruler.

(iv) Draw a ray making a suitable acute angle $\angle$ BAX with AB.

(iii) Along AX, draw 5 (= 3 + 2) arcs intersecting the rays AX at A_1, A_2, A_3, A_4 and A_5 such that

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$$

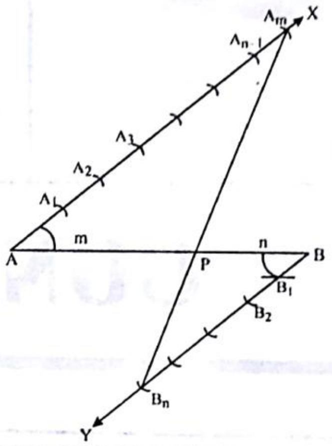
(iv) Join BA_5.

(v) Through A_3 draw a line A_3P parallel to A_5B by making $\angle AA_3P = \angle AA_5B$, intersecting AB at point P.

The point P so obtained is the required point, which divides AB internally in the ratio 3 : 2.

Alternative method for division of a line segment internally in a given ratio :

Use the following steps to divide a given line segment AB internally in a given ratio m : n, where m and n are natural numbers.



Steps of construction :

(i) Draw a line segment AB of given length.

(ii) Draw a ray AX making a suitable acute angle $\angle$ BAX with AB.

(iii) Draw ray BY, on opposite side of AX with respect to AB, parallel to AX by making an angle $\angle$ ABY equal to $\angle$ BAX.

(iv) Draw arcs intersecting the ray AX at $A_1, A_2, \ldots\ldots, A_m$, and ray BY at $B_1, B_2, \ldots\ldots\ldots, B$ such that

$$AA_1 = A_1A_2 = \ldots\ldots = A_{m-1}A_m = BB_1 = B_1B_2 = \ldots\ldots = B_{n-1}B_n$$

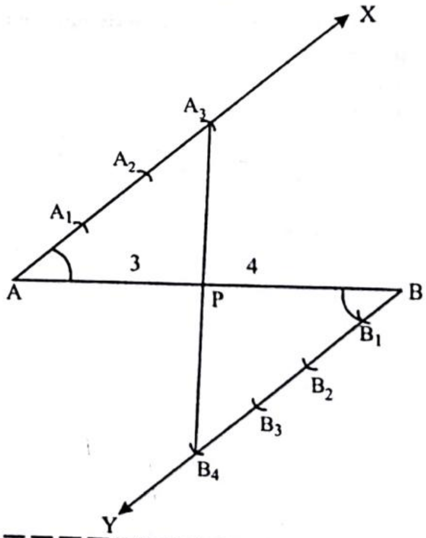
(v) Join A_mB_n. Suppose it intersects AB at P.

The point P is the required point dividing AB in the ratio m : n internally.

ILLUSTRATION 11.2

Divide a line segment of length 6 cm internally in the ratio 3 : 4.

SOLUTION:



Steps of construction :

(i) Draw a line segment AB of length 6 cm.

(ii) Draw a ray AX making a suitable acute angle $\angle$ BAX with AB.

(iii) Draw a ray BY parallel to AX by making $\angle$ ABY equal to $\angle$ BAX.

(iv) Draw three arcs intersecting the ray AX at A_1, A_2, A_3 and 4 arcs intersecting the ray BY at B_1, B_2, B_3, B_4 such that

$$AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_3B_4.$$

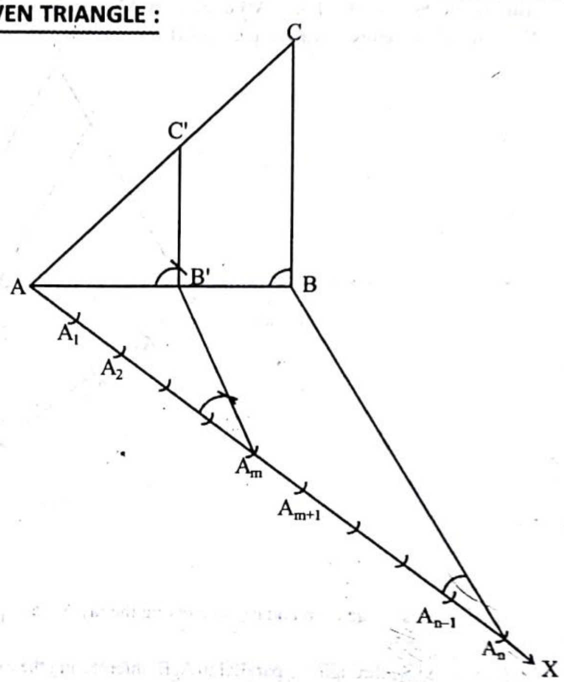
(v) Join A_3B_4. Suppose it intersects AB at a point P.

Then, P is the point dividing AB internally in the ratio 3 : 4.

CONSTRUCTIONS OF A TRIANGLE SIMILAR TO A GIVEN TRIANGLE :

Scale factor :

The ratio m : n of the side of the triangle to be constructed with the corresponding sides of the given triangle is known as their scale factor.



(I) Steps of construction when m < n :

(i) Construct the given triangle ABC by using the given data.

(ii) Take any one of the three side of the given triangle as base. Let AB be the base of the given triangle.

(iii) At one end, say A, of the base AB draw a ray AX making a suitable acute angle with AB below the base AB.

(iv) Along AX mark n points $A_1, A_2, A_3, ..., A_n$ such that

$$AA_1 = A_1A_2 = ...= A_{n-1}A_n.$$

(v) Join A_nB.

(vi) Draw A_mB' parallel to A_nB which meets AB at B'.

(vii) From B', draw B'C' || BC meeting AC at C'.

Triangle AB'C' is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of Δ ABC.

ILLUSTRATION 11.3

Construct a Δ ABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm. Now, construct a triangle similar to Δ ABC such that each of its sides is two-third of the corresponding sides of Δ ABC.

SOLUTION :

Steps of construction :

(i) Draw a line segment AB = 5 cm.

(ii) With A as centre and radius = 7 cm, draw an arc above AB.

(iii) With B as centre and radius = 6 cm, draw another arc, intersecting the arc drawn in step(ii) at C.

(iv) Join AC and BC to obtain Δ ABC.

(v) Below AB, draw a ray AX making a suitable acute angle with AB on opposite side of C with respect to AB.

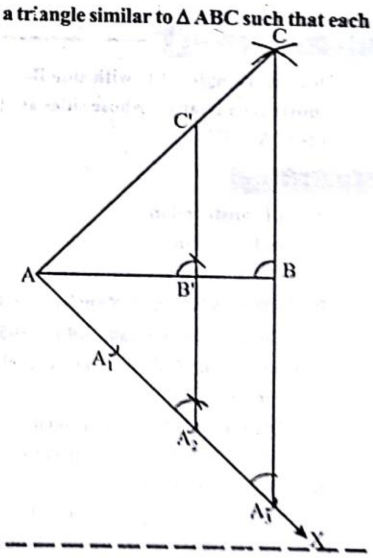
(vi) Draw three arcs (greater of 2 and 3 in 2/3) intersecting the ray AX at A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$.

(vii) Join A_3B.

(viii) Draw $A_2B' \parallel A_3B$, meeting AB at B'.

(ix) From B', draw B'C' || BC, meeting AC at C'.

AB'C' is the required triangle, each of the whose sides is two-third of the corresponding sides of Δ ABC.

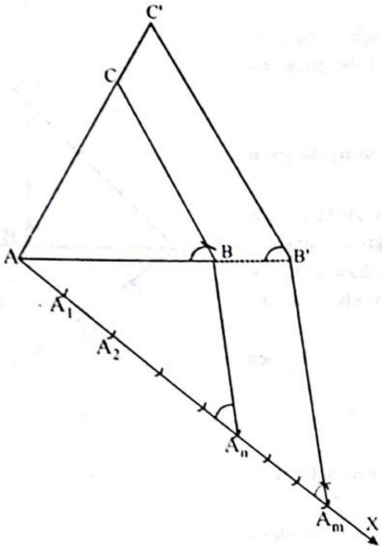


(II) Steps of construction when m > n :

(i) Construct the given triangle by using the given data.

(ii) Take any one of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

(iii) At one end, say A, of base AB draw a ray AX making a suitable acute angle with base AB. on the opposite side of the vertex C with respect to AB.



(iv) Draw arcs (large of m and n) intersecting the ray AX at $A_1, A_2, A_3, ..., A_m$ such that $AA_1 = A_1A_2 = ... = A_{m-1}A_m$.

(v) Join A_n to B

(vi) Draw a line through A_m parallel to A_nB, intersecting the extended line segment AB at B'.

(vii) Draw a line through B' parallel to BC intersecting the extended line segment AC at C'

(viii) Δ AB'C' so obtained is the required triangle.

ILLUSTRATION 11.4

Draw a triangle ABC with side BC = 7 cm. ∠B = 45°, ∠A = 105°. Construct a triangle whose sides are (4/3) times the corresponding side of Δ ABC.

SOLUTION:

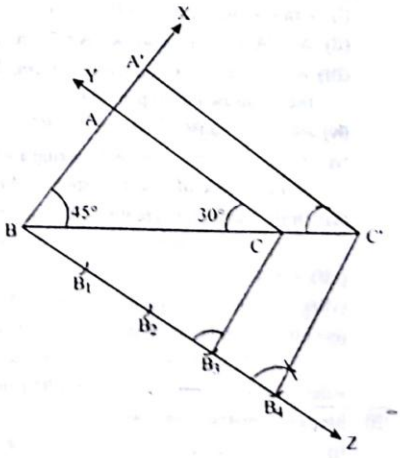
Steps of construction :

(i) Draw BC = 7 cm.

(ii) Draw a ray BX and CY such that ∠CBX = 45° and

$\angle BCY = 180° - (45° + 105°) = 30°$

Suppose BX and CY intersect each other at A. Δ ABC so obtained is the given triangle..



(iii) Draw a ray BZ making a suitable acute angle with BC on opposite side of vertex A with respect to BC .

(iv) Draw four (greater of 4 and 3 in 4/3) arcs intersecting the ray BZ at B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(v) Join B_3 to C and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C'.

(vi) Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Triangle A'BC' so obtained is the required triangle.

CONSTRUCTION OF TANGENT TO A CIRCLE :

(a) To draw the tangent to a circle at a given point on it, when the centre of the circle is known :

Given : A circle with centre O and a point P on it.

Required : To draw the tangent to the circle at P.

Steps of construction :

(i) Join OP.

(ii) Draw a line AB perpendicular to OP at the point P.

APB is the required tangent at P.

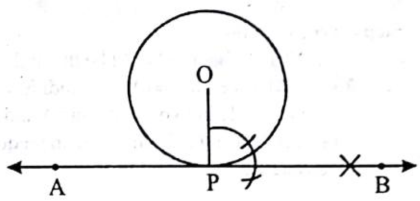


ILLUSTRATION 11.5

Draw a circle of diameter AB = 6 cm. with centre O. Through A or B draw tangent to the circle.

SOLUTION:

Given : A circle of diameter AB with centre O.

Required : To draw tangent to the circle at B or A.

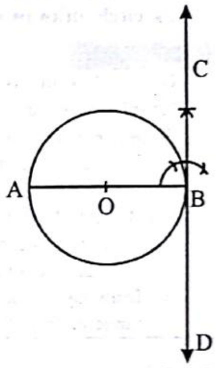
Steps of construction :

(i) With O as centre and radius equal to 3 cm. ($6 \div 2$) draw a circle.

(ii) Draw a diameter AOB.

(iii) Draw $CD \perp AB$ at B.

So, CD is the required tangent.



(b) To draw the tangent to a circle at a given point on it, when the centre of the circle is not known.

Given : A circle and a point P on it.

Required : To draw the tangent to the circle at P.

Steps of construction :

(i) Draw any chord PQ through P and join P and Q to a point R in major arc $\widehat{PQ}$ (or minor arc PQ).

a line AB such that

(ii) Draw $\angle QPB$ equal to $\angle PRQ$ and on opposite side of R with respect to chord PQ.

The line APB will be a tangent to the circle at P.

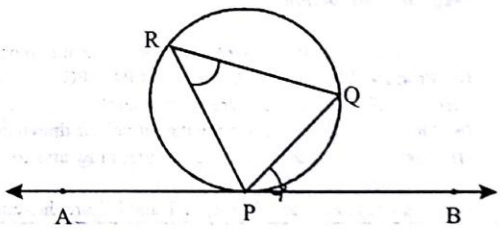


ILLUSTRATION 11.6

Draw a circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction.

SOLUTION:

Given : A circle of radius 4.5 cm. P is a point on the circle.

Required : To draw a tangent to a circle at P.

Steps of construction :

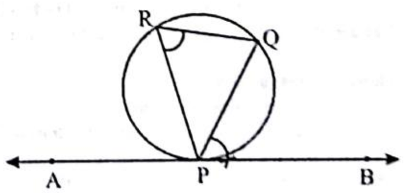
(i) Draw a circle of radius 4.5 cm.

(ii) Draw any chord PQ through P from the given point P on the circle.

(iii) Take a point R on the circle and join PR and PQ.

(iv) A line AB through the point P, such that $\angle QPB = \angle PRQ$ and on opposite side of chord PQ with respect to point R.

(v) Thus, APB is the required tangent.



(c) To draw the tangent to a circle from a point outside it (external point), when its centre is known.

Given : A circle with centre O and a point P outside it.

Required : To construct the tangents to the circle from point P.

Steps of construction :

(i) Join OP and bisect it. Let M be the mid point of OP.

(ii) Taking M as centre and MO as radius, draw a circle to intersect the given circle in two points, say A and B.

(iii) Draw rays PA and PB. These are the required tangents from P to given circle.

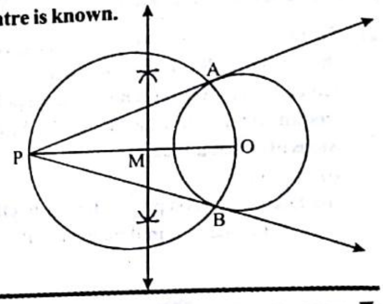


ILLUSTRATION 11.7

Draw a circle of radius 2.5 cm. From a point P, 6 cm. apart from the centre of the circle, draw two tangents to the circle.

SOLUTION:

Given : A point P is at a distance of 6 cm. from the centre of a circle or radius 2.5 cm.

Required : To draw two tangents to the circle from the given point P.

Steps of construction :

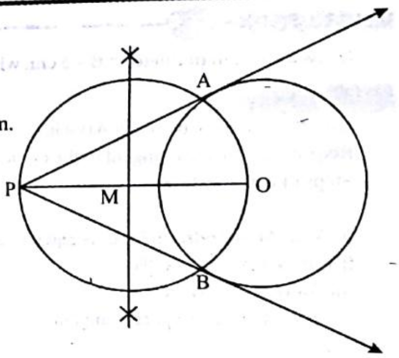
(i) Draw a line segment OP of length 6 cm.

(ii) With centre O and radius equal to 2.5 cm, draw a circle.

(iii) Bisect OP. Let M be mid-point of OP.

(iv) Taking M as centre and MO as radius draw a circle to intersect the given circle in two points, say A and B.

(v) Draw rays PA and PB. These are the required tangents from P to the given circle.



(d) To draw tangents to a circle from a point outside it (when its centre is not known)

Given : A circle and a point P outside it.

Required : To draw tangents from a point P outside the circle.

Steps of construction :

(i) Draw a secant PAB to intersect the circle at two points A and B.

(ii) Produce AP to a point C, such that PA = PC.

(iii) With BC as a diameter, draw a semicircle.

(iv) Draw PD $\perp$ CB, intersecting the semicircle drawn in step (iii)at D.

(v) Taking PD as radius and P as centre, draw arcs to intersect the given circle at T and T'.

(vi) Draw rays PT and PT'. Rays PT and PT' are the required tangents.

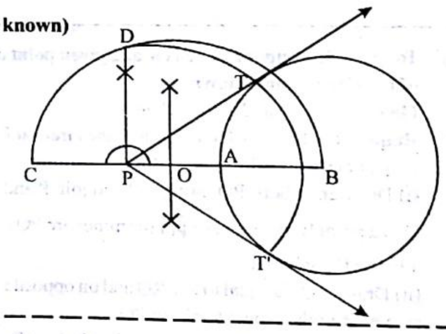


ILLUSTRATION 11.8

Draw a circle of a radius 3 cm. From a point P, outside the circle draw two tangents to the circle without using the centre of the circle.

SOLUTION:

Given : P is a point outside the circle of radius 3 cm.

Required : To draw two tangents to the circle from the point P, without the use of centre of the circle.

Steps of construction :

(i) Draw a circle of radius 3 cm.

(ii) Take a point P outside the circle and draw a secant PAB, intersecting the circles at A and B.

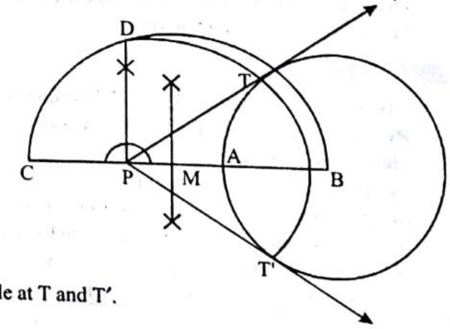
(iii) Produce AP to C, such that AP = CP.

(iv) Draw a semicircle with CB as a diameter.

(v) Draw PD $\perp$ CB, intersecting the semicircle drawn in step (iv) at D.

(v) Taking PD as radius and P as centre, draw two arcs to intersect the circle at T and T'.

(vi) Draw rays PT and PT', which the required tangents.



MISCELLANEOUS SOLVED EXAMPLES

1. Draw a triangle ABC with side BC = 8 cm. ∠ B = 45°, ∠ A = 75°. Construct a triangle whose sides are (4/3) times the corresponding side of Δ ABC.

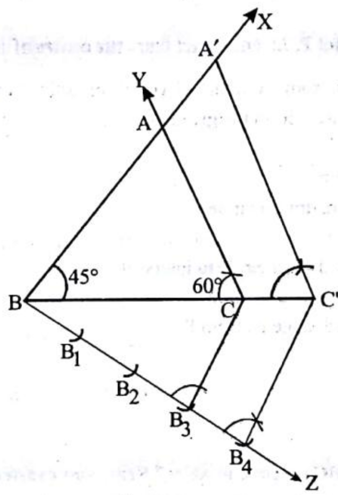
Sol. Steps of construction :

(i) Draw BC = 8 cm.

(ii) Draw a ray BX and CY such that ∠ CBX= 45° and

$\angle BCY = 180° - (45° + 75°) = 60°$

Suppose BC and CY intersect at A. Δ ABC so obtained is the given triangle.



(iii) Draw a ray BZ making an acute angle with BC on opposite side of vertex A with respect to BC .

(iv) Draw four (greater of 4 and 3 in 4/3) arcs intersecting the rays BZ at B_1, B_2, B_3, B_4 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(v) Join B_3 to C and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C′.

(vi) Draw a line through C′ parallel to CA intersecting the extended line segment BA at A'.

Triangle A'BC' so obtained is the required triangle.

2. Draw a circle of diameter 4 cm. with centre O. Draw a diameter AOB. Through A or B draw tangent to the circle.

Sol. Given : A circle with centre O.

Required : To draw tangent to the circle at B or A.

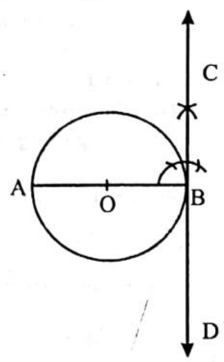
Steps of construction :

(i) With O as centre and radius equal to 2 cm. (4 ÷ 2) draw a circle.

(ii) Draw a diameter AOB.

(iii) Draw CD ⊥ AB at B.

So, CD is the required tangent.



3. Draw a circle of radius 3 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction.

Sol. Given : A circle of radius 3 cm. P is a point on the circle.

Required : To draw a tangent to a circle at P.

Steps of construction :

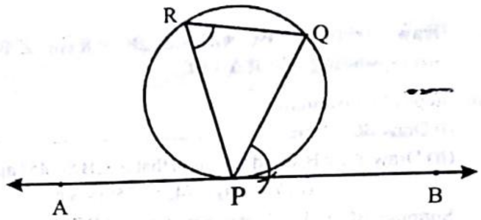
(i) Draw a circle of radius 3 cm.

(ii) Draw a chord PQ, from the given point of the circle.

(iii) Take a point R on the circle and join PR and PQ.

(iv) Draw a line AB through the point P, such that $\angle QPB = \angle PRQ$ and on opposite side of chord PQ with respect to point R.

(v) Thus, APB is the required tangent.



4. Draw a circle of radius 3 cm. From a point P, 7.5 cm. apart from the centre of the circle, draw two tangents to the circle.

Sol. Given : A point P is at a distance of 7.5 cm from the centre of a circle of radius 3 cm.

Required : To draw two tangents to the circle from the given point P.

Steps of construction :

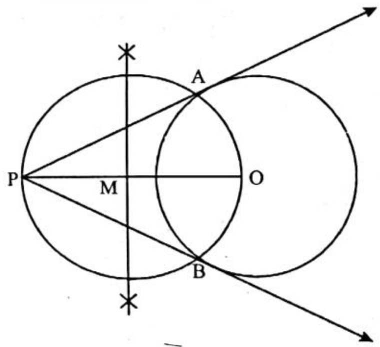
(i) Draw a line segment OP of length 7.5 cm.

(ii) With centre O and radius equal to 3 cm, draw a circle.

(iii) Bisect OP. Let M be mid-point of OP.

(iv) Taking M as centre and MO as radius draw a circle to intersect the circle in two points, say A and B.

(v) Join PA and PB. These are the required tangents from P



5. Construct a ΔABC in which AB = 6 cm, BC = 5 cm and AC = 7.5 cm. Now construct a triangle similar to ΔABC such that each of its sides is third-fourth of the corresponding sides of ΔABC. Also prove your assertion.

Sol. Steps of construction :

(i) Draw a line segment BC = 5 cm.

(ii) With B as centre and radius = AB = 6 cm, draw an arc.

(iii) With C as centre and radius = AC = 7.5 cm, draw another arc, interescting the arc drawn in step (ii) at the point A.

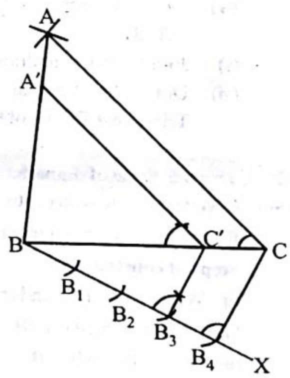
(iv) Join AB and AC to obtain ΔABC.

(v) Below BC draw ray BX, making an acute angle $\angle CBX$.

(vi) Draw four arcs intersecting the ray BX at B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

(vii) Join B_4C.

(viii) Since we have to construct a triangle each of whose sides is third-fourth of the corresponding sides of ΔABC. Hence draw $B_3C' \parallel B_4C$.



(ix) From C′, draw $C'A' \parallel CA$, intersecting BA at point A′. Then A′BC′ is the required triangle.

6. Construct a triangle similar to a given equilateral ΔPQR with side 5 cm such that each of its sides is $\frac{6}{7}$th of the corresponding side of ΔPQR.

Sol. Steps of construction :

(i) Draw a line segment QR = 5 cm

(ii) With Q as centre and radius = 5 cm, draw an arc.

(iii) With R as centre and radius = 5 cm, draw another arc intersecting the arc drawn in step (ii) at the point P.

(iv) Join PQ and PR to obtain ΔPQR.

(v) Below QR, draw ray QX making an acute angle $\angle RQX$.

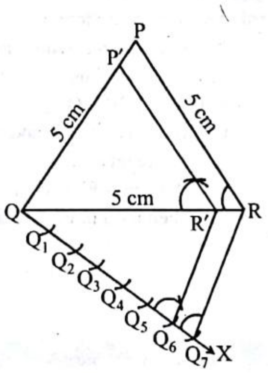
(vi) Draw seven arcs intersecting the ray QX at $Q_1, Q_2 .... Q_7$, such that $QQ_1 = Q_1Q_2 = Q_2Q_3 = ... = Q_6Q_7$

(vii) Join Q_7R.

(viii) Draw $Q_6R' \parallel Q_7R$, intersecting QR at R'

(ix) From R' draw $R'P' \parallel RP$, intersecting the QP at P'.

We get required ΔQAB similar to ΔPQR.



7. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{7}{5}$th of the corresponding sides of triangle ABC.

Sol. Steps of construction :

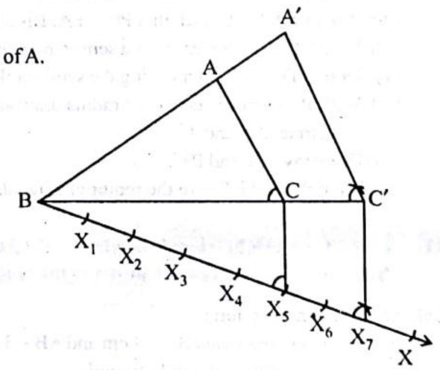
(i) Draw a ray BX making a suitable acute angle with BC on opposite side of A.

(ii) Draw seven arcs intersecting the ray BX at $X_1, X_2, ..., X_7$ such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5 = X_5X_6 = X_6X_7$

(iii) Join CX_5 and draw a line X_7C' parallel to X_5C intersects BC producted at C'.

(iv) Draw a line C'A' parallel to CA to intersect BA producted at A'.

Then A'BC' will be the required triangle.



8. Construct a triangle ABC whose sides are 7.5 cm, 7 cm and 6.5 cm. Construct atriangle similar to ABC and with sides $\frac{2}{3}$rd of the corresponding sides of triangle ΔABC.

Sol. Steps of construction :

(i) Take BC = 7.5 cm. With B as centre and radius 7 cm draw an arc.

(ii) With C as centre and radius 6.5 cm, draw arc intersecting the arc drawn in step (i) at point A. Then ΔABC is the required triangle.

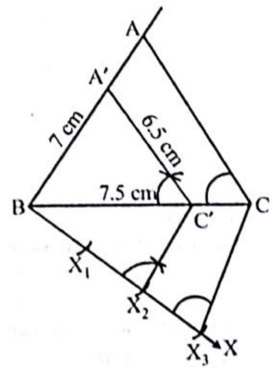
(ii) Draw a ray BX making an acute angle with BC opposite side of A with respect to BC.

(iii) Draw three arcs intersecting the ray BX at X_1, X_2 and X_3 such that $BX_1 = X_1X_2 = X_2X_3$.

(iv) Join X_3C.

(v) Draw $X_2C' \parallel X_3C$.

(vi) Draw $C'A' \parallel CA$. Then $\Delta A'BC'$ is the required triangle.



9. Draw a circle with centre O and radius 4 cm. Take a point P outside the circle at a distance of 9 cm from its centre. Draw two tangents to the circle from the point P.

Sol. Steps of construction :

(i) Draw a circle with centre O and radius 4 cm.

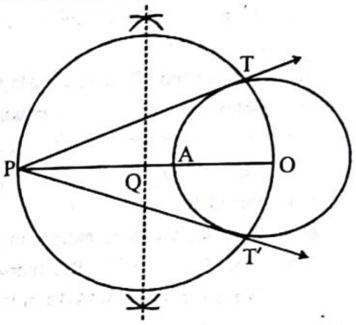
(ii) Draw a radius OA and produce it to P such that OP = 9 cm.

(iii) Bisect OP at Q.

(iv) With Q as centre and QP as radius, draw a circle to intersect the given circle at T two points and T′.

(v) Draw rays PT and PT′.

Then PT and PT′ are the required tangents to the given circle from the point P.



10. Draw a circle of radius 2.5 cm. Taking a point P outside it. Without using the centre, draw two tangents to the circle from the point P.

Sol. Steps of construction :

(i) Draw a circle of radius 2.5 cm. Take a point P outside it.

(ii) Through P draw a secant PAB to meet the circle at A and B.

(iii) Produce AP to C such that PC = PA. Bisect CB at Q.

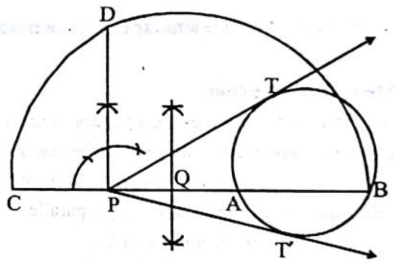
(iv) With CB as diameter, draw a semi-circle. (centre Q)

(v) Draw PD ⊥ CB, intersecting the semi-circle at point D.

(vi) With P as centre and PD as radius draw an arc to intersects the circle at T and T′.

(vii) Draw rays PT and PT′.

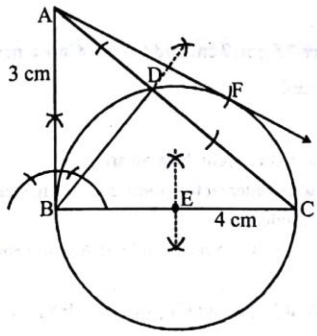
Then PT and PT′ are the required tangents to the circle from P.



11. Let ABC be a right triangle in which AB = 3 cm, BC = 4 cm and ∠B = 90°. BD is the perpendicular from B to AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

Sol. Steps of construction :

(i) Draw line segments BC = 4 cm and AB = 3 cm perpendicular to BC. Join AC. ABC is the right triangle.



(ii) Draw BD ⊥ AC

(iii) Draw perpendicular bisector of BC, which intersects BC at E.

(iv) Draw a circle with centre E and radius EB (or EC) clearly if passes through the point B, C and D. Now AB is the tangent to the circle drawn at B

(v) With centre A and radius equal to AB, draw an arc intersecting the circle at F (other than the point B).

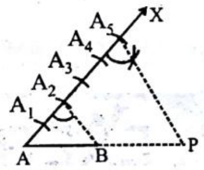
(vi) Draw ray AF.

1 EXERCISE

FIB Fill in the Blanks

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. A unique triangle can be constructed when its three .... are known (sides, angles)
2. If the radius of a circle is 4.5 cm, then the distance between two parallel tangents to the circle is ....
3. A triangle can be constructed when its two sides and .... angle is known. (included, any one)
4. A triangle can be constructed when the sum of lengths of any two sides is .... then the length of the third side.
5. A line segment of 7 cm length has been divided externally in the ratio 5 : 3. Its steps of construction is given below :



There are some blank boxes in the steps of constructions. Fill them, so that steps of construction will be completed.

(i) Draw line segment AB = 7 cm

(ii) [ ]

(iii) Draw five arcs intersecting the rays AX at A_1, A_2, A_3, A_4 and A_5 such that $AA_1 = A_1A_2 = ... = A_4A_5$

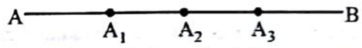
(iv) [ ]

(v) Through A_5, draw a line. $A_5P \parallel A_2B$, intersecting AB produced at P.

T/F True / False

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. By geometrical construction, it is possible to divide a line segment in the ratio $2+\sqrt{3} : 2-\sqrt{3}$
2. By geometrical construction, it is possible to divide a line segment in the ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$.
3. A pair of tangents can be constructed from a point P to a circle of radius 3.5 cm at a distance of 3m from the centre.
4. In the figure, if $AA_1 = A_1A_2 = A_2A_3 = A_3B$, then mark the given statements as true or false.



(i) A_1 divides AB in the ratio 1: 3

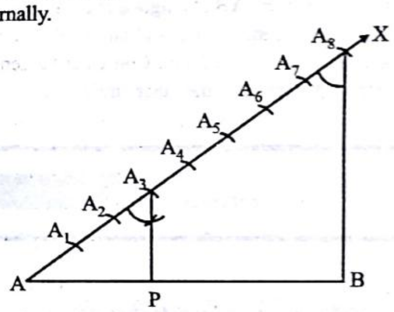
(ii) A_2 divides AB in the ratio 1: 3

(iii) A_3 divides AB in the ratio 3 : 1.

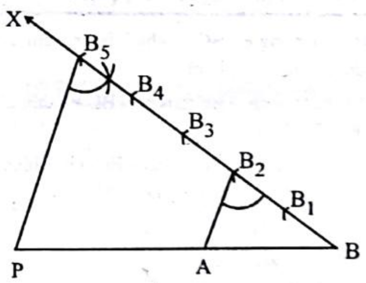
VSAQ Very Short Answer Questions

DIRECTIONS : *Give answer in one word or one sentence.*

1. Divide a line segment of length 10 cm in the ratio of 3 : 5 internally.



2. Divide a line segment of 7 cm length externally in the ratio of 3 : (v)



3. Take a point P at a distance of 8 cm from the centre of a circle of radius 3 cm and from P draw two tangents PA and PB to the circle. Measure the lengths of each tangent.
4. Draw a pair of tangents to a circle of radius 2.8 cm, which are inclined to each other at 60°.
5. Draw a circle with centre O and radius 2.2 cm. Take a point P on it. Draw a tangent to the circle at the point P.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle ABC = 60°$. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.
2. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 30°.
3. Draw a circle of radius 3 cm. Take two points P and Q on one of its both sides extended diameter each at a distance of 7 cm from the centre. Draw tangents to the circle from these two points P and Q.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

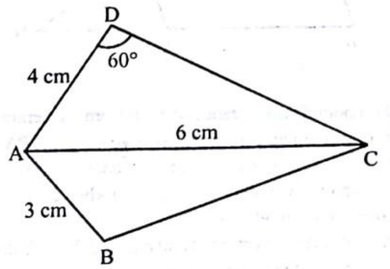
1. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.
2. Construct a Δ ABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Now construct a triangle similar to ΔABC such that each of its sides is two-third of the corresponding sides of ΔABC. Also, prove your assertion.
3. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are 3/2 of the corresponding sides of the isosceles triangle.
4. Construct two tangents to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.
5. Given a triangle ABC in which AB = 2.8 cm, BC = 4.3 cm and $\angle B = 90°$. BD is the perpendicular from B to AC. A circle is passing through points B, C and D. Construct the tangents from A to the circle.
6. Draw right triangle in which the sides (other than hypotenuse) are of lengths 3 cm and 4 cm. then construct another similar triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.
7. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

2 EXERCISE

HOTS HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Construct a triangle ABC in which BC = 6 cm, $\angle A = 60°$ and altitude through A is 4 cm.
2. Construct a triangle ABC in which BC = 7 cm, $\angle A = 60°$ and median AT is 5 cm.
3. Construct a cyclic quadrilateral ABCD in which AB = 3 cm, AD = 4 cm, AC = 6 cm and $\angle D = 60°$.



4. Find the mean proportional of two given line segments of lengths p cm and q cm.
5. Construct a square whose area is equal to the area of a given rectangle ABCD.
6. Construct a pentagon siimilar to the given pentagon ABCDE. Whose side corresponding to the side AB of pentagon ABCDEF is of length equal to the length of a given line segment XY.
7. Let PQR be a right triangle in which PQ = 6 cm, QR = 8 cm and $\angle Q = 90°$. QS is the perpendicular from Q on PR. The circle through Q, R, S is drawn. Construct the tangents from P to the circle passing through Q, R and S.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. sides
2. 9 cm
3. included
4. greater
5. (ii) Draw a ray AX making an acute $\angle BAX$.
   (iv) Join A_2 to B.

TRUE / FALSE

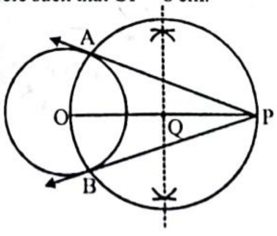
1. False
2. True
3. False
4. (i) True (ii) False (iii) True

VERY SHORT ANSWER QUESTIONS :

1. **Steps of construction :**
   (i) Draw a line segment AB = 10 cm.
   (ii) Draw a ray AX making an acute $\angle BAX$.
   (iii) Draw 8 = (3 + 5) arcs intersecting the ray AX at points $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ and A_8 such that $AA_1 = A_1A_2 = ... = A_7A_{(viii)}$
   (iv) Join A_8B.
   (v) Through A_3, draw a line $A_3P \parallel A_8B$, intersecting AB at P.
   The point P so obtained is the required point, which divides AB internally into the ratio 3 : (v)

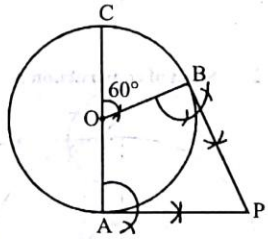
2. **Step of construction :**
   (i) Draw the line segment AB = 7 cm.
   (ii) Draw a ray BX making an acute $\angle ABX$.
   (iii) Draw 5 arcs intersecting the rays BX at points B_1, B_2, B_3, B_4 and B_5 such that $BB_1 = B_1B_2 = ... = B_4B_{(v)}$
   (iv) Join B_2A
   (v) Through B_5, draw a line $B_5P \parallel B_2A$, intersecting BA produced at P. The point P so obtained is the required point, which divides AB externally in the ratio 3 : (v)

3. **Steps of construction :**
   (i) Draw a circle with radius 3 cm and centre O. Take a point P outside the circle such that OP = 8 cm.
   (ii) Bisect OP at Q.
   (iii) With Q as centre and QP as radius, draw a circle which intersects the given circle at two points A and B.



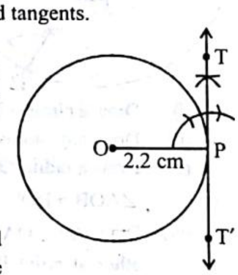
   (iv) Join PA and PB.
   PA and PB are the required tangents. On measurement PA = 7.4 cm, PB = 7.4 cm.

4. **Steps of construction :**
   (i) Draw a circle with centre O and radius = 2.8 cm
   (ii) Draw any diameter AOC.
   (iii) Construct $\angle BOC$ = 60°, meeting the circle at B.



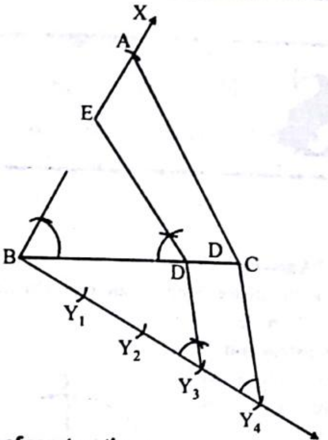
   (iv) At A and B, draw perpendiculars to OA and OB respectively intersecting each other at point P.
   PA and PB are the required tangents.

5. **Steps of construction :**
   (i) Draw a circle with centre O and radius 2.2 cm. Take a point P on it.
   (ii) Join OP.
   (iii) Draw TT′ perpendicular to OP at P.
   Then TPT′ is the desired tangnet to the circle at the point P.

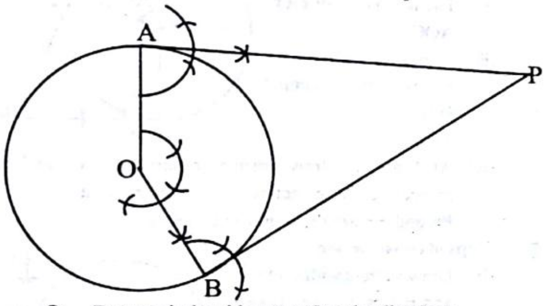


SHORT ANSWER QUESTIONS :

1. **Steps of construction :**
   (i) Draw BC = 6.5 cm
   (ii) Draw a ray BX making an angle 60° with BC
   (iii) With centre B and radius 4.5 cm draw an arc intersecting the ray BX at A.
   (iv) Join AC. Then ΔABC is the required triangle.
   (v) Draw a ray BY making a suitable acute angle with BC on opposite side of ray AX with respect to BC
   (vi) Draw three arcs intersecting the ray BY at Y_1, Y_2, Y_3 and Y_4 such that $BY_1 = Y_1Y_2 = Y_2Y_3 = Y_3Y_4$
   (vii) Join Y_4C.
   (viii) Draw Y_3D parallel to Y_4C intersecting the BC at D.
   (ix) Draw a line DE||AC intersecting BA at E.
   Then required triangle BDE.



2. **Steps of construction :**



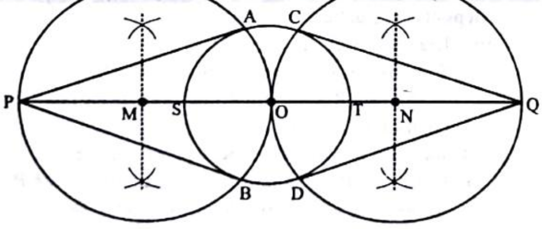
(i) Draw a circle with centre O and radius 5 cm.

(ii) Draw any radius OA.

(iii) Draw a radius OB such that $\angle AOB = 180° - 30° = 150°$.

(iv) Draw $AP \perp OA$ and $BP \perp OB$, Which intersect each other at point P. Thus PA and PB are the required tangents such that $\angle APB = 30°$

3.



Steps of construction :

(i) Draw a circle with O as centre and of radius 3 cm.

(ii) Draw a diameter ST and extend it to both the sides to P and Q such that PO = OQ = 7 cm.

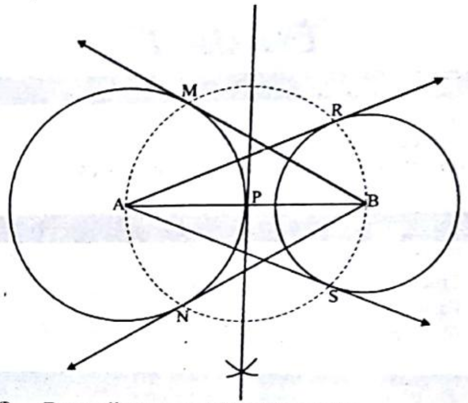
(iii) Draw perpendicular bisectors of PO and OQ. Let M and N be the mid-points of PO and OQ respectively.

(iv) Taking M as centre and MP as radius, draw a circle which meets the given circle at A and B. Similarly taking N as centre and NQ as radius, draw a circle which meets the given circle at C and D.

(v) Join PA, PB, QC and QD.
PA, PB, QC and QD are the required tangents.

LONG ANSWER QUESTIONS :

1. **Steps of construction :**



(i) Draw a line segment AB of length 8 cm.

(ii) With A as centre, draw a circle of radius 4 cm.

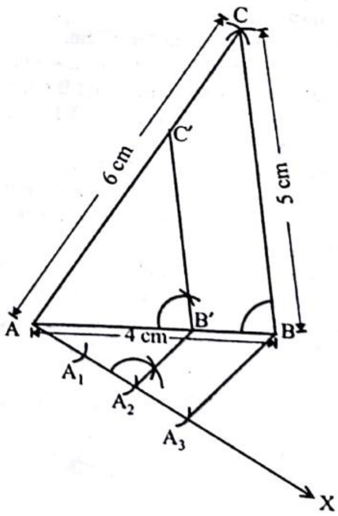
(iii) With B as centre, draw another circle of radius 3 cm.

(iv) Draw the perpendicular bisector of AB. Let P be the mid-point of AB.

(v) With P as centre and PA as radius, draw a circle which intersects the circle with centre A at M and N; and the circle with centre B at R and S.

(vi) Draw rays BM, BN, AR and AS.
These rays BM, BN, AR and AS are required tangents.

2. **Steps of construction :**



(i) Draw a line segment AB = 4 cm.

(ii) With A as centre and radius = 6 cm, draw an arc above AB.

(iii) With B as centre and radius = 5 cm, draw another arc, intersect the arc drawn in step (ii) at C.

(iv) Join AC and BC to obtain ΔABC.

(v) Below AB, draw a ray AX making an acute angle with AB.

(vi) Draw three arcs intersecting the ray AX at points A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$

(vii) Join A_3B.

(viii) From point A_2, draw $A_2B' \parallel A_3B$ meeting AB at B′.

(ix) From B′, draw $B'C' \parallel BC$, meeting AC at C′.
AB′C′ is the required triangle.

Justification : In ΔABC, $BC \parallel B'C'$,

$\angle BAC = \angle B'AC'$ and $\angle ABC = \angle AB'C'$

So, $\Delta ABC \sim \Delta AB'C'$

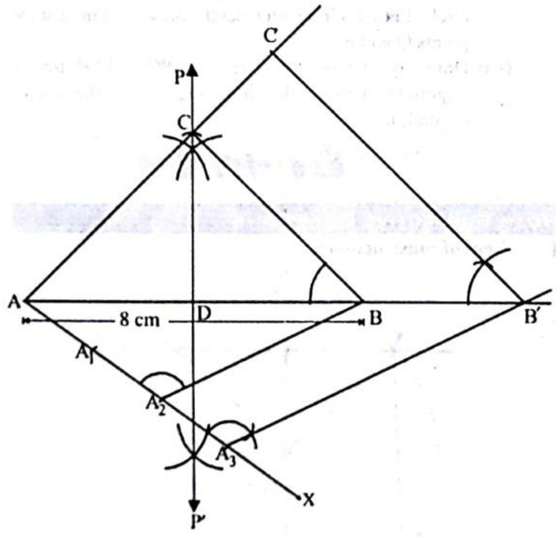
$$\therefore \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{AB'}{AB} = \frac{2}{3}.$$

3. Steps of construction :

(i) Draw a line segment AB = 8 cm.

(ii) Draw perpendicular bisector PP′ of AB, which intersects AB at D.

(iii) Taking centre as D and radius of 4 cm, draw an arc, intersecting the ray DP at C.



(iv) Join AC and BC. ΔABC is the required triangle.

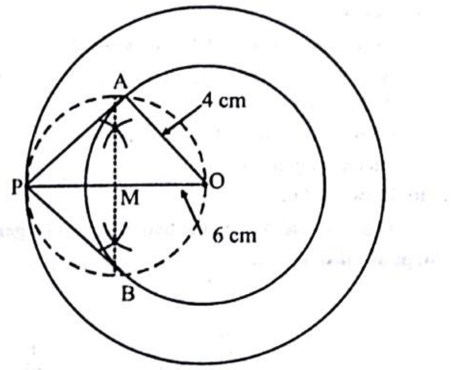
(v) Draw a ray AX making an acute angle with AB below AB. Draw three arcs intersecting the ray AX at points A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$

(vi) Join A_2B.

(vii) Draw a line A_3B' from point A_3 parallel to A_2B which meets AB produced at B′.

(viii) From B′ draw a line B′C′ parallel to BC which meets AC produced at C′.
$\Delta AB'C'$ is the required triangle.

4. Steps of construction :



(i) Draw two concentric circles with centre O and of radii 4 cm and 6 cm.

(ii) Let P be any point on the larger circle.

(iii) Join OP.

(iv) Draw the right bisector of OP. Let M be the mid-point of OP.

(v) Taking M as centre and radius OM, draw a circle which intersects the smaller circle at points A and B.

(vi) Join PA and PB.
These PA and PB are the required tangents.

(vii) On measuring with scale PA = 4.5 cm and PB = 4.5 cm

Calculation : Since the tangent at any point of a circle and radius through the point of contact are perpendicular to each other, therefore $\angle OAP = 90°$.

Now in right triangle ΔOAP,

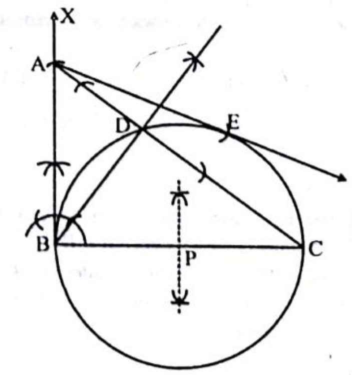
$OP^2 = OA^2 + PA^2$

$\Rightarrow PA^2 = OP^2 - OA^2 = 6^2 - 4^2 = 36 - 16 = 20$

$\therefore PA = 2\sqrt{5} = 2 \times 2.236 = 4.5$ cm. (approximate)

Hence, the measurement is the same as by the calculation.

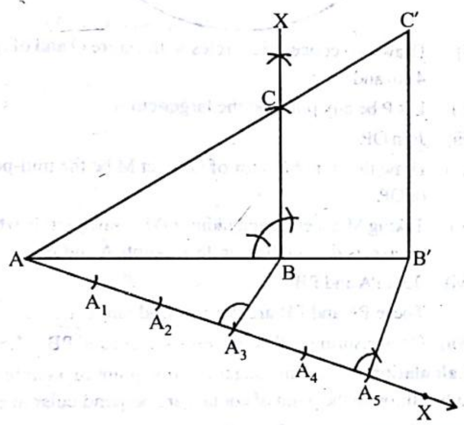
5. Steps of construction :



(i) Draw a line segment BC of length 4.3 cm.
(ii) At point B of BC, draw perpendicular ray BX to BC.
(iii) With centre B and radius 3 cm draw an arc intersectings the ray BX at A.
(iv) Join A and C.
(v) Draw perpendicular BD from B to AC.
(vi) With BC as a diameter draw a circle which passes through B, C and D.
(vii) With A as centre and radius AB (= 3 cm) draw an arc intersecting the circle at E.
(viii) Draw ray AE.

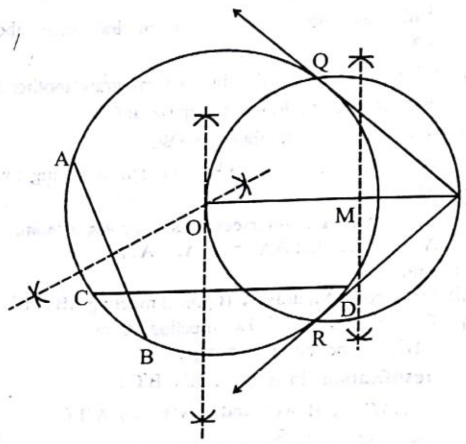
Thus AB and AE are the two required tangents.

6. Steps of construction :



(i) Draw AB = 4 cm
(ii) Draw ray BX perpendicular to AB.
(iii) With centre B and radius 3cm draw an arc intersecting the ray BX at C.
(iv) Join AC, we get ΔABC with the given data.
(v) Draw any ray AX making an acute angle with AB on the side opposite to vertex C with respect to AB.
(vi) Locate 5 points (the greater of 5 and 3 in $\frac{5}{3}$) A_1, A_2, A_3, A_4, and A_5 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$
(vii) Join A_3B.
(viii) Through A_5, draw a line parallel to A_3B that intersects the extended line segment AB at B′.
(ix) Through B′, draw a line parallel to BC to meet the extended line segment AC at C′. Then AB′C′ is the required triangle.

7. Steps of construction :

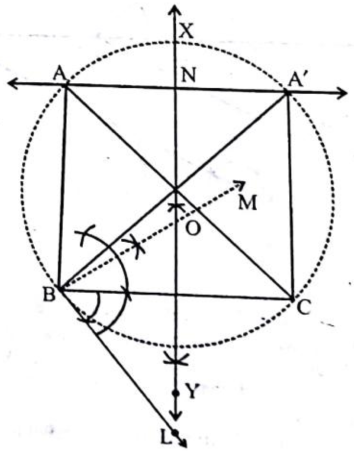


(i) Draw a circle with the help of a bangle.
(ii) Take any two non-parallel chords AB and CD of this circle.
(iii) Draw perpendicular bisectors of the chords AB and CD. Let these bisectors meet at the point O, then O is the centre of the circle.
(iv) Take any point P outside the circle.
(v) Join OP and draw its perpendicular bisector to meet OP at M
(vi) With M as centre and OM (or MP) as radius, draw a circle. Let this circle intersect the previous circle at the points Q and R.
(vii) Draw rays PQ and PR. Then rays PQ and PR are the required tangents to the circle (drawn with the help of a bangle).

Exercise 2

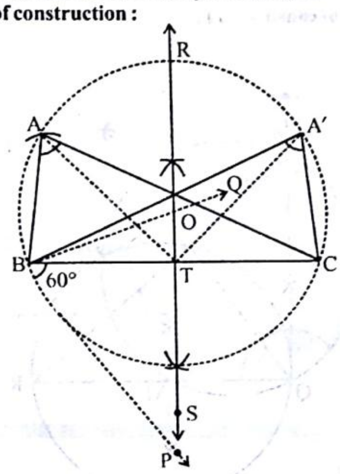
HOTS SUBJECTIVE QUESTIONS :

1. Steps of construction :



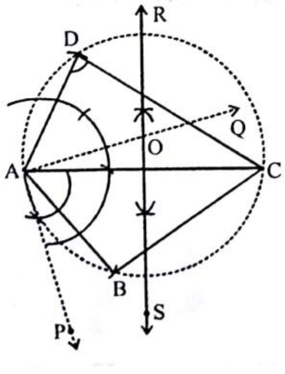
(i) Draw a line segment, BC = 6 cm.
(ii) Draw ray BL below BC such that $\angle CBL = 60°$
(iii) Draw ray BM, perpendicular to BL
(iv) Draw perpendicular bisector XY of BC which intersects ray BM at the point O and BC at point P..
(v) With O as centre and OB as radius draw a circle.
(vi) With centre P and radius 4 cm draw an arc intersecting the ray PX at N.
(vii) Through N, draw a line perpendicular to XY which intersects the circle at points A and A′.
(viii) Join AB, AC and A′B, A′C.
(ix) Now, ABC or A′BC is required triangle.

2. Steps of construction :



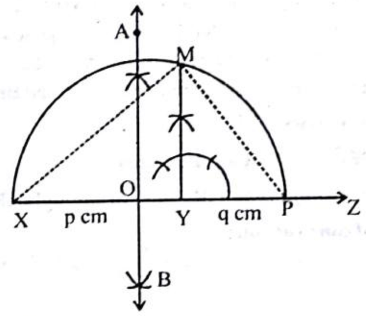
(i) Draw a line segment, BC = 7 cm.
(ii) Draw ray BP below BC such that $\angle CBP = 65°$
(iii) Draw ray BQ above BC such that $\angle PBQ = 90°$
(iv) Draw a perpendicular bisector RS of BC which intersects BC at point T.
(v) Mark the intersecting point of RS and ray BQ as O.
(vi) With O as centre and OB as radius draw a circle.
(vii) With T as centre radius as 5 cm, draw two arcs intersecting the circle at the points A and A′.
(viii) Join AB, AC, A′B and A′C.
(ix) Now, ABC or A′BC is required triangle.

3. Steps of construction.



(i) Draw a line segment, AC = 6 cm.
(ii) Draw ray AP below AC such that $\angle CAP = 60°$
(iii) Draw ray AQ above AC such that $\angle PAQ = 90°$
(iv) Draw a perpendicular bisector RS of AC which intersects ray AQ at the point O.
(v) With O as centre and OA as radius draw a circle.
(vi) With A as centre, radius as 4 cm, draw an arc above AC which intersects the circle at the point D.
(vii) With A as centre, 3cm as radius draw an arc below AC which intersects the circle at point B.
(viii) Join AB and BC, CD and AD.
ABCD is the required quadrilateral.

4. Steps of construction :

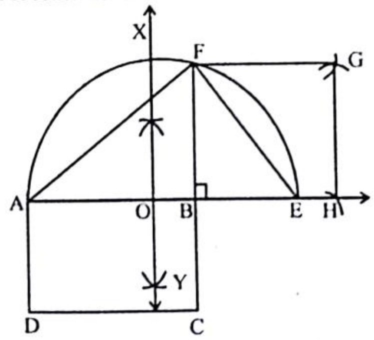


(i) Draw a ray XZ.
(ii) Mark two point Y and P on ray XZ such that XY = p cm and YP = q cm.
(iii) Draw the perpendicular bisector AB of XP which meets XZ at the point O.
(iv) With O as centre and OX as radius draw a semi circle above XZ.
(v) Draw YM perpendicular to XZ which meets semicircle at point M.
(vi) Now MY, is the mean proportional of XY and YP i.e., p and q.

Proof: $\Delta XYM \sim MYP$

$$\Rightarrow \frac{XY}{MY} = \frac{MY}{PY} \Rightarrow MY = \sqrt{(XY)(YP)} = \sqrt{pq}$$

5. Steps of construction :



(i) Produce the side AB of rectangle ABCD to the point E such that BE = BC.

(ii) Draw perpendicular bisector XY of AE, which intersects AE at O.

(iii) With centre O and radius OA, draw a semi-circle above AE.

(iv) Produce CB, which intersects the semicircle drawn in step (iii) at F. BF is the mean proportion of AB and BE.

(v) With centre B and radius BF, draw an arc intersecting the AE produced at point H.

(vi) With centre H and radius equal to BF, draw an arc above BH.

(vii) With centre F and radius equal to BF, draw an arc intersecting the arc drawn in step (vi) at G.

(viii) Join FG and GH. Thus BFGH is the square whose area is equal to the area of the given rectangle.

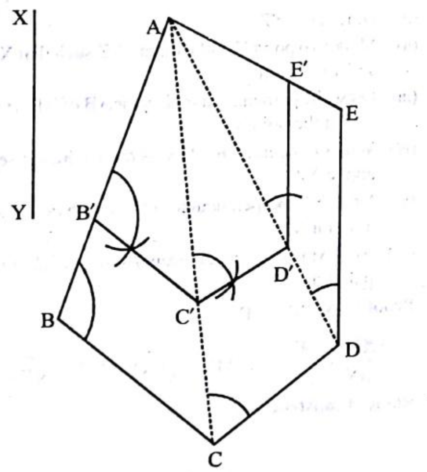
Proof : BF is the mean proportional of AB and BE.

$\Rightarrow$ $(BF)^2 = AB \times BE$

$\Rightarrow$ $(BF)^2 = AB \times BC$ ($\because$ BC = BE)

$\Rightarrow$ Area of square BFGH = Area of rectangle ABCD.

6. Steps of construction :



(i) ABCDE is a given pentagon and XY is the given line segment.

(ii) With centre A and radius equal to the length of XY, draw an arc intersecting the side AB of pentagon ABCDEA at B′.

(iii) Join AC and AD.

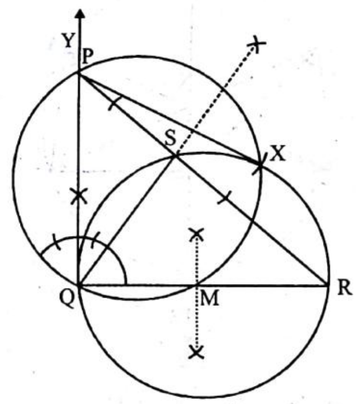
(iv) Draw B′C′ parallel to BC which meets AC at point C′.

(v) Draw C′D′ parallel to CD which meets AD at point D′.

(vi) Draw D′E′ parallel to DE which meets AE at point E′.

AB′C′D′E′ is the required pentagon.

7. Steps of construction :



(i) Draw Δ PQR, such that PQ = 6 cm, QR = 8 cm and $\angle Q = 90°$

(ii) Draw QS ⊥ PR

(iii) Bisect QR. Let the mid-point of QR be M.

(iv) Draw a circle taking M as centre and radius equal to QM, which will pass through Q, R and S.

(v) With centre P and radius equal to PQ, draw an arc intersecting the circle drawn in step (iv) at point X.

(vi) Draw ray PX. Thus PQ and PX are required tangents.

chapter 12

AREAS RELATED TO CIRCLES

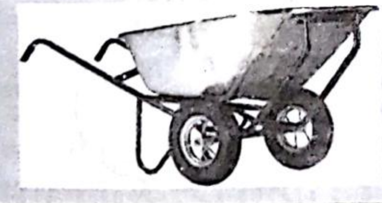
Introduction

Many objects that we come across in our daily life are related to the circular shape in some form or the other. Cycle wheels, wheel barrow (thela), dartboard, round cake, papad, drain cover, various designs, bangles, brooches, circular paths, washers, flower beds, etc. are some examples of such objects. So, the problem of finding perimeters and areas related to circular figures is of great practical importance.

Washer

Wheel

Cake



(wheel barrow)

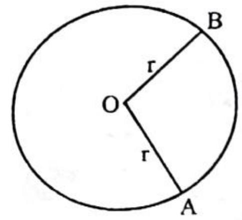
CIRCLE AND ITS RELATED TERMS :

Circle :

A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.

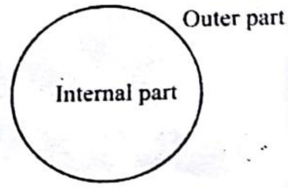
Radius :

A line segment joining the centre of the circle to a point on the circle is called its radius.
In Figure, there is a circle with centre O and one of its radius is OA.
OB is another radius of the same circle.
Thus All radii (plural of radius) of a circle are equal.
The length of the radius of a circle is generally denoted by the letter 'r'.
It is customary to write radius instead of the length of the radius.

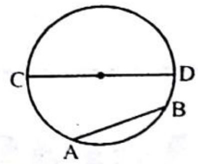


Internal and Outer part :

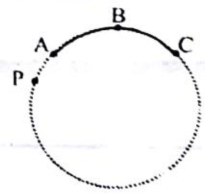
A closed geometric figure in the plane divides the plane into three parts namely, the inner part of the figure, the figure and the outer part. In Figure the shaded portion is the inner part of the circle, the boundary is the circle and the unshaded portion is the outer part of the circle.



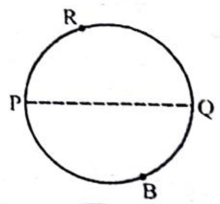
Chord : A line segment joining any two points of a circle is called a chord.
A chord passing though the centre of circle is called its diameter.
AB and CD both are chords but the chord CD passes through the centre.
Hence CD is the diameter also.
Diameter of a circle = twice the radius of the circle.
Note : Diameter is the longest chord of a circle.



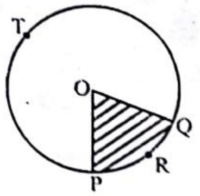
Arc : A part of a circle is called an arc.
In Figure, AC is an arc and is denoted by arc ABC or $\overset{\frown}{ABC}$



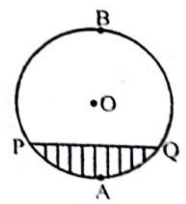
Semicircle : A diameter of a circle divides a circle into two equal arcs, each known as a semicircle. In Figure, PQ is a diameter and $\overset{\frown}{PRQ}$ is a semicircle and so is $\overset{\frown}{PBQ}$.



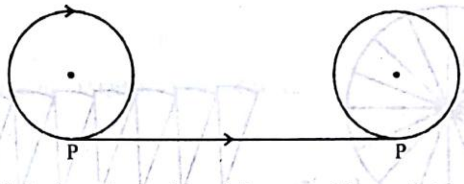
Sector : The region bounded by an arc of a circle and two radii at its end points is called a sector. In the figure, the shaded portion is a sector formed by the arc PRQ and the unshaded portion is a sector formed by the arc PTQ.



Segment : A chord divides the interior of a circle into two parts, each called a segment. The segment in which the centre of the circle does not lie is called minor segment and the segment in which the centre of the circle lies is called major segment. In the figure, the shaded region $PAQP$ and the unshaded region $PBQP$ are both segments of the circle. $PAQP$ is called a minor segment and $PBQP$ is called a major segment.



Circumference : Take a wheel and mark a point P on the wheel where it touches the ground. Rotate the wheel along a line till the point P comes back on the ground. Measure the distance between the Ist and last position of P along the line. This distance is equal to the circumference of the circle. Thus, The length of the boundary of a circle is the circumference of the circle.



Consider different circles and measure their circumference(s) and diameters. Observe that in each case the ratio of the circumference to diameter turns out to be the same.

The ratio of the circumference of a circle to its diameter is always a constant. This constant is universally denoted by Greek letter π.

Therefore, $\frac{c}{d} = \frac{c}{2r} = \pi$, where c is the circumference of the circle, d its diameter and r is its radius.

An approximate value of π is $\frac{22}{7}$.

[Aryabhata–I (476 AD), a famous Indian Mathematician gave a more accurate value of π which is 3.1416. In fact this number π is an irrational number.]

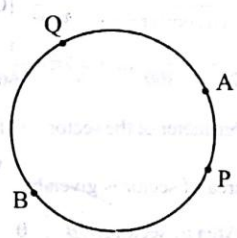
MEASUREMENT OF AN ARC OF A CIRCLE :

Consider an arc PAQ of a circle (Fig). To measure its length we put a thread along PAQ and then measure the length of the thread with the help of a scale.

Similarly, you may measure the length of the arc PBQ.

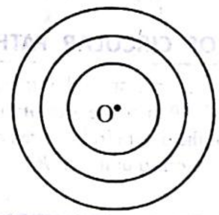
Minor arc : An arc of a circle whose length is less than a semicircular arc of the same circle is called a minor arc. PAQ a minor arc.

Major arc : An arc of a circle whose length is greater than a semicircular arc of the same circle is called a major arc. In figure, arc PBQ is a major arc.



CONCENTRIC CIRCLES :

Circles having the same centre but different radii are called concentric circles.

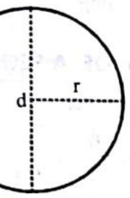


CONGRUENT CIRCLES AND ARCS :

Two circles or arcs are said to be congruent if we can superpose (place) one over the other such that they cover each other completely.

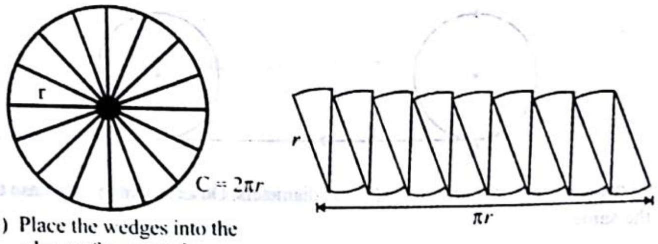
AREA OF A CIRCLE :

Cut a circle into an even number of equally sized pie-shaped wedges, whose pointed end is at the centre of the circle. Arrange these wedges so that they lie next to each other and alternate pointed up and down and up again until all the wedges are used. The resulting figure will resemble a rectangle. As the number of wedges increases, a cleaner picture more closely resembling a rectangle would form.



Transforming a Circle into Crude rectangle
(1) Cut the circle into wedges.

Since half of the wedges are pointed up and half are pointed down, the rectangle will have a base equal to half of the circumference. The base, or half the circumference, would be $\frac{1}{2}(2\pi r) = \pi r$.

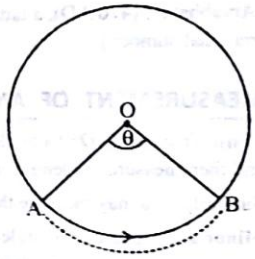


(2) Place the wedges into the shape of a rectangle

The height of the rectangle would be the radius r of the circle.
Hence, area of the circle = Area of the rectangle = $(\pi r)(r) = \pi r^2$.

PERIMETER AND AREA OF SECTOR OF A CIRCLE :

Two radii OA and OB enclose a portion of the circular region making central angle θ. The region is called a sector of the circle. In Fig., sector AOB is the sector with central angle θ. Let 'l' be the length of arc AB.



The, $\dfrac{\widehat{AB}}{\text{circumference}} = \dfrac{\theta}{360°}$ [Corresponding arcs subtend proportional central angles]

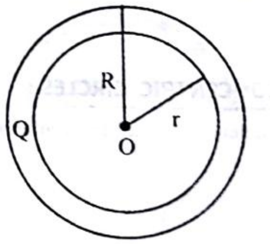
or $\dfrac{l}{2\pi r} = \dfrac{\theta}{360}$ or $l = 2\pi r.\dfrac{\theta}{360} = \dfrac{\pi r\theta}{180}$

∴ Perimeter of the sector $= OA + OB + AB = 2r + \dfrac{\pi r\theta}{180}$

Also area of sector is given by $\dfrac{\text{Area of sector } AOB}{\text{Area of the circle}} = \dfrac{\theta}{360}$

$\dfrac{\text{Area of sector } AOB}{\pi r^2} = \dfrac{\theta}{360}$ or Area of sector $OAB = \pi r^2 \dfrac{\theta}{360}$

AREA OF CIRCULAR PATH :



If we have a circular field of radius 'r', surrounded by a path of uniform width 'd' and $r + d = R$ then, the area of the circular path
= Area of the outer circle − Area of the inner circle
$= (\pi R^2 - \pi r^2)$ sq unit $= \pi(R^2 - r^2)$ sq unit

AREAS OF COMBINATIONS OF PLANE FIGURES :

We come across various combination of plane figures in our daily life and also in the form of various interesting designs like Flower beds, drain covers, window designs, designs on table covers. We illustrate the process of calculating areas of these figures through some examples.

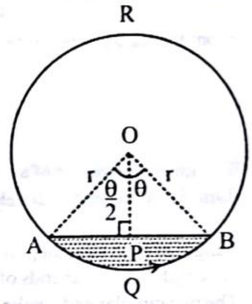
AREA OF A SEGMENT :

In right ΔOPA,

$$\sin\left(\frac{\theta}{2}\right) = \frac{AP}{OA} \Rightarrow \sin\left(\frac{\theta}{2}\right) = \frac{AP}{r}$$

$$\Rightarrow AP = r\sin\left(\frac{\theta}{2}\right),\ AB = 2AP = 2r\sin\left(\frac{\theta}{2}\right) \qquad \text{....(i)}$$

and $\cos\left(\frac{\theta}{2}\right) = \frac{OP}{OA} \Rightarrow \cos\left(\frac{\theta}{2}\right) = \frac{OP}{r}$



$\Rightarrow \quad OP = r\cos\left(\frac{\theta}{2}\right)$ ....(ii)

Area of minor segment $ABPA$ = (Area of sector OAQBO) – (Area of ΔOAB)

$$= \frac{\pi r^2\theta}{360} - \frac{1}{2} \times AB \times OP$$

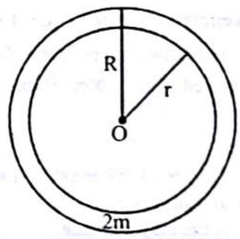
$$= \frac{\pi r^2\theta}{360} - \frac{1}{2} \times 2r\sin\left(\frac{\theta}{2}\right) \times r\cos\left(\frac{\theta}{2}\right) = \frac{\pi r^2\theta}{360} - r\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

Area of major segment $APBRA$ = (Area of the circle) – (Area of minor segment $ABPA$)

$$= \pi r^2 - \frac{\pi r^2\theta}{360} + r\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right).$$

MISCELLANEOUS SOLVED EXAMPLES

1. A path of width 2 meters runs around a circular plot whose circumference is $75\frac{3}{7}$ metres. Find (i) the area of path. (ii) the cost of gravelling the path at ₹ 7 per square meter.



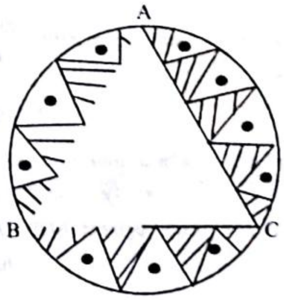
Sol. Here, $2\pi r = 75\frac{3}{7}$ i.e., $2 \times \frac{22}{7} \times r = \frac{528}{7}$

or $r = \frac{528}{7} \times \frac{1}{2} \times \frac{7}{22} = 12$ i.e., the radius of the plot is 12 m.

(i) Area of path $= \pi(R^2 - r^2) = \frac{22}{7}(14^2 - 12^2) = \frac{22}{7} \times 52\text{m}^2 = \frac{1144}{7}$ sq.m.

(ii) Cost $= ₹\left(\frac{1144}{7} \times 7\right) = ₹\,1144$

2. In a circular table cover of radius 32 cm., a design is formed leaving an equilateral triangle ABC in the middle, as shown in figure. Find the area of the design (shaded region).



Sol. Let O be the centre of the circular table cover of radius 32 cm.

ΔABC is an equilateral triangle.

Join OA, OB and OC.

Then, $\angle AOB = \angle BOC = \angle COA = 120°$

In ΔOAB, $OA = OB$

Draw $OM \perp AB$

$\Rightarrow \quad \angle AOM = \angle BOM = 60°$

Now, $\frac{AM}{OA} = \sin 60° \Rightarrow \frac{AM}{32} = \frac{\sqrt{3}}{2}$

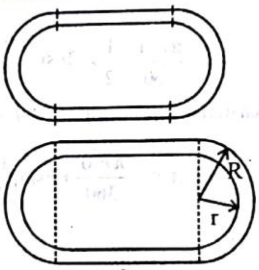
$\therefore \quad AM = 16\sqrt{3} \Rightarrow AM = 32\sqrt{3}$ cm.

Hence, area of $\Delta ABC = \frac{\sqrt{3}}{4}(32\sqrt{3})^2$ sq.cm. $= 768\sqrt{3}$ sq.cm.

From the figure, Area of the design = Area of the circle – Area of ΔABC

$$= [\pi(32)^2 - 768\sqrt{3}] \text{ sq. cm} = \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{sq. cm.}$$

3. The inner perimeter of a racetrack (figure) is 400m and the outer perimeter is 488m. The length of each straight portion is 90m. Find the cost of developing the track at the rate of ₹12.50 /m².



Sol. Length of each straight portion = 90m, Outer perimeter of the track = 488 m

$\therefore$ length of circular ends of outer perimeter $= 488 - 90 \times 2 = 308$ m

The two circular ends make a complete circle

$\therefore\ 2\pi R = 308 \Rightarrow R = \frac{308}{2\times 22}\times 7 = 49$m

Inner perimeter of the track = 400m

$\therefore$ length of circular ends of inner perimeter $= 400 - 90 \times 2 = 220$m

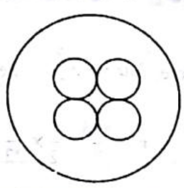
$\therefore\ 2\pi r = 220 \Rightarrow r = \frac{220}{2\times 22}\times 7 = 35$m

$\therefore$ width of the track $= R - r = 49 - 35 = 14$m $\therefore$ Area of straight portion on the track $= 2\,(90 \times 14) = 2520\ \text{m}^2$

Area of circular parts $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r) = \frac{22}{7}(49+35)(49-35) = \frac{22}{7}\times 84\times 14 = 3696\ \text{m}^2$

$\therefore$ total area of track $= 2520 + 3696 = 6216\ \text{m}^2$ $\therefore$ cost of developing track $= 6216 \times 12.50 =$ ₹ 77,700

4. From a circular sheet of paper with a radius 20 cm, four circles of radius 5cm each are cut out. Find the ratio of the uncut to the cut portion is.



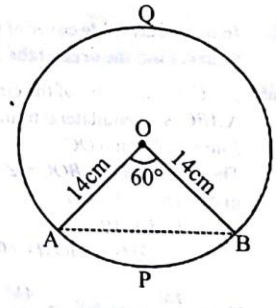
Sol. Area of sheet of paper with radius 20 cm. $= \pi(20)^2 = 400\pi\ \text{cm}^2$

Area of 4 circles of radius 5 cm. $= 4 \times \pi(5)^2 = 100\pi\ \text{cm}^2$

$\therefore$ Area of remaining portion $= 400\pi - 100\pi = 300\pi\ \text{cm}^2$

$\therefore$ Required ratio $= 300\pi : 100\pi = 3 : 1$

5. A chord of circle 14 cm. makes an angle of 60° at the centre of the circle. Find :
(i) area of minor sector (ii) area of the minor segment
(iii) area of the major sector (iv) area of the major segment



Sol. Given : $r = 14$ cm, $\theta = 60°$

(i) Area of minor sector $OAPB = \frac{\theta}{360}(\pi r^2) = \frac{60}{360}\times 3.14\times 14\times 14 = 102.57\ \text{cm}^2$

(ii) Area of minor segment $APB = \frac{\pi r^2\theta}{360} - \frac{r^2}{2}\sin\theta = 102.57 - \frac{14\times 14}{2}\sin 60°$

$$= 102.57 - 98\times\frac{\sqrt{3}}{2} = 17.80\ \text{cm}^2$$

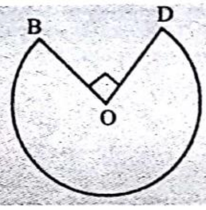
(iii) Area of major sector = Area of circle – Area of minor sector $OAPB$

$= \pi(14)^2 - 102.57 = 615.44 - 102.57 = 512.87\ \text{cm}^2$

(iv) Area of major segment AQB = Area of circle – Area of minor segment APB

$= 615.44 - 17.80 = 597.64\ \text{cm}^2$

6. The shape of the top of a table in a restaurant is that of a segment of a circle with centre O and $\angle BOD = 90°$
If $BO = OD = 60$ cm, find :
(i) the area of the top of the table.
(ii) the perimeter of the table. Take $\pi = 3.14$

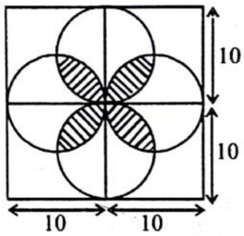


Sol. Given that $\angle BOD = 90°$, so the top of the table is $\frac{3}{4}$ of a circle of radius 60 cm.

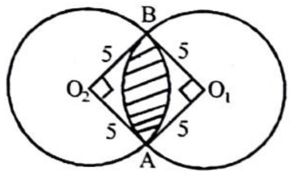
(i) Area of the top of the table $= \frac{3}{4}\pi r^2 \text{cm}^2 = \frac{3}{4} \times 3.14 \times (60)^2 \text{ cm}^2 = 8478 \text{ cm}^2$.

(ii) Perimeter of the table $= \left(\frac{3}{4} \times 2\pi r + 2r\right)\text{cm} = \left(\frac{3}{4} \times 2 \times 3.14 \times 60 + 2 \times 60\right)\text{cm} = (90 \times 3.14 + 120)\text{ cm} = 402.6 \text{ cm}$.

7. Find the area of the shaded region. [All the circles shown in the figure are congruent.]



Sol. It is clear that any two congruent circles intersect orthogonally. In the figure, below :
O_1AO_2B is a square,
Since, $\angle AO_1B = 90°$, $\angle AO_2B = 90°$, $\angle O_2BO_1 = 90°$ and $\angle O_2AO_1 = 90°$
Sides $O_1A = O_1B = O_2A = O_2B = 5$ unit.



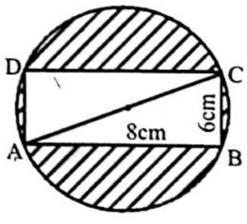
In the figure O_1A and O_1B should be perpendicular to each other.
Also O_2A and O_2B should be perpendicular to each other.
Consider the area of shaded region in this figure
$= \text{Area (sector } O_1AB) + \text{Area (sector of } O_2BA) - \text{Area } (\square O_1AO_2B)$

$$= \frac{1}{4}\pi(5)^2 + \frac{1}{4}\pi(5)^2 - (5)^2 = \frac{1}{2}\pi(5)^2 - (5)^2 = 25(\pi/2 - 1).$$

Hence, the required area = 4 times the area calculated above $= 4 \times 25(\pi/2 - 1) = 100(\pi/2 - 1) = 57$ sq. unit.

8. The sides of a rectangle inscribed in a circle are 8 cm. and 6 cm. Find the difference of the area of the circle and the rectangle.

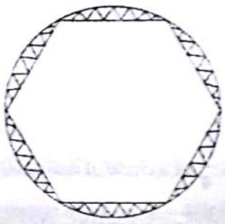
Sol. Let $ABCD$ be the given rectangle.
The diagonal AC is the diameter of the circle
$AC^2 = 8^2 + 6^2 = 64 + 36 = 100$
$\therefore AC = 10$ cm
$\therefore$ Radius of the circle = 5 cm



$\therefore$ Area of the circle $= \pi r^2 = \frac{22}{7} \times 5 \times 5 = \frac{550}{7} = 78.57 \text{cm}^2$

Area of the rectangle $= 8 \times 6 = 48 \text{ cm}^2$ $\quad\therefore$ Difference of the areas $= 78.57 - 48 = 30.57 \text{ cm}^2$

9. A round table cover six equal designs, as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Re. 0.35/cm^2.

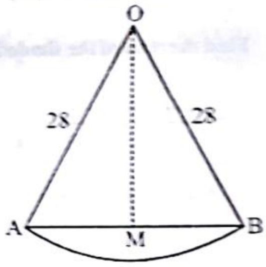


Sol. One design = area of one segment

Central angle for one sector = $\dfrac{360^\circ}{6} = 60^\circ$, Area of sector $OAB = \dfrac{60^\circ}{360^\circ} \times \pi(28^2) = \dfrac{1}{6} \times 3.14 \times (28)^2 = 410.3\text{ cm}^2$

As, $\angle AOB = 60^\circ$ and $OA = OB$, ΔAOB is equilateral

$\text{Ar}(\Delta AOB) = \dfrac{\sqrt{3}}{4}(28)^2 = 333.2$

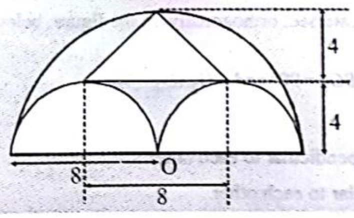


$\Rightarrow$ Area of one segment $= 410.3 - 333.2 = 77.1\text{ cm}^2$

Area of six segments $= 77.1 \times 6 = 462.6\text{ cm}^2$

$\Rightarrow$ Total cost $= 462.6 \times 0.35 =$ ₹ 161.91

10. Find the area of the shaded region in the diagram below where the given triangle is isosceles with vertices of base lying on axis of the radius perpendicular to the diameters of the two small semicircles.



Sol. The figure is self explanatory.

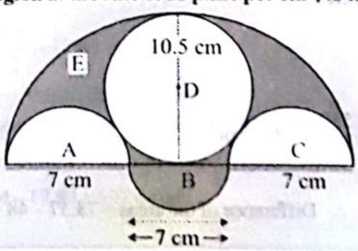
Shaded area = Area of big semicircle of radius 8 unit – Area of 2 semi-circles of radius 4 unit – Area of triangle with base 8 unit & height 4 unit

$$= \frac{\pi 8^2}{2} - \left(\frac{2\pi 4^2}{2} + \frac{1}{2} \times 8 \times 4\right) = 32\pi - 16\pi - 16 = 16(\pi - 1).$$

11. In figure, there are three semicircles, A, B and C having diameter 7 cm each, and another semicircle E having a circle D with diameter 10.5 cm are shown. Calculate:

(i) the area of the shaded region.

(ii) the cost of painting the shaded region at the rate of 25 paise per cm^2, to the nearest rupee.



Sol. (i) Area of the shaded region = Area of semicircle with diameter 21 cm – Area of two semicircle with diameter 7 cm – Area of circle with diameter 10.5 cm + Area of semi circle with diameter 7 cm.

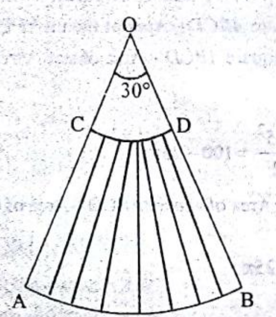
So, area of the shaded region = $\frac{1}{2}\pi\left(\frac{21}{2}\right)^2 - 2\times\frac{1}{2}\times\pi\left(\frac{7}{2}\right)^2 - \pi\times\left(\frac{21}{4}\right)^2 + \frac{1}{2}\pi\left(\frac{7}{2}\right)^2$.

$$=\frac{1}{2}\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}-\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}-\frac{22}{7}\times\frac{21}{4}\times\frac{21}{4}+\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}=\frac{22}{7}\left[\frac{21\times21}{8}-\frac{49}{4}-\frac{21\times21}{16}+\frac{49}{8}\right]$$

$$=\frac{22}{7}\left[\frac{21\times21\times2-21\times2}{16}-\frac{49}{8}\right]=\frac{22}{7}\left[\frac{21\times21}{16}-\frac{49}{8}\right]=\frac{22}{7}\times\left[\frac{21\times21-49\times2}{16}\right]=67.38\text{ cm}^2$$

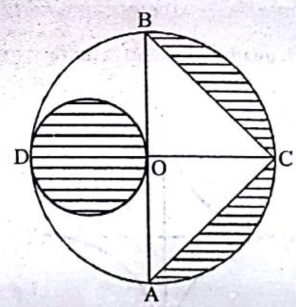
(ii) Cost of painting of shaded region = 0.25 × 67.38 = ₹ 16.84

12. In figure, *AB* and *CD* are arcs of two concentric circles of radii 21 cm respectively and 7 cm and centre *O*. If $\angle AOB = 30°$, find the area of the shaded region.



Sol. The required area = ar (*OAB*) – ar (*OCD*) = $\frac{30°}{360°}\times\pi(21)^2-\frac{30°}{360°}\times\pi(7)^2=\frac{\pi}{12}\left[21^2-7^2\right]=\frac{1}{12}\times\frac{22}{7}\times28\times14=102.67\text{ cm}^2$

13. *AB* and *CD* are two perpendicular diameters of a circle with centre *O*, *OD* is the diameter of smaller circle. If *OA* = 7 cm, find the area of the shaded region in figure.



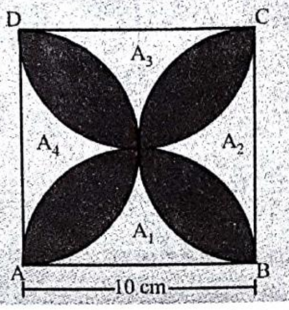
Sol. $OA = OD = 7$ cm.

The required area = Area of the circle with diameter *OD* + Area of semicircle *ACB* – Area of ΔACB

$$=\pi\left(\frac{7}{2}\right)^2+\frac{1}{2}\pi(7)^2-\frac{1}{2}AB.OC=\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}+\frac{1}{2}\times\frac{22}{7}\times7\times7-\frac{1}{2}\times14\times7$$

$$=\frac{11\times7}{2}+11\times7-7\times7=66.5\text{ cm}^2$$

14. In figure, *ABCD* is a square of side 10 cm. and semicircles are drawn with each side of the square as diameter. Find area of the shaded region.



Sol. We mark the area of unshaded regions as $A_1, A_2\, A_3$ and A_4 as shown in the figure.

Area of the shaded region = Area of the square $ABCD$ – Area of the unshaded portion $(A_1+A_2+A_3+A_4)$.

Area of unshaded region $A_1 + A_3$ = Area of square $ABCD$ – Area of semi circle on BC as diameter – Area of semi-circle on AD as diameter.

$$=10^2-\frac{\pi\cdot 5^2}{2}-\frac{\pi\cdot 5^2}{2}=100-25\pi.$$

Similarly, area of unshaded region A_2+A_4 = Area of square $ABCD$ – Area of the semi-circles on diameter AB and DC.

So, Area $A_2+A_4=10^2-2\times\frac{\pi\times 5^2}{2}=100-25\pi.$

So, Area $A_1+A_2+A_3+A_4=200-50\pi.$

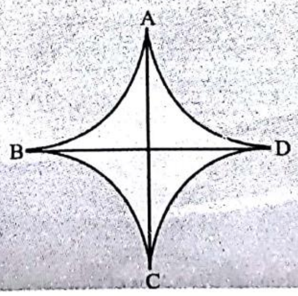
Area of the shaded region = Area of square $ABCD$ – Area $(A_1+A_2+A_3+A_4)$

$= 10^2-(200-50\pi)=100-200+50\pi.$

So, Area (shaded region) $= 50\pi-100=50(\pi-2).$

So, Area (shaded region) $= 50(3.14-2)=50\times 1.14=57\text{ cm}^2.$

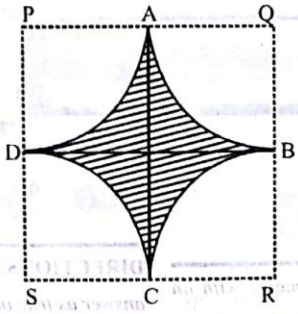
15. Calculate the area of the shaded portion. The quadrants shown in the figure are each of radius 7 cm. $\left(\text{Take } \pi=\frac{22}{7}\right)$



Sol. Draw square $PQRS$ as shown.

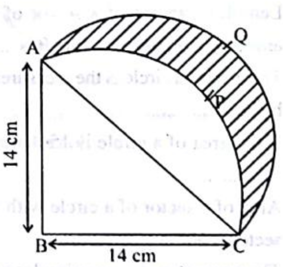
Here $PQ=QR=PS=14$ cm

Area of the shaded portion = Area of the square $PQRS$ – Area of four equal quadrants



$= 14\times14-4\times\frac{1}{4}\pi\times(7)^2 = 196-\frac{22}{7}\times7\times7 = 196-154 = 42\text{ cm}^2$.

16. *ABCP* is a quadrant of a circle of radius 14 cm. With *AC* as diameter, a semicircle is drawn. Find the area of the shaded portion. (fig.)



Sol. In right angled triangle *ABC*, We have

$AC^2 = AB^2 + BC^2$

$AC^2 = 14^2 + 14^2$

$AC = \sqrt{2\times14^2} = 14\sqrt{2}$ cm.

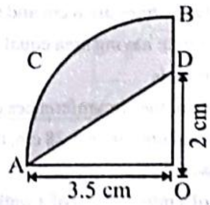
Now required area = Area $APCQA$ = Area $ACQA$ – Area $ACPA$

= Area $ACQA$ – (Area $ABCPA$ – Area ΔABC)

$= \frac{1}{2}\times\pi\times\left(\frac{14\sqrt{2}}{2}\right)^2 - \left[\frac{1}{4}\times\pi(14)^2+\frac{1}{2}\times14\times14\right]$

$= \frac{1}{2}\times\frac{22}{7}\times7\sqrt{2}\times7\sqrt{2}-\frac{1}{4}\times\frac{22}{7}\times14\times14+7\times14 = 154-154+98 = 98\text{ cm}^2$.

17. In the figure, *OACB* represents a quadrant of a circle. The radius *OA* = 3.5 cm, *OD* = 2 cm. Calculate the area of the shaded portion.



Sol. Area of quadrant $= \frac{1}{4}\pi r^2 = \frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = 9.63\text{ cm}^2$

Area of $\Delta AOD = \frac{1}{2}\times OA\times OD = \frac{1}{2}\times3.5\times2 = 3.5\text{ cm}^2$

$\therefore$ Area of shaded portion $= 9.63-3.5 = 6.13\text{ cm}^2$

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. A line segment whose end points lie on the circle is called .... to the circle.
2. If the area of a sector of a circle which subtends an angle of 60° at the centre is $\frac{77}{3}$ cm^2, then the radius of the circle is................
3. Circumference of a circle is ....
4. Area of a circle is ....
5. Length of an arc of a sector of a circle with radius r and angle with degree measure θ is ....
6. The area of a circle is the measurement of the region enclosed by its ....
7. If the area of a circle is 154 cm^2, then its circumference is ....
8. Area of a sector of a circle with radius 6 cm if angle of the sector is 60° is ....
9. The perimeter of a semicircular protractor of diameter 14 cm is ....
10. The area of the circular ring included between two concentric circles of radii 14 cm and 10.5 cm is ....
11. The perimeter of a sector of a circle of radius r cm and of central angle θ (in degrees) is ....
12. The radii of two circles are 8 cm and 6 cm respectively. The radius of the circle having area equal to the sum of the areas of the two circles is ....
13. The difference of the circumferences of two circles is 66 cm. If the diameter of one circle is 28 cm, then the diameter of the other circle is ....
14. If the length of a minute hand of a wall clock is 7 cm, then the area swept by it in 30 minutes is ....
15. Area of a sector of a circle with radius r and angle with degrees measure θ is ....
16. The sum of the circumferences of two circles is 110 cm. If the diameter of one circle is 28 cm, then the diameter of the other circle is ....

T/F True / False :

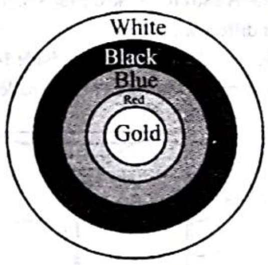
DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The radii of two circles are 19 cm and 9 cm respectively. The radius of the circle which has circumference equal to the sum of the circumferences of the two circles is 26 cm.
2. The wheels of a car are of diameter 80 cm each. It will make 4300 complete revolutions in 10 minutes when the car is travelling at a speed of 66 km per hour.
3. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre the length of the arc is 22 cm.
4. If the circumference of a circle is 88 cm, then its radius is 14 cm.
5. The length of an arc of a sector of a circle of radius r units and of centre angle θ is $\frac{\theta}{360^\circ}\times\pi r^2$.
6. If an arc of a circle of radius 14 cm subtends an angle of 60° at the centre, then the length of the arc is $\frac{44}{3}$ cm.
7. If a sector of a circle of diameter 21 cm subtends an angle of 120° at the centre, then its area is 85.5 cm^2.
8. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. The total area cleaned at each sweep of the blades is 1255 cm.
9. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. The area of the sea over which the ships are warned is 190 km^2 (app.).
10. The area of the circular ring included between two concentric circles of radii 14 cm and 10.5 cm is 269.5 cm^2.
11. The length of a rope by which a cow must be tethered in order that it may be able to graze of an area of 616cm^2 is 18m.

MTF Match the Following :

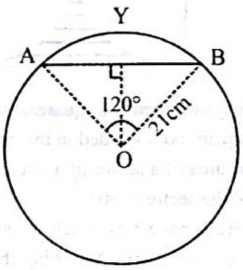
DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm. wide, then match the two columns I and II:



Column I	Column II
(A) Area of Gold score	(p) 1039.5
(B) Area of Red score	(q) 1732.5
(C) Area of Blue score	(r) 346.5
(D) Area of Black score	(s) 2425.5

2. For circle shown, match the column



Column I	Column II
(A) Area of segment AYB	(p) $\frac{441}{4}\sqrt{3}$
(B) Area of sector $OAYB$	(q) $\frac{21}{4}(88-21\sqrt{3})$
(C) Area of $\Delta\, OAB$	(r) 462
(D) OM	(s) 21/2

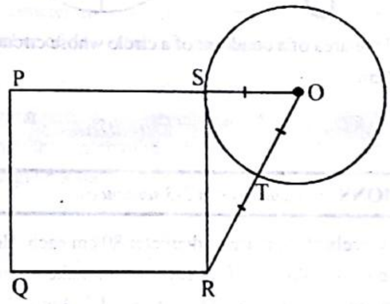
3. Two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56m. If the centre of each circular flowered bed is the point of intersection O of the diagonals of the square lawn, then match the column.

Column I	Column II
(A) area of ΔOAB	(p) 4032
(B) area of flower bed	(q) 784
(C) area of sector OAB	(r) 448
(D) Total area	(s) 1232

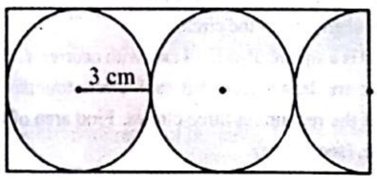
VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Find the area of sector of a circle whose radius is 6 cm, when
   (a) the angle at the centre is 35°
   (b) the length of arc is 22 cm
2. The radius of a wheel is 42 cm. How many revolutions will it make in going 26.4 km ?
3. The diameter of a cycle wheel is 28 cm. How many revolution will it make in moving 13.2 km ?
4. A wire of length 26.4 cm. is bent in the shape of a circle. Find the area of the circle formed.
5. The radii of two circles are 8 cm. and 6 cm. Find the radius of the circle having area equal to the sum of the areas of the two circles.
6. $PQRS$ is a square. SR is a tangent (at point S) to the circle with centre O and $TR = OS$. Then, find the ratio of area of the circle to the area of the square.



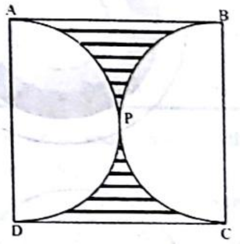
7. In the given figure, find the area of the unshaded portion within the rectangle. (Take $\pi = 3.14$)



8. A rectangular sheet of cardboard is 14 cm × 7 cm. Two equal circular holes are cut out touching each other, having largest possible area. Find the area of the carboard left out.

9. The area of a circular ring enclosed between two concentric circles is 286 cm^2. Find the radii of the two circles, given that their difference is 7 cm.

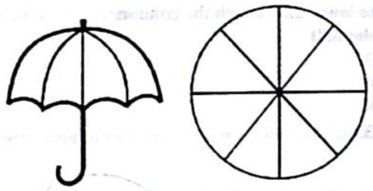
10. In figure, $ABCD$ is a square of side 14 cm, APD and BPC are semi-circles. Find area of the shaded region.



11. A circle is inscribed in a square of side 14 cm. Find the area of the square not included in the circle.

12. Find the area of a sector of a circle with radius 6 cm if the angle of the sector is 60°.

13. An umbrella has 8 ribs which are equally spaced (shown in the adjoining figure). Assuming the umbrella to be a flat circule of radius 45cm, find the area between its two consecutives ribs.



14. Find the area of a quadrant of a circle whose circumference is 22 cm.

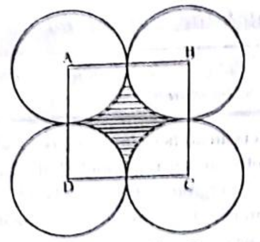
Short Answer Questions

DIRECTIONS : *Give answer in 2-3 sentences.*

1. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km/hr?

2. A chord of a circle of radius 15 cm. subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

3. $ABCD$ is a square of side 14 cm. with centres A, B, C, D four circles are drawn such that each circle touches externally two of the remaining three circles. Find area of the shaded region. (see figure)

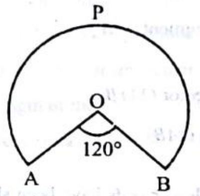


4. Four equal circles each of radius 5 cm touch one another. Find the area between them.

5. The sum of diameters of two circles is 2.8 m and the difference of their circumferences is 0.88 m. Find the radii of the two circles.

6. The length of the minute hand of a clock is 14 cm long. Find the area swept by the minute hand in 5 minutes.

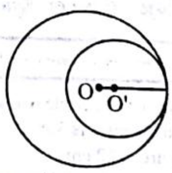
7. In the adjoining figure, $OAPB$ is a sector of a circle of radius 3.5 cm with centre at O. If $\angle AOB = 120°$, find the length $OAPB$.



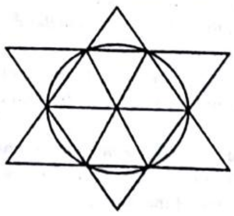
8. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

(use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

9. Two circles touch internally and their centres are O and O' as shown. The sum of their areas is 180π and the distance between their centres is 6 cm. What is the diameter of the larger circle?



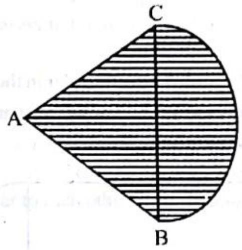
10. A circle of radius 'a' is divided into 6 equal sectors. An equilateral triangle is drawn on the chord of each sector to lie outside the circle. Find the area of the resulting figure.



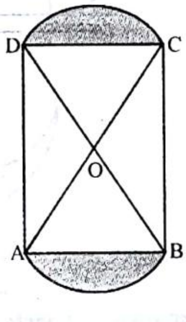
11. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

12. If the length of an arc of a sector of a circle with radius r is l, then show that the area of the sector is $\frac{1}{2}lr$.

13. In an equilateral ΔABC of side 14 cm, side BC is the diameter of a semicircle as shown in the figure below. Find the area of the shaded region. $\left(\text{Take } \pi = \frac{22}{7} \text{ and } \sqrt{3} = 1.732\right)$



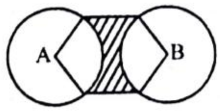
14. In the adjoining figure, two circular flower beds have been shown on the two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.



15. A wire bent in the form of a circle of radius 42 cm is cut and again bent in the form of a square. Find the ratio of the areas of the regions enclosed by the circle and the square.

16. The radius of a circle is 7 cm. A chord of length $\sqrt{98}$ cm. is drawn in the circle. Find the area of the minor segment.

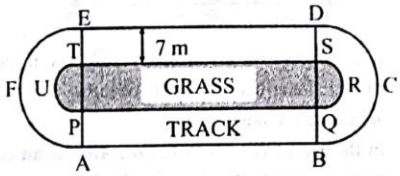
17. What is tne area of the shaded region shown below, if the radius of each circle is equal to the side of the hexagon, which in turn is equal to 6 cm, and A and B are the centres of the circles ?



LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. The figure given below shows a running track surrounding a grassed enclosure $PQRSTU$. The enclosure consists of rectangle $PQST$ with a semicircular region at each end. $PQ = 200$, $PT = 70$ m.



Calculate :

(i) the area of the grasssed enclosure in m^2.

(ii) the outer perimeter $ABCDEF$ of the track

$\left(\text{Take } \pi = \frac{22}{7}\right)$

2. A chord AB of a circle of radius 10 cm. makes a right angle at the centre of the circle. Find the area of the major and minor segments. (Take $\pi = 3.14$)

3. The area of an equilateral triangle is 17,320.5 cm^2. About each vertex of the triangle is described a circle whose radius is equal to half the side of the triangle. Find the area of the portion of the triangle which is not included in the circle ($\pi = 3.14$)

4. A sheet is 11 cm long and 2 cm wide. Circular pieces 0.5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared.

5. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular point as centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of triangle not included in the circles

(Take $\sqrt{3} = 1.73$)

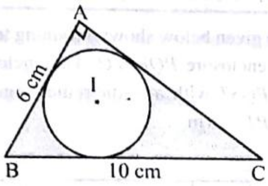
6. A conical vessel has a hemispherical lid. The total height when closed is 30 cm and the maximum girth is 44 cm. Find (1) its volume (2) its total surface area (Take $\pi = 22/7$)

7. AC and BD are two perpendicular diameters of a circle $ABCD$. Given that the area of the shaded portion is 308 cm^2, calculate:

(i) the length of AC, and

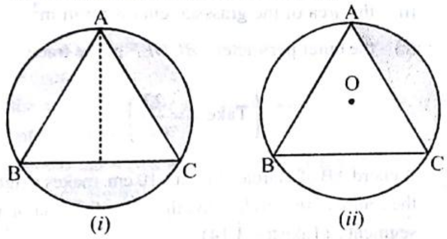
(ii) the circumference of the circle $\left(\text{Take } \pi = \frac{22}{7}\right)$.

8. In the adjoining figure, ABC is a right angled triangle at A. Find the area of the shaded region if $AB = 6$ cm, $BC = 10$ cm and I is the centre of incircle of ΔABC.

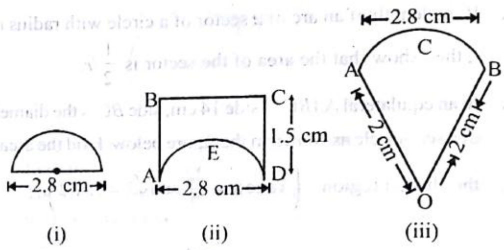


9. (a) In the figure (*i*) given below, ABC is an equilateral triangle inscribed in a circle of radius 32 cm. Find the area of the shaded region.

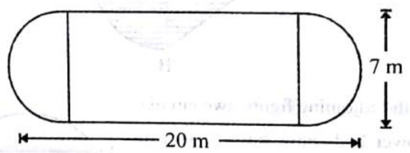
(b) In the figure (ii) given below, ABC is an equilateral triangle inscribed in a circle of radius 4 cm with centre O. Find the area of the shaded region.



10. An ant is moving around a few food particles of different shapes scattered on the floor. For which food particle the ant would have to take a longer round?



11. The shape of a garden is rectangular in the middle and semi-circular at the ends as shown in the diagram. Find the area and perimeter of this garden.



2 EXERCISE

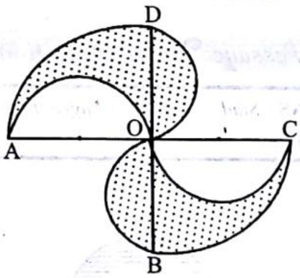
MCQ Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The sum of the areas of two circles, which touch each other externally, is 153 π. If the sum of their radii is 15, than the ratio of the larger to the smaller radius is

(a) 4 : 1 (b) 2 : 1
(c) 3 : 1 (d) None of these

2. In the given figure below, the boundary of the shaded region comprises of four semicircles and two quarter circles. If $OA = OB = OC = OD = 7$ cm and the straight lines AC and BD are perpendicular to each other, then the length of the boundary is



(a) 68 cm (b) 49 cm
(c) 66 cm (d) 44 cm

3. A race track is in the form of a ring whose inner and outer circumference are 437m and 503m respectively. The area of the track is

(a) 66 sq. cm (b) 4935 sq. cm
(c) 9870 sq. cm (d) None of these

4. A circle of maximum possible size is cut from a square sheet of board. Subsequently, a square of maximum possible size is cut from the resultant circle. Area of the final square will be

(a) 75% of the size of the original square
(b) 50% of the size of the original square
(c) 75% of the size of the circle
(d) 25% of the size of the original square

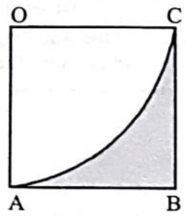
5. Given XY has been divided into 5 congruent segments and semicircles have been drawn. But suppose XY were divided into millions of congruent segments and semicircles were drawn, what would the sum of the lengths of the arcs be ?

(a) $2YX$ (b) $5XY$
(c) XY (d) None of these

6. If the sum of the circumferences of two circles with diameters d_1 and d_2 is equal to the circumference of a circle of diameter d, then

(a) $d_1^2 + d_2^2 = d^2$ (b) $d_1 + d_2 = d$
(c) $d_1 + d_2 > d$ (d) $d_1 + d_2 < d$

7. In the adjoining figure, $OABC$ is a square of side 7 cm. OAC is a quadrant of a circle with O as centre. The area of the shaded region is



(a) 10.5 cm^2
(b) 38.5 cm^2
(c) 49 cm^2
(d) 11.5 cm^2

8. The area of a circular ring formed by two concentric circles whose radii are 5.7 cm and 4.3 cm respectively is (Take $\pi = 3.1416$)

(a) 43.98 sq.cm (b) 53.67 sq. cm
(c) 47.24 sq.cm (d) 38.54 sq.cm

9. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240°. If another circle of the area same as the sector is formed, then radius of the new circle is

(a) 79.5 cm (b) 81.6 cm
(c) 83.4 cm (d) 88.5 cm

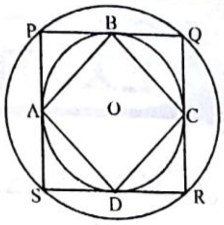
10. The area of a sector of angle p (in degrees) of a circle with radius R is

(a) $\frac{p}{360} \times 2\pi R$ (b) $\frac{p}{180} \times \pi R^2$
(c) $\frac{p}{720} \times 2\pi R$ (d) $\frac{p}{720} \times 2\pi R^2$

11. If the sector of a circle of diameter 10 cm subtends an angle of 144° at the centre, then the length of the arc of the sector is

(a) 2π cm (b) 4π cm
(c) 5π cm (d) 6π cm

12. The figure below shows two concentric circles with centre O. $PQRS$ is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at point B, C, D and A. The ratio of the perimeter of the outer circle to that of polygon $ABCD$ is



(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{2}$

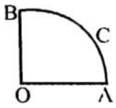
(c) $\frac{\pi}{2}$ (d) π

13. If a circular grass lawn of 35m in radius has a path 7m wide running around it on the outside, then the area of the path is

(a) 1450 m^2 (b) 1576 m^2

(c) 1694 m^2 (d) 3368 m^2

14. In the adjoining figure, *OACB* is a quadrant of a circle of radius 7 cm. The perimeter of the quadrant is



(a) 11 m (b) 18 m

(c) 25 m (d) 36 m

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. If the radius of a circle is $\frac{7}{\sqrt{\pi}}$ cm, then the area of the circle is equal to

(a) $\frac{49}{\pi}$ cm^2 (b) $\frac{154}{\pi}$ cm^2

(c) 154 cm^2 (d) 49 cm^2

2. If the ratio of the areas of the two circles is 25 : 16, then the ratio of their circumferences is.

(a) $\frac{625}{500}$ (b) $\frac{4}{5}$

(c) $\frac{5}{4}$ (d) $\frac{500}{625}$

3. If the sector of *a* circle of diameter 14cm subtends an angle of 30° at the centre, then its area is

(a) 49π (b) $\frac{49\pi}{12}$

(c) $\frac{242}{3\pi}$ (d) $\frac{121}{3\pi}$

4. Which of the following is/are correct?

(a) Area of a circle with radius 6 cm, if angle of sector is 60°, is $\frac{132}{14}$ cm^2.

(b) If *a* chord of circle of radius 14 cm makes an angle of 60° at the centre of the circle, then area of major sector is 512.87 cm^2.

(c) The ratio between the circumference and area of a circle of radius 5 cm is 2 : 5.

(d) Area of a circle whose radius is 6 cm, when the length of the arc is 22 cm, is 66 cm^2.

5. Which of the following is/are correct.

(a) A chord divides the interior of a circle into two parts.

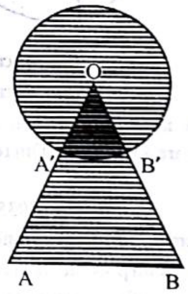
(b) An arc of a circle whose length is less than that of a semicircle of the same circle is a called a minor arc.

(c) Circles having the same centre but different radii are called concentric circles.

(d) A line segment joining any two points of a circle is called an arc.

Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*



In the above given figure, *a* circular arc of radius 6 cm has been drawn with vertex *O* of an equilateral triangle *OAB* of side 12 cm as centre.

1. The area of the sector *A'OB'* is

(a) 6π cm^2 (b) π cm^2

(c) 6 cm^2 (d) $(6+\pi)$ cm^2

2. The area of the shaded region is

(a) 156.522 cm^2 (b) 156.552 cm^2

(c) 165.552 cm^2 (d) 561.552 cm^2

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** If the circumference of a circle is 176 cm, then its radius is 28 cm.
Reason : Circumference $= 2\pi \times$ radius

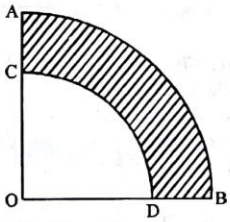
2. **Assertion :** If the outer and inner diameter of a circular path is $10m$ and $6m$, then area of the path is $16\pi\, m^2$.
Reason : If R and r be the radius of outer and inner circular path respectively then area of path $= \pi(R^2 - r^2)$

3. **Assertion :** If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is 40 cm^2.
Reason : Circumference of the circle = length of the wire.

MMQ Multiple Matching Questions :

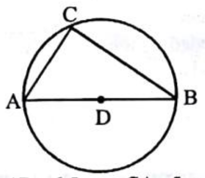
DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r, s......) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1. **Column-I** **Column-II**

(A) 

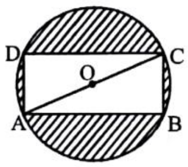
$OA = 26\text{m}, OC = 23\text{m}$, Area = ?

(p) 30 cm^2

(B) 

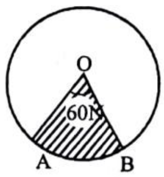
$AD = 6.5$ cm, $CA = 5$ cm, Area of ΔABC = ?

(q) 115 m^2

(C) 

$BC = 6$ cm, $AB = 8$ cm, Area of the shaded region = ?

(r) 35 cm^2

(D) 

$OB = 14.8$ m, Area of the shaded region = ?

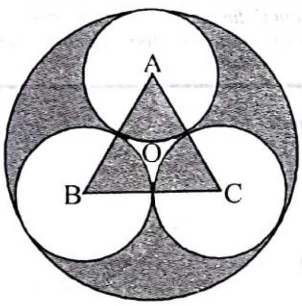
(s) 0.003 m^2

(t) 30 m^2

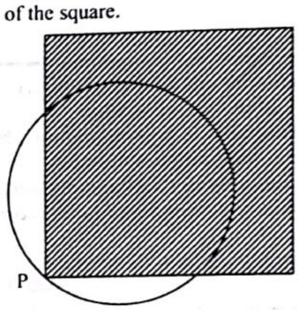
HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

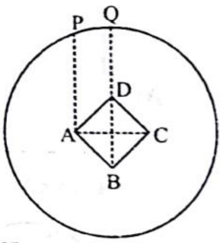
1. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 square centimeters then find the area (in square centimeters) of the triangle ABC.
2. In figure, three circles of radius 2 cm touch one another externally. These circle are circumscribed by a circle of radius R cm. Find the value of R and the area of the shaded region in terms of π and $\sqrt{3}$.



3. A bucket is in the shape of a frustrum of a right-circular cone with a base radius of 20 cm and height of 35 cm. Semi vertical angle of the cone of which frustrum is a part, is 45°. It contains water whose height is 10 cm. A solid iron ball of radius $5\sqrt[3]{74}$ cm is dropped into the bucket.
   (i) What is the amount of water in the bucket (in cc)?
   (ii) After the ball is dropped into the bucket, find the height of the water in the bucket.
4. A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square.



   (i) Find the area of the part of the circle (round punch) falling outside the square sheet.
   (ii) Find the proportion of the sheet area that remains after punching.
5. In an equilateral triangle of side 18 cm, *a* circle is inscribed touching its sides. Find the area of the remaining portion of the triangle.
6. In the adjoining figure, $ABCD$ is a square drawn inside a circle with centre O. The centre of the square coincides with O and the diagonal AC is horizontal. If AP, DQ are vertical and $AP = 45$ cm, $DQ = 25$ cm, find
   (i) the radius of the circle.
   (ii) the side of the square.
   (iii) the area of the shaded region
   Take $\sqrt{2} = 1.41$ and $\pi = 3.14$



SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. chord
2. 7 cm^2

$$\left[\frac{60}{360}\times\frac{22}{7}\times r^2=\frac{77}{3}\Rightarrow r^2=49\Rightarrow 4=7\right]$$

3. $2\pi r$. 4. πr^2 5. $\frac{\theta}{360}\times 2\pi r$
6. boundary 7. 44 cm 8. 132/7 cm^2
9. 36 cm 10. 269.5 cm^2
11. $\left(\frac{\theta}{360}\times 2\pi r^n+2r\right)$ cm
12. 10 cm. 13. 7 cm
14. 77 cm^2 15. $\frac{\theta}{360}\times\pi r^2$ 16. 7 cm

TRUE / FALSE

1. True 2. False 3 True
4. True 5. False 6. True
7. False 8 True 9. True
10. True 11. False

MATCH THE FOLLOWING :

1. (A) $\rightarrow$ r; (B) $\rightarrow$ p; (C) $\rightarrow$ q; (d) $\rightarrow$ s
2. (A) $\rightarrow$ q; (B) $\rightarrow$ r; (C) $\rightarrow$ p; (d) $\rightarrow$ s
3. (A) $\rightarrow$ q; (B) $\rightarrow$ r; (C) $\rightarrow$ s; (d) $\rightarrow$ p

VERY SHORT ANSWER QUESTIONS :

1. (a) Area of sector

$$=\pi r^2.\frac{\theta}{360°}=\frac{22}{7}\times 6\times 6\times\frac{35}{360}\text{cm}^2=11\text{ sq.cm.}$$

(b) Here length of arc $\ell=22$ cm.

$$\therefore\ 2\pi r\frac{\theta}{360°}=22\text{cm.}$$

Area of sector $=\pi r^2.\frac{\theta}{360°}=\frac{1}{2}r.2\pi r\frac{\theta}{360°}$

$$=\frac{1}{2}r.\ell=\frac{1}{2}\times 6\times 22\text{ sq.cm}=66\text{ sq.cm}$$

2. Distance travelled in one revolution = Circumference of the wheel

$$=2\pi r=2\times\frac{22}{7}\times 42\text{cm.}=264\text{ cm}$$

$\therefore$ No. of revolutions required to travel 26.4 km

$$=\frac{26.4\times 1000\times 100}{264}=10000$$

3. Distance travelled by the wheel in one revolution $=2\pi r$

$$=2\times\frac{22}{7}\times\frac{28}{2}=88\text{cm.}$$

and the total distance covered by the wheel

$=13.2\times 1000\times 100$ cm $=13,20,000$ cm

$\therefore$ Number of revolutions made by the wheel

$$=\frac{1320000}{88}=15,000$$

4. Given that a wire of length 26.4 cm. is bent in the shape of a circle of radius r.

$\therefore$ Circumferene of the circle = 26.4 cm.

$$\Rightarrow\ 2\pi r=26.4\ \Rightarrow\ r=\frac{26.4}{2\pi}=\frac{26.4\times 7}{2\times 22}=4.2\text{ cm.}$$

$$\therefore\text{ Area of the circle}=\pi r^2=\frac{22}{7}(4.2)^2=55.44\text{ cm}^2$$

5. Total area of two circles $=\pi(8)^2+\pi(6)^2=100\pi$ cm^2

Let the radius of the required circle be r.

$\Rightarrow\pi r^2=100\pi\Rightarrow\quad r^2=100\Rightarrow r=1$ cm.

6. Let the radius of the circle be r, then area of the circle is πr^2.

Now, $\quad OR=OT+TR=r+r=2r$

[$\because TR=OS=r$ (given)]

Now, by pythagorus theorem in ΔORS,

$$SR=\sqrt{(2r)^2-r^2}=\sqrt{3}r$$

So, area of square $PQRS=3r^2$

$$\therefore\text{ Required ratio}=\frac{\pi r^2}{3r^2}=\frac{\pi}{3}.$$

7. Area of unshaded portion

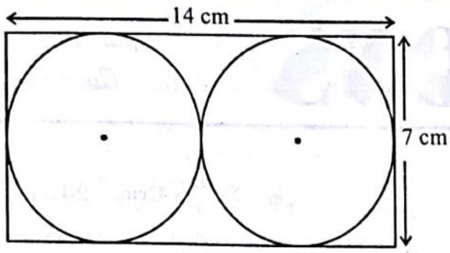
$=$ Area of rectangle – Area of $2\frac{1}{2}$ circles

$=15\times 6-\frac{5}{2}\times 3.14\times 3^2$

$=90-70.65$

$=19.35$ cm^2

8. $21\ cm^2$



[**Hint.** Radius of largest circle $= \frac{7}{2}$ cm

Required area $= \left[14\times 7 - 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\right] cm^2$

9. 10 cm, 3 cm;

[**Hint.** $R - r = 7,\ \frac{22}{7}(R^2 - r^2) = 286 \Rightarrow R^2 - r^2 = 13\times 7$

$\Rightarrow (R+r)(R-r) = 91 \Rightarrow (R+r)\times 7 = 91 \Rightarrow R + r = 13.]$

10. Area of the shaded region

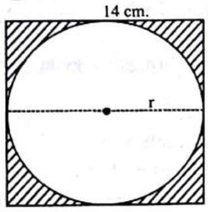
= Area of the square – Area of two semicircles

= Area of square – Area of circle $= 14\times 14 - \pi(7)^2$

[$\because$ side of square = 14 cm, Radius of circle = 7 cm.]

$= 196 - \frac{22}{7}\times 7\times 7 = 196 - 154 = 42\ cm^2$

11. $r = \frac{14}{2} = 7$cm



Area of shaded region

$= (\text{side})^2 - \pi r^2 = (14)^2 - \frac{22}{7}\times 7\times 7 = 196 - 154 = 42\ cm^2$

12. $\frac{132}{7}\ cm^2$;

[**Hint.** Area $= \frac{60}{360}\times\frac{22}{7}\times 6^2\ cm^2 = \frac{132}{7}\ cm^2$]

13. $\frac{22275}{28}\ cm^2$

[**Hint.** Required area $= \left(\frac{1}{8}\times\frac{22}{7}\times(45)^2\right)cm^2$]

14. $\frac{77}{8}\ cm^2$;

[**Hint.** If radius of circle is r cm, then

$2\times\frac{22}{7}\times r = 22 \Rightarrow r = \frac{7}{2}$

$\therefore$ Required area $= \frac{1}{4}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\ cm^2 = \frac{77}{8}\ cm^2$.]

SHORT ANSWER QUESTIONS :

1. 4375;

[**Hint.** Distance covered in 10 minutes

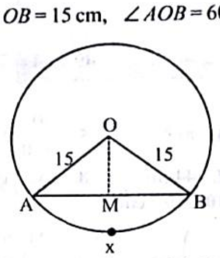
$= \left(\frac{10}{60}\times 66\right)$km = 11 km

$= (11\times 1000\times 100)$ cm

$\therefore$ Distance covered in one revolution $= \pi d = \left(\frac{22}{7}\times 80\right)$cm

$\therefore$ No. of revolutions $= \frac{11\times 1000\times 100\times 7}{22\times 80} = 4375$.]

2. Here, $OA = OB = 15$ cm, $\angle AOB = 60°$ (figure)



As, $OA = OB$, we must have $\angle OAM = 60° = \angle OBM$

Area of sector AOB

$= \frac{60°}{360°}\times\pi(15)^2 = \frac{3.14\times 225}{6} = 117.75\ cm^2$

As $\Delta\ OAB$ is equilateral Ar ($\Delta\ OAB$)

$= \frac{\sqrt{3}}{4}(15)^2 = \frac{1.73\times 225}{4} = 97.31$

Now, area of minor segment $= 117.75 - 97.31 = 20.44\ cm^2$

Area of the major segment $= \pi(15)^2 - 20.44$

$= 3.14\times 225 - 20.44 = 686.06\ cm^2$

3. Here, side of square = 14 cm and

radius of each circle = 7 cm

Area of the shaded region

= Area of square – Area of four quadrants of the circle

= Area of square – Area of one full circle

$= 14\times 14 - \pi\times(7)^2$

$= 196 - \frac{22}{7}\times 7\times 7 = 196 - 154 = 42\ cm^2$

4. $21.5\ cm^2$

5. 77 cm, 63 cm;

[**Hint.** $2r_1 + 2r_2 = 2.8 \Rightarrow r_1 + r_2 = 1.4$

$2\times\frac{22}{7}(r_1 - r_2) = 0.88 \Rightarrow r_1 - r_2 = 0.14]$

6. $\frac{154}{3}\ cm^2$;

[**Hint.** The angle through which the minute hand rotates in 5 minutes $= \frac{5}{60}\times 360° = 30°$

$\therefore$ The area swept $= \frac{30}{360}\times\frac{22}{7}\times 14^2\ \text{cm}^2 = \frac{154}{3}\ \text{cm}^2$]

7. $\frac{65}{3}$ cm;

[**Hint.** Length of arc $APB = \frac{240}{360}\times 2\times\frac{22}{7}\times\frac{7}{2} = \frac{44}{3}$ cm

Required length $= \left(\frac{7}{2}+\frac{7}{2}+\frac{44}{3}\right)\text{cm} = \frac{65}{3}$ cm.]

8. $20.4375\ \text{cm}^2$; $686.0625\ \text{cm}^2$

9. Let r_1 = radius of bigger circle,
r_2 = radius of smaller circle.

As given $\pi r_1^2 + \pi r_2^2 = 180\pi$. ...(1)

$\Rightarrow r_1^2 + r_2^2 = 180$ and

distance between centers i.e.,

$r_1 - r_2 = 6 \Rightarrow r_2 = r_1 - 6$

From the equation (1), $r_1^2 + (r_1 - 6)^2 = 180$

$\Rightarrow r_1^2 + (r_1^2 - 12r_1 + 36) = 180$

$\Rightarrow 2r_1^2 - 12r_1 + 36 = 180 \Rightarrow 2r_1^2 - 12r_1 - 144 = 0$

$\Rightarrow r_1^2 - 6r_1 - 72 = 0$

$\Rightarrow (r_1 - 12)(r_1 + 6) = 0$. Hence, $r_1 = 12$ cm and $d_1 = 24$ cm.

So, diameter of the bigger circle = 24 cm.

10. Each chord of the circle with radius a, subtend an angle of 60° at the centre; and make an equilateral triangle with centre of circle as vertex. So, each side = a.

Since, on each chord an equilateral triangle is made, there are total 12 such triangles.

Area of one equilateral triangle $= \frac{\sqrt{3}}{4}a^2$.

Total area $= \frac{\sqrt{3}}{4}a^2 \times 12 = 3\sqrt{3}a^2$.

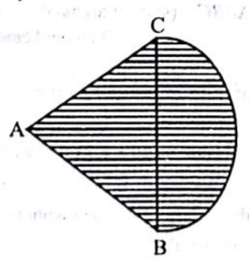
11. $88.44\ \text{cm}^2$.

12. If the sector makes an angle θ (in degrees) at the centre of the circle, then

$$l = \frac{?}{360}\times 2pr \quad\Rightarrow\quad \frac{?}{360}\times pr = \frac{1}{2}l$$

Now, area of sector $= \frac{?}{360}\times pr^2 = \frac{?}{360}\times pr\times r = \frac{1}{2}lr$

13. Area of shaded region
= Area of equilateral ΔABC + Area of the semicircle



$= \frac{\sqrt{3}}{4}\times 14\times 14 + \frac{1}{2}\times\frac{22}{7}\times 7\times 7$

$= \sqrt{3}\times 7\times 7 + \frac{11}{7}\times 7\times 7$

$= \sqrt{3}\times 49 + 77$

$= 1.732\times 49 + 77$

$= 161.87\ \text{cm}^2$

14. Let OA = x metres,

In ΔOAB, $\angle AOB = 90° \Rightarrow x^2 + x^2 = 56^2 \Rightarrow x^2 = 28\times 56$

Required area = 2 (area of sector OAB + area of ΔBOC)

$= 2\left(\frac{1}{4}\pi x^2 + \frac{1}{2}x\times x\right)\text{m}^2$

$= \left(\frac{1}{2}\times\frac{22}{7}\times 28\times 56 + 28\times 56\right)\text{m}^2$

$= 56(44+28)\text{m}^2 = 56\times 72\ \text{m}^2 = 4032\ \text{m}^2$

15. Length of wire $= 2\pi\times 42\text{cm} = 84\pi$ cm.

side of the square $= \left(\frac{1}{4}\times 84\pi\right) = 21\pi$ cm

$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi\times 42^2}{(21\pi)^2} = \frac{4}{\pi} = \frac{4\times 7}{22} = \frac{14}{11}$.

16. $14\ \text{cm}^2$

17. Interior angle of hexagon $= \frac{(2\times 6-4)}{6}\times 90 = 120°$

Area of hexagon $= \frac{1}{2}\times 6\times 6\times\frac{\sqrt{3}}{2}\times 6 = 54\sqrt{3}$ sq. cm

Area of a hexagon occupied by circles

$= 2\times\frac{120}{360}\times\pi\times 6^2 = 24\pi$ sq. cm

Area of a shaded region

$= 54\sqrt{3} - 24\pi = 6(9\sqrt{3} - 4\pi)$ sq. cm.

LONG ANSWER QUESTIONS :

1. (i) Area of the grassed enclosure

$= PQ\times PT + 2\times\frac{1}{2}\times\pi\times r^2$

$\left[\because r = \frac{PT}{2} = \frac{70}{2} = 35\text{m}\right]$

$= 200\times 70 + \frac{22}{7}\times 35\times 35$

$= 14000 + 3850$

$= 17850\ \text{m}^2$.

(ii) Outer perimeter of $ABCDEF$

$= AFE + ED + DCB + BA$

$= 2AFE + 2ED$

$= 2\times\frac{1}{2}(2\pi r)+2\times 200$

(where $r = 35+7 = 42$ m)

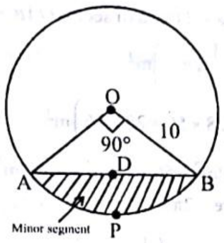
$= 2\times\frac{1}{2}\times\frac{2\times 22\times 42}{7}+400$ m

$= 12\times 22+400$ m

$= (264+400)$ m

$= 664$ m.

2. Radius of circle, $r = 10$ cm.



Area of minor segment $= \left[\frac{\pi r^2\theta}{360}-\frac{1}{2}r^2\sin\theta\right]$ where θ is the angle subtended at the centre of the circle.

$= \frac{3.14\times 10\times 10\times 90}{360}-\frac{1}{2}\times 10\times 10\times \sin 90°$

$= \frac{314}{4}-\frac{1}{2}(100)\times 1 = 78.5-50 = 28.5\text{ cm}^2$

Area of circle $= \pi r^2 = 3.14\times 10\times 10 = 314\text{ cm}^2$

Area of major segment = Area of circle − Area of minor segment

$\therefore$ Area of major segment $= (314-28.5)\text{ cm}^2 = 285.5\text{ cm}^2$

3. Let the radius of the circle be r.

The side of triangle $= 2r$

$\therefore$ Area of triangle $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(2r)^2 = \sqrt{3}r^2$

But area $= 17320.5\text{ m}^2$ [Given]

$\therefore\quad \sqrt{3}r^2 = 17320.5$

$\therefore\quad r^2 = \frac{17320.5}{\sqrt{3}} = \frac{17320.5}{1.732} = 10{,}000$

$\therefore\quad r = \sqrt{10{,}000} = 100$m

Area of the 3 sectors in the triangle

$= \frac{1}{2}$ (area of circle) $= \frac{1}{2}\times 3.14\times 100^2$

$= 1.57\times 10{,}000 = 15{,}700\text{ m}^2$

$\therefore$ Area of triangle not included in circles

$= 17{,}320.5 - 15700 = 1620.5\text{ m}^2$

4. Given : length of the sheet = 11 cm

Breadth of the sheet = 2cm

Diameter of the circular piece = 0.5 cm

Radius of the circular piece $= \frac{0.5}{2} = 0.25$ cm

Now, area of the sheet = length × breadth

$= 11\times 2 = 22\text{ cm}^2$.

Area of a circular disc $= \pi r^2$

$= \frac{22}{7}\times(0.25)^2\text{ cm}^2$

Number of circular discs formed

$= \frac{\text{Area of the sheet}}{\text{Area of one disc}}$

$= \frac{22}{\frac{22}{7}\times(0.25)^2}$

$= \frac{22\times 7}{22\times 0.0625} = 112$

Hence, 112 discs can be formed.

5. Let ABC be an equilateral triangle with side a, then

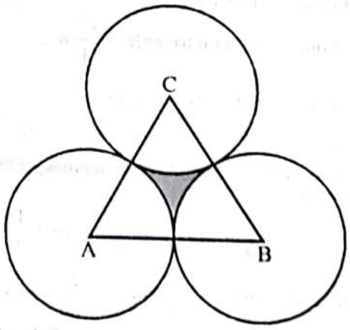
$\frac{\sqrt{3}}{4}a^2 = 49\sqrt{3}\text{ cm}^2$

$\therefore\quad a^2 = 4\times 49\text{ cm}^2 \Rightarrow a = 14$ cm

Each angle of equilateral $\Delta ABC = 60°$.

Circles with radius $\frac{1}{2}$ of side of ΔABC

i.e., $\frac{1}{2}$ of 14 cm i.e., 7 cm are drawn with angular points.



A, B and C as centres.

Required area = area of shaded region

= area of ΔABC − (sum of areas of 3 sectors with radius 7 cm and central angle of 60°)

$= 49\sqrt{3}\text{ cm}^2 - 3\times\frac{60}{360}\times\frac{22}{7}\times 7^2\text{ cm}^2$

$= \left(49\sqrt{3}-77\right)\text{cm}^2 = (49\times 1.73-77)\text{cm}^2$

$= 7.77\text{cm}^2$.

6. Let r cm be the radius of the hemisphere and h cm be the height of the conical part.

Length of the girth $= 2\pi r = 44$ cm.

$r = \dfrac{44 \times 7}{22 \times 2}$ cm $= 7$cm

$\Rightarrow h + r = 30$ cm $\Rightarrow h = 30 - 7$ cm. $= 23$ cm.

Vol. of the vessel = Vol. of hemisphere + Vol. of cone

$= \dfrac{2}{3}\pi r^3 + \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi r^2(2r + h)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 7^2 \times (2 \times 7 + 23)$ c.c. $= 1899.33$ c.c.

Slant height, $\ell = \sqrt{23^2 + 7^2}$ cm $= \sqrt{578}$ cm $= 24.04$ cm.

Total surface area = curved surface of hemisphere + curved surface of cone $= 2\pi r^2 + \pi r\ell = \pi r\,(2r + \ell)$

$= \dfrac{22}{7} \times 7 \times (14 + 24.04)$ sq. cm $= 836.88$ sq. cm.

7. Let $AO = OC = r$. Then :

(i) Area of the shaded portion
= Area of quadrant AOB + Area of quadrant COD

$\Rightarrow \dfrac{\pi r^2}{4} + \dfrac{\pi r^2}{4} = 308$ (Given)

$\Rightarrow \dfrac{2\pi r^2}{4} = 308$

$\Rightarrow \dfrac{\pi r^2}{4} = \dfrac{308}{2}$

$\Rightarrow r^2 = \dfrac{308 \times 4 \times 7}{2 \times 22} = 196$

$\Rightarrow r = \sqrt{196} = 14$ cm

$\therefore AC = 2r = 2 \times 14 = 28$ cm

(ii) Circumference $= 2\pi r = 2 \times \dfrac{22}{7} \times 14 = 88$ cm.

8. $BC^2 = AB^2 + AC^2 \Rightarrow 10^2 = 6^2 + AC^2 \Rightarrow AC = 8$ cm.

Join AI, BI and CI. Let r cm be the radius of incircle.

Area of ΔIAB + area of ΔIBC + area of ΔICA = area of ΔABC.

$\dfrac{1}{2}(6 \times r + 10 \times r + 8 \times r) = \dfrac{1}{2} \times 6 \times 8 \Rightarrow r = 2$

Required area = area of ΔABC − area of incircle

$= \left(\dfrac{1}{2} \times 6 \times 8 - \dfrac{22}{7} \times 2^2\right)$ cm^2

9. (a) R (radius of circumcircle) $= \dfrac{2}{3}h$ (height of Δ) and $h = \dfrac{\sqrt{3}}{2}a$ (side of Δ)

$\Rightarrow R = \dfrac{2}{3} \times \dfrac{\sqrt{3}}{2}a = \dfrac{1}{\sqrt{3}}a \Rightarrow 32 = \dfrac{1}{\sqrt{3}}a \Rightarrow a = 32\sqrt{3}$

Required area = area of circle − area of ΔABC

$= \pi \times 32^2 - \dfrac{\sqrt{3}}{4}\left(32\sqrt{3}\right)^2$

(b) $\angle AOC = 2.\angle ABC = 2 \times 60° = 120°$

$\Rightarrow$ required area $= \dfrac{1}{3}$ (area of circle − area of ΔABC).

10. (i) We have,

Diameter of the semi-circular particle = 2.8 cm

$\therefore r$ = Radius of the semi-circular particle = 1.4 cm

Path traced by the ant

$= \pi r + 2r = \left(\dfrac{22}{7} \times 1.4 + 2 \times 1.4\right)$cm

$= (22 \times 0.2 + 2.8)$ cm $= (4.4 + 2.8)$ cm $= 7.2$ cm

(ii) Clearly,

Path traced by the ant $= AB + BC + CD +$ arc AED

$= 1.5$ cm $+ 2.8$ cm $+ 1.5$ cm $+ (\pi \times 1.4)$ cm

$= \left(1.5 + 2.8 + 1.5 + \dfrac{22}{7} \times 1.4\right)$cm

$= (5.8 + 4.4)$ cm $= 10.2$ cm

(iii) We have,

Path traced by the ant $= OA +$ arc $ACB + OB$

$= (2 + \pi \times 1.4 + 2)$ cm

$= \left(4 + \dfrac{22}{7} \times 1.4\right)$cm $= (4 + 4.4)$ cm $= 8.4$ cm

Clearly, for (iii) food article ant will have to take a longer round.

11. We have,

Diameter of circular ends = 7 m

$\therefore r$ = Radius of circular ends $= \dfrac{7}{2}$ m $= 3.5$ m

Also,

l = Length of the rectangle $= \{20 - (3.5 + 3.5)\}$ m $= (20 - 7)$ m $= 13$ m

b = Width of the rectangle = 7 m

Perimeter of the garden $= 2l + \pi r + \pi r$

$= 2l + 2\pi r$

$= \left\{2 \times 13 + 2 \times \dfrac{22}{7} \times 7\right\}$m $= (26 + 44)$ m $= 70$ m

Area of the garden

= Area of rectangular part + Area of semi-circular ends

$= l \times b + \dfrac{1}{2}(\pi r^2) + \dfrac{1}{2}(\pi r^2)$

$= l \times b + \pi r^2$

$= \left\{13 \times 7 + \dfrac{22}{7} \times 7^2\right\}m^2 = (91 + 154)$ m$^2 = 245$m^2

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (a) Let the radii of the 2 circles be r_1 and r_2, then

$r_1 + r_2 = 15$ (given) ..... (1)

and $\pi r_1^2 + \pi r_2^2 = 153\pi$ (given) ..... (2)

$\Rightarrow \pi_1^2 + \pi_2^2 = 153$ $\therefore r_1^2 + (15 - r_1^2) = 153$

On solving, we get

$r_1 = 12,\ r_2 = 3$

Required ratio = 12 : 3 = 4 : 1

2. (c) Boundary of shaded region = Circumference of four semicircles (two circles, $r = 7/2$) + Circumference of two quarter circles (one semi-circles, $r = 7$)

$\Rightarrow (2 \times 2\pi r) + \pi \times 2r = (4 \times 22/7 \times 7/2) + (22/7 \times 7)$

$= 44 + 22 = 66$ cm.

3. (b) $2\pi r_1 = 503$ and $2\pi r_2 = 437$

$\therefore\ r_1 = \dfrac{503}{2\pi}$ and $r_2 = \dfrac{437}{2\pi}$

Area of ring = $\pi (r_1 + r_2)(r_1 - r_2)$

$= p\left(\dfrac{503+437}{2p}\right)\left(\dfrac{503-437}{2p}\right)$

$= \dfrac{940}{2}\left(\dfrac{66}{2\pi}\right) = 235 \times \dfrac{66}{2} \times 7 = 235 \times 21 = 4935$ sq. cm.

4. (b) Let the side of the original square be 'a' units. Therefore, the area of the original square = a^2 units. The diameter of the circle of maximum possible dimension that is cut from the square will be 'a' units. The diagonal of the square of maximum possible dimension that can be cut from the circle will be 'a' units.

If the diagonal of the final square is 'a' units, then its area $a^2/2$ units.

Therefore, the area of the new square will be 50% of the area of the original square.

5. (c) Should be XY since you divide XY into millions of congruent portions, each portion which is the diameter of the semicircle is very small. So the sum of all the arcs should be XY.

6. (b) $\pi d_1 + \pi d_2 = \pi d \Rightarrow d_1 + d_2 = d$

7. (a) Required area = $\left(7^2 - \dfrac{1}{4} \times \dfrac{22}{7} \times 7^2\right)$ cm^2

$= (49 - 38.5)$ cm^2.

8. (a) Let the radii of the outer and inner circles be r_1 and r_2 respectively; we have

Area = $\pi r_1^2 - \pi r_2^2 = \pi (r_1^2 - r_2^2)$

$= \pi (r_1 - r_2)(r_1 + r_2)$

$= \pi (5.7 - 4.3)(5.7 + 4.3) = \pi \times 1.4 \times 10$ sq. cm

$= 3.1416 \times 14$sq. cm. = 43.98 sq. cms.

9. (b) Area of sector = $240/360 \times \pi (100)^2 = 20933$ cm^2.

Let r be the radius of the new circle, then

$20933 = \pi r^2 \Rightarrow r = \sqrt{\dfrac{20933}{\pi}} = 81.6$ cm.

10. (d) **11.** (b)

12. (c) Joining B to O and C to O

Let the radius of the outer cirlce be r

$\therefore$ perimeter = $2\pi r$

But $OQ = BC = r$ [diagonals of the square BQCO)

$\therefore$ Perimeter of $ABCD = 4$r.

Hence, ratio = $\dfrac{2\pi r}{4r} = \dfrac{\pi}{2}$

13. (c) Radius of outer concentric circle = (35 + 7) m = 42 m.

Area of path = $\pi (42^2 - 35^2)$ m^2 = $\dfrac{22}{7}(42^2 - 35^2)$m^2]

14. (c) Perimeter = $\dfrac{1}{4} \times 2\pi r + 2r = \left(\dfrac{1}{2} \times \dfrac{22}{7} \times 7 + 2 \times 7\right)$cm

= 25 cm

MORE THAN ONE CORRECT :

1. (b, d)

Area of the circle = $\pi\left(\dfrac{7}{\sqrt{\pi}}\right)^2 = \dfrac{\pi(49)}{\pi} = 49$ cm^2.

Now, consider $\dfrac{154}{\pi} = \dfrac{154 \times 7}{22} = 49$

2. (a, c)

$\dfrac{\pi r_1^2}{\pi r_2^2} = \dfrac{25}{16}$

$\dfrac{r_1}{r_2} = \dfrac{5}{4}$

$\dfrac{2\pi r_1}{2\pi r_2} = \dfrac{5}{4} = \dfrac{5 \times 125}{4 \times 125} = \dfrac{625}{500}$

3. (b, d)

Area = $\dfrac{\theta}{360°} \times \pi r^2 = \dfrac{30}{360} \times \pi (7)^2 = \dfrac{49\pi}{12}$

Now, $\dfrac{121}{3\pi} = \dfrac{121 \times 7 \times \pi}{3 \times 22 \times \pi} = \dfrac{121}{3\pi}$

4. (b, c, d)

(a) Area = $\dfrac{?}{360} \times pr^2 = \dfrac{60}{360} \times \dfrac{22}{7} \times (6)^2 = \dfrac{132}{7}$ cm^2

(b) Area of minor sector = $\dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{60}{360} \times \dfrac{22}{7} \times 14 \times 14 = 102.57$ cm^2

Area of major sector

= Area of circle – Area of minor sector

$= \dfrac{22}{7}(14)^2 - 102.57$

$= 615.44 - 102.57 = 512.87$ cm^2

(c) $\frac{C}{A} = \frac{2\pi(5)}{\pi(5)^2} = \frac{2}{5}$ ⇒ length of arc = 22

(d) ⇒ $\left(\frac{\theta}{360}\right)2\pi r = 22$

$\therefore$ Area of sector $= \left(\frac{\theta}{360}\right)\pi r^2 = \left(\frac{\theta}{360}\right)\frac{\pi r}{2}(2r)$

$= \left(\frac{\theta}{360}\right)2\pi r\left(\frac{r}{2}\right) = \frac{22\times 6}{2} = 66\text{ cm}^2$

5. (a, b, c)

PASSAGE BASED QUESTIONS :

PASSAGE-I

1. (a) $\angle AOB = 60°$

Area of the sector $A'OB' = \frac{60}{360}\pi(6)^2 = 6\pi\text{ cm}^2$

2. (b) Area of shaded region

= area of circle + area of ΔOAB – area of sector $A'OB'$

$= \pi(6)^2 + \frac{\sqrt{3}}{4}(12)^2 - 6\pi = 36\pi + \frac{\sqrt{3}}{4}(144) - 6\pi$

$= 94.2 + 62.352 = 156.552\text{ cm}^2.$

ASSERTION & REASON :

1. (a) Both assertion and reason are correct. Also Reason is the correct explanation of the assertion.

$C = 2\times\frac{22}{7}\times r = 176$

$r = \frac{176\times 7}{2\times 22} = 28\text{ cm.}$

2. (a) Both assertion and reason are correct. Also, Reason is the correct explanation of the assertion.

Area of the path $= \pi\left[\left(\frac{10}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right]$

$= \pi(25-9) = 16\pi$

3. (d) Assertion is not correct, but reason is true.

$2\pi r = 22$

$r = 3.5\text{ cm}$

$\therefore$ Area of the circle $= \frac{22}{7}\times 3.5\times 3.5 = 38.5\text{ cm}^2.$

MULTIPLE MATCHING QUESTIONS :

(A) → q; (B) → p, s; (C) → p, s; (D) → q

(A) Area of the shaded region $= \frac{90}{360}\pi\left[(26)^2 - (23)^2\right]$

$= \frac{1}{4}\times\frac{22}{7}\left[(26)^2 - (23)^2\right] = 115.5 \simeq 115\text{ m}^2$

(B) $AB = 2\times AD = 2\times 6.5 = 13\text{ cm}$

$BC = \sqrt{(AB)^2 - (AC)^2} = \sqrt{(13)^2 - (5)^2} = 12\text{ cm}$

Area $(\Delta ABC) = \frac{1}{2}\times AC\times BC = \frac{1}{2}(5)(12) = 30\text{ cm}^2$

$= 0.003\text{ m}^2$

(C) $(AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$

$AC = 10\text{ cm}$

Area of the shaded region

= (area of the circle) – (area of the rectangle $ABCD$)

$= \left[\frac{22}{7}\times\left(\frac{10}{2}\right)^2\right] - (8\times 6)$

$= (78.57 - 48) = 30.57\text{cm}^2$

(D) Area of the shaded region $= \frac{60}{360}\times\frac{22}{7}\times(14.8)^2$

$= 114.7 \simeq 115\text{ m}^2$

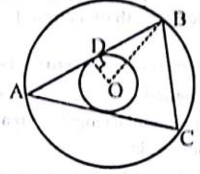
HOTS SUBJECTIVE QUESTIONS :

1. Area of outer circle $= \pi.OB^2$

and area of inner circle $= \pi\times OD^2$

As given $\pi\times OB^2 = 4\times\pi OD^2$ ⇒ $\frac{OD^2}{OB^2} = \frac{1}{4}$ ⇒ $\frac{OD}{OB} = \frac{1}{2}$

$\sin\angle OBD = \frac{OD}{OB} = \frac{1}{2}$ $\therefore \angle OBD = 30°$



⇒ $\angle ABC = 60°$. [Since, $\angle ABC = 2\angle OBD$]

Hence, ΔABC is equilateral.

The next step is to find side AB and OD.

Since $\pi r^2 = 12$, *we get*, $r = OB = \sqrt{\frac{12}{\pi}}$ *and* $OD = \frac{1}{2}\sqrt{\frac{12}{\pi}}$

Using Pythagoras theorem, we get,

$DB^2 = OB^2 - OD^2 = \frac{12}{\pi} - \frac{1}{4}\times\frac{12}{\pi} = \frac{9}{\pi}$; ⇒ $DB = \frac{3}{\sqrt{\pi}}$

Hence, side $AB = 2\times\frac{3}{\sqrt{\pi}} = \frac{6}{\sqrt{\pi}}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}\times\text{side}^2 = \frac{\sqrt{3}}{4}\times\frac{36}{\pi} = \frac{9\sqrt{3}}{\pi}$.

2. Radius of the bigger circle = R.

Radius of each of the three inscribed circle = 2 cm. A, B, C are centres of these circles. ABC is an equilateral triangle of each side = 2 + 2 = 4 cm.

Centre O of the bigger circle is also the centre of the equilateral triangle ABC.

$AO = R - 4$, and $AO = \frac{2}{3}$ rd of the median of the ΔABC.

Median $= \frac{4\sqrt{3}}{2} = 2\sqrt{3}$, So, $AO = \frac{2}{3} \times 2\sqrt{3}$.

$AO = \frac{4\sqrt{3}}{3} = R - 2 \Rightarrow R = \frac{4\sqrt{3}}{3} + 2$

$\Rightarrow R = \frac{6+4\sqrt{3}}{3} = \frac{2(3+2\sqrt{3})}{3}$

Area of the shaded region = Area of the bigger circle – Total area of the unshaded portion.

Area of the bigger circle $= \pi \times R^2 = \pi\left\{\frac{2(3+2\sqrt{3})}{3}\right\}^2$

Area of equilateral $\Delta ABC = \frac{\sqrt{3}}{4} \times 4^2 = 4\sqrt{3}$.

Area of three shaded sectors $= 3 \times \frac{1}{6}\pi \times 2^2 = 2\pi$.

[Since, each sector is $\frac{1}{6}$ th of a circle.]

Area of three circles (radius = 2 cm) $= 3 \times \pi \times 2^2$

Total area of unshaded portion = Area of unshaded region in the centre of equilateral triangle + Area of unshaded region in the three smaller circle.

So, area (unshaded) = (Area of equilateral triangle–Area of three shaded sectors) + (Area of three circles–area of three shaded sectors)

So, total area of unshaded sector

$= \left(4\sqrt{3} - 2p\right) + \left(3 \times p \times 2^2 - 2p\right)$

$= 4\sqrt{3} - 2p + 12p - 2p = 4\sqrt{3} + 8p$

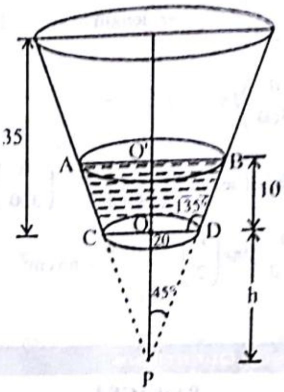
$\Rightarrow$ Area (shaded region) $= p\left\{\frac{2(3+2\sqrt{3})}{3}\right\}^2 - 4\sqrt{3} - 8p$

$= p\left\{\frac{4(9+12+12\sqrt{3})}{9}\right\} - 4\sqrt{3} - 8p$.

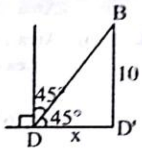
$= p\left\{\frac{84+48\sqrt{3}}{9}\right\} - 4\sqrt{3} - 8p = \left(\frac{84+48\sqrt{3}-72}{9}\right)p - 4\sqrt{3}$.

$= \left(\frac{12+48\sqrt{3}}{9}\right)\pi - 4\sqrt{3} = \left[\left(\frac{4+16\sqrt{3}}{3}\right)\pi - 4\sqrt{3}\right]$ sq. units.

3. (i)



Draw a perpendicular from B to the horizontal extended line CDD'.



$\tan 45° = 1 = \frac{BD'}{x} = \frac{10}{x} \Rightarrow x = 10$

$\therefore O'B = OD + 10 = 20 + 10 = 30$.

In ΔPOD and $PO'B$

$\frac{OD}{O'B} = \frac{OP}{O'P} \Rightarrow \frac{20}{30} = \frac{h}{10+h} \Rightarrow 10h = 200$ or $h = 20$ cm

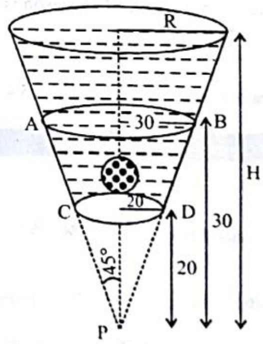
Amount of water in bucket = Volume of cone ABP – Volume of cone CDP

$= \frac{1}{3}\pi[O'B^2(OO'+h) - OD^2(h)]$

$= \frac{1}{3}p[30^2 \times 30 - 20^2 \times 20]$

$= \frac{1}{3}p[27000 - 8000] = 19000\frac{p}{3}$.

(ii) Volume of ball $= \frac{4}{3}p(5\sqrt[3]{74})^3 = \frac{4}{3}p \times 125 \times 74$



Let the height reached to H level from point P. This increase is due to the volume of the ball.

Base radius at that point = $R = H$

($\because$ Base angle of cone is 45°)

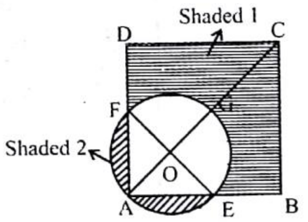
Volume of cone till height H – Volume of cone till height B = Volume of ball

$$\Rightarrow \frac{1}{3}\pi[H^2 \times H - 30^2 \times 30] = \frac{4}{3}\pi \times 125 \times 74$$

$$\Rightarrow H^3 - 27000 = 37000 \Rightarrow H = \sqrt[3]{64000} = 40\text{ cm}.$$

$\therefore$ Height of water in the bucket = 40 – 20 = 20 cm.

4. (i)



In ΔFAE, $\angle FAE = 90°$ as it is an angle of a square. Further EF will be the diameter of the circle as an angle subtended by a diameter on the circumference of a circle = 90°. So, EF will pass through the centre O.

In ΔFOA and ΔAOE

$OF = OE$ (radius of circle)

$AO = AO$ (common)

$\angle FAO = \angle EAO = 45°$

So $\Delta FOA \sim \Delta AOE \Rightarrow AF = AE$

Area of $\Delta AFE = \frac{1}{2}\text{base} \times \text{height} = \frac{1}{2}AF^2$

$AF^2 + AE^2 = FE^2 = 2^2 = 4$

(Pythagorus theorem)

or $AF^2 = 2 \Rightarrow$ Area of $\Delta AFE = \frac{1}{2} \times 2 = 1$

Area of the part of the circle falling outside the square sheet

= Area of shaded region 2 $= \frac{\pi r^2}{2}$ – area of

$\Delta AFE = \frac{\pi r^2}{2} - 1 = \frac{\pi \times 1^2}{2} - 1 = \frac{\pi - 2}{2}$ [$\because r = 1$]

(ii) Area of the white portion of circle =

$$\pi r^2 - \frac{\pi - 2}{2} = \frac{2\pi - (\pi - 2)}{2} = \frac{\pi + 2}{2}$$

$\therefore$ Area of shaded region 1 = Area of sq. $ABCD$ – Area of white portion of circle

$$= 2^2 - \frac{\pi + 2}{2} = \frac{6 - \pi}{2}$$

Hence, required proportion of the sheet

$$= \frac{\left(\frac{6-\pi}{2}\right)}{4} = \frac{6 - \pi}{8}$$

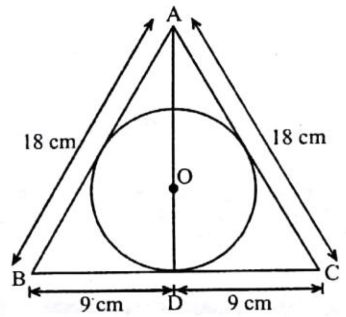
5. Let ABC be an equilateral triangle of side 18 cm

$\Rightarrow AB = BC = CA = 18$ cm

Draw $AD \perp BC$

$\Delta ADB = \Delta ADC$ [By R.H.S. criterion of congruence]

$\Rightarrow BD = CD$ [CPCT]



$\Rightarrow BD = CD = \frac{1}{2}BC = \frac{1}{2} \times 18\text{ cm} = 9\text{ cm}.$

Since, the centre of the inscribed circle coincides with the centroid of the equilateral triangle.

$\therefore$ radius of incirlce $OD = \frac{1}{3}AD$ [$\because AO : OD = 2 : 1$].

Now, in right ΔABD

$AB^2 = AD^2 + BD^2$

[By pythagoras theorem]

$\Rightarrow (18)^2 = AD^2 + 9^2 \Rightarrow AD^2 = (18)^2 - 9^2 = 324 - 81 = 243$

$\Rightarrow AD = +\sqrt{81 \times 3} = 9\sqrt{3}$ cm $\Rightarrow OD = \frac{1}{3} \times 9\sqrt{3} = 3\sqrt{3}$

Area of incircle $= \pi r^2 = \frac{22}{7} \times (3\sqrt{3})\text{ cm}^2$

$\left[\because r = OD = \frac{1}{3}AD = \frac{1}{3} \times 9\sqrt{3} = 3\sqrt{3}\text{ cm}\right]$

$$= \frac{22}{7} \times 27\text{ cm}^2 = 84.86\text{ cm}^2$$

Area of equilateral ΔABC

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}\times(18)^2 = \sqrt{3}\times 81\text{ cm}^2$

$= 1.732\times 81 = 140.29\text{ cm}^2$.

Hence area of the remaining portion of the triangle $= (140.29 - 84.86)\text{cm}^2 = 55.43\text{ cm}^2$.

6. (i) Let $OA = x$ cm, then $OD = x$ cm.

Radius of circle $= OP = OQ = OD + DQ = (x+25)$ cm

In ΔPAO, $\angle PAO = 90°$. By Pythagoras theorem,

$OP^2 = AP^2 + OA^2$

$\Rightarrow (x+25)^2 = 45^2 + x^2 \Rightarrow x = 28.$

$\therefore$ Radius of circle $= (28+25)$ cm $= 53$ cm.

(ii) In ΔAOD, $\angle AOD = 90°$

$\Rightarrow AD^2 = OA^2 + OD^2 = x^2 + x^2$

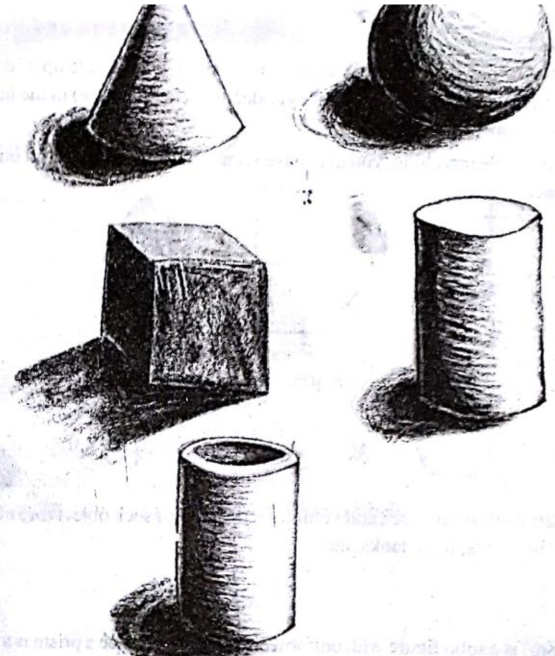
$\Rightarrow AD^2 = 2x^2 \Rightarrow AD = \sqrt{2}x \Rightarrow AD = 28\sqrt{2}$ cm

$\Rightarrow AD = 28\times 1.41$ cm $= 39.48$ cm.

(iii) Required area = area of circle – area of square $ABCD$

$= \left(3.14\times 53^2 - \left(28\sqrt{2}\right)^2\right)\text{cm}^2.$

chapter 13



SURFACE AREAS AND VOLUMES

Introduction

In earlier classes you have studied little about solids like cuboid, cone, cylinder, sphere etc.

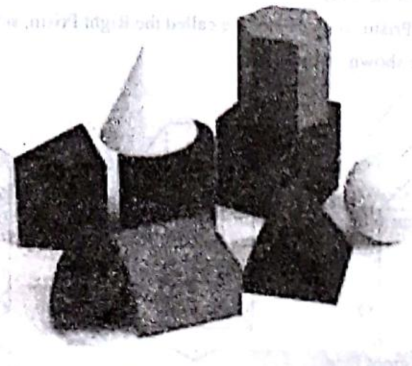


Figure : Various solids

In this chapter, you will study to find the surface area and volume of these solids.

In our day-to-day life, we come across a number of solids made up of combinations of two or more of the basic solids. You must have seen test tube (combination of cylinder and a hemisphere) in the laboratory, top (lattu) in play ground (combination of cone and a hemisphere).

Similarly, while travelling, you may have seen some big and beautiful buildings or monuments made up of a combination of solids mentioned above.



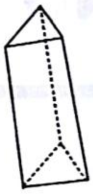
Calculation of surface area and volumes (capacity) of such objects are of great importance in our practical life, like in manufacturing of furnitures, toys, tanks, etc.

PRISM :

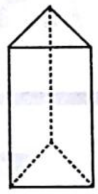
A 'Prism' is a solid figure with uniform cross section. Hence a prism is a three dimensional solid in which top and bottom faces are identical planes. The lateral faces of the prism are rectangles. The top and bottom faces are also called bases.

Right Prism:

A prism is called Right Prism, if its lateral edges are perpendicular to the top and bottom faces (or both bases).



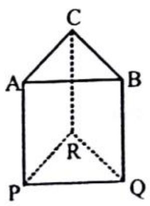
(i) Triangular Prism [Triangular bases but lateral edges are not perpendicular to the bases]



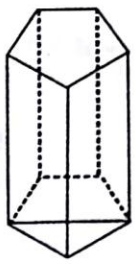
(ii) Triangular Right Prism [Triangular bases and lateral edges are perpendicular to the bases]

Here we shall discussed the Right Prisms only. Hence we called the Right Prism, simply as Prism.

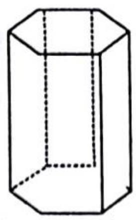
Some important types of Prism are shown below :



Triangular prism (Top and bottom face are triangular)

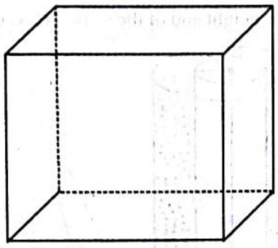


Pentagonal Prism (Top and bottom faces are pentagonal)

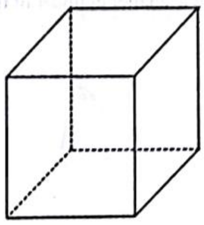


Hexagonal prism (Top and bottom faces are hexagonal)

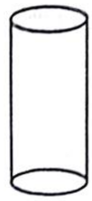
Square prism or Cuboid (Top and bottom faces are squares)



Cuboid (Top and bottom faces are rectangles)



Cube (All faces are square)



Cylinder (Topand bottom faces in faces are circles)

In each Prism,

(i) The number of lateral faces = the number of sides of the a base.

(ii) The number of edges of a prism = (number of sides of the a base) × 3.

(iii) The sum of the lengths of the edges = 2 (perimeter of base) + (number of lateral sides) × height

Lateral surface area (LSA) of a prism :

L.S.A = (Perimeter of a base) × height = ph

Total surface area (TSA) of a prism :

T.S.A = L.S.A + 2 (area of a base)

Volume of a prism :

Volume = Area of a base × height = Ah.

(i) Cube:

In a right prism, if the top and bottom faces (or bases) are square, then it is called a cube.

In a cube, all the three dimensions i.e. its length, breadth and height are equal.

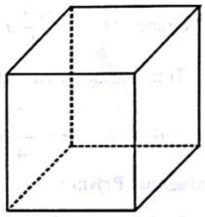
If 'a' be the edge of a cube, then

Volume = a^3

Total surface area = $6a^2$

Diagonal of a cube = $\sqrt{3}$ × edge = $\sqrt{3}\,a$

Edge of a cube = $\sqrt[3]{\text{volume}}$



(ii) Cuboid:

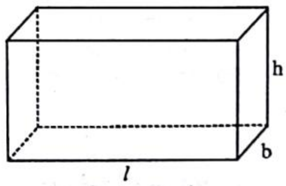
In a right prism, if the bases are rectangles, then it is called a cuboid.

If 'l' be the length, 'b' be the breadth and 'h' be the height (or depth) of a cuboid, then

Volume = length × breadth × height = $l \times b \times h$

Total surface area = $2(lb + bh + hl)$

Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$



(iii) Cylinder:

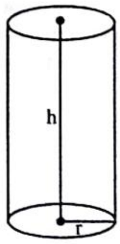
In a right prism, if the top and bottom faces (or bases) are circles, then it is called a cylinder. Thus a cylinder has two congruent and parallel circular planes which are connected by a curved srface. Each of the circular planes is called a base of the cylinder.

(a) Solid Cylinder: A solid cylinder is the region bounded by a cylinder i.e. if there is no air or vacuume inside the cylinder, then the cylinder is called solid cylinder.

If 'r' is the radius of the base 'h' is the height of the cylinder, then

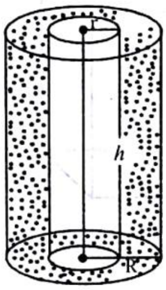
(i) Area of curved surface = $2\pi rh$ (ii) Total surface area = $2\pi r(h + r)$

(iii) Volume of the cylinder = $\pi r^2 h$



(b) Hollow Cylinder :

A cylinder from which a smaller cylinder of the same height and of the same axis is cut out is called a hollow cylinder.



A line passing through the top and bottom faces (or bases) is called the axis of the cylinder.

If 'r' & 'R' are the internal & external radii respectively and 'h' is the height, then

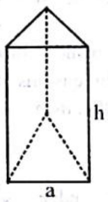
(i) Volume of the hollow cylinder $= (\pi R^2 - \pi r^2)h = \pi(R^2 - r^2)h$

(ii) Area of the curved surface $= 2\pi Rh + 2\pi rh = 2\pi(R + r)h$

(iii) Total surface area = area of curved surface + 2 (area of a base)

$$= 2\pi(R+r)h + 2\pi(R^2 - r^2) = 2\pi(R+r)h + 2\pi(R+r)(R-r) = 2\pi(R+r)(h+R-r)$$

(iv) Triangular Prism :



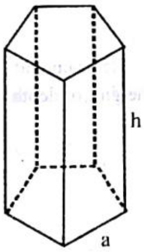
(i) Volume $= A \times h = \dfrac{\sqrt{3}}{4}a^2 \times h$

(ii) Total surface area = lateral surface area + sum of areas of two ends

$$= ah + ah + ah + \frac{\sqrt{3}}{4}a^2 = 3ah + \frac{\sqrt{3}}{2}a^2$$

(v) Pentagonal Prism :

(i) Surface area of pentagon $= \sqrt{3}a^2$



(ii) Volume $= \sqrt{3}a^2 \times h$

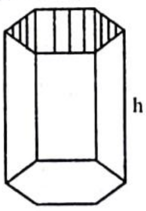
(iii) Total surface area $= 5ah + 2\sqrt{3}a^2$.

(vi) Hexagonal Prism :

(i) Surface Area of Hexagon $= 2.5981\, a^2$

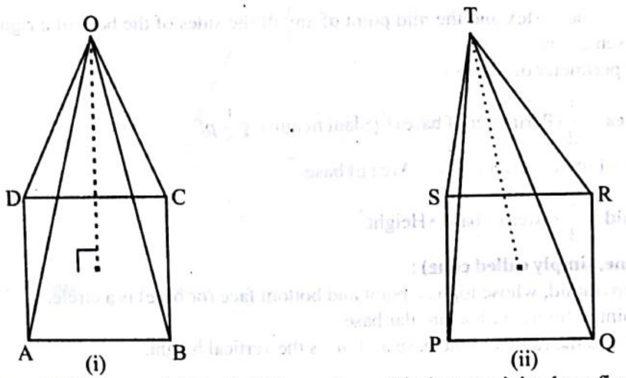
(ii) Volume $= A \times h = 2.5981 . a^2 \times h$

(iii) Total surface area $= 6ah + 2 \times 2.5981 a^2$



PYRAMID :

A 'Pyramid' is a three- dimensional figure made of a plane base (bottom face) and triangular lateral faces that meet at the vertex.

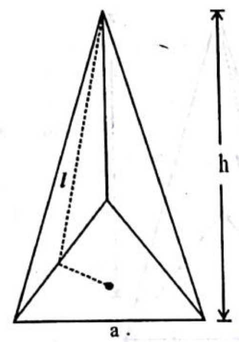


In the above figure, the base $ABCD$ is a quadrilateral. All the vertices of the base are joined to a fixed point O in space, by straight lines. The resultant solid obtained is called a pyramid.

Right pyramid : If the base of a pyramid is a regular polygon or circle and if the line joining the vertex to the centre of the base is perpendicular to the base, then the pyramid is called a right pyramid. In the above two figures, (i) is the Right Pyramid where as figure (ii) is not a Right Pyramid. Here we shall discussed the Right Pyramid only. Hence, we called the Right Pyramid, simply as Pyramid.

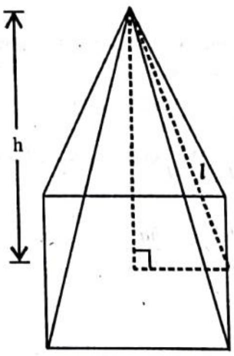
Types of Pyramid : Some important types of pyramid are shown below:

(i)



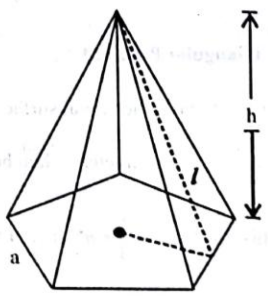
Triangular Pyramid (Base in an equilateral triangle)

(ii)



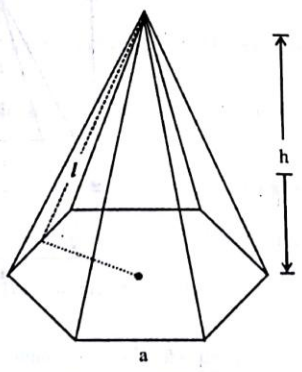
Square Pyramid (Base is a square)

(iii)



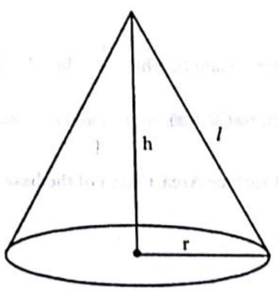
Pentagonal Pyramid (Base is a regular pentagon)

(iv)



Hexagonal Pyramid (Base is a regular hexagon)

(v)



Cone

In each Pyramid,

(i) The length of the line segment joining the vertex to the centre of the base is called the height of the pyramid and is represented by '*h*'.

(ii) The distance between the vertex and the mid point of any of the sides of the base of a right pyramid is called its slant height and is represented by '*l*'.

(iii) For a pyramid with perimeter of base $= p$

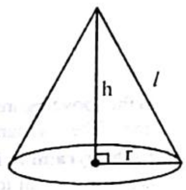
Lateral surface area $= \frac{1}{2}$(Perimeter of base)× (Slant height) $= \frac{1}{2}pl$

Total surface area = Lateral surface area + Area of base

Volume of a pyramid $= \frac{1}{3}$ ×Area of base ×Height.

(i) Cone (Right circular cone, simply called cone):

A cone is a kind of right pyramid, whose top is a point and bottom face (or base) is a circle.
Thus a cone is a solid pointed figure with a circular base.
If '*l*' is the slant height, '*r*' is the radius of the base and '*h*' is the vertical height.



(i) $l^2 = h^2 + r^2$

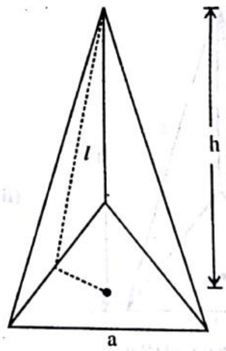
$l = \sqrt{h^2 + r^2}$ $\qquad$ $h = \sqrt{l^2 - r^2}$ $\qquad$ $r = \sqrt{l^2 - h^2}$

(ii) Area of curved surface $= \pi r l = \pi r\left(\sqrt{h^2 + r^2}\right)$

(iii) Total surface area = curved surface area + area of the base $= \pi r l + \pi r^2 = \pi r(l + r)$

(iv) Volume of the cone $= \frac{1}{3}\pi r^2 h$

(ii) Triangular Pyramid :



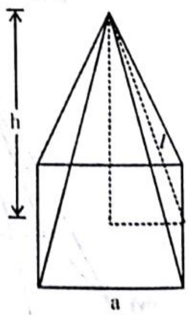
(i) Area of the lateral surface of the pyramid

$= \frac{1}{2} \times$ perimeter × slant height $= \frac{1}{2} \times 3a \times l = \frac{3}{2}al$

(ii) Volume $= \frac{1}{3} \times h \times$ area of base $= \frac{1}{3} \times h \times \frac{\sqrt{3}}{4}a^2 = \frac{ha^2}{4\sqrt{3}}$

(iii) Total Area of the pyramid $= \frac{1}{2}3al + \frac{\sqrt{3}}{4}a^2$

(iii) Square Pyramid :



(i) Volume $= \frac{1}{3}h \times$ area of base $= \frac{1}{3}h \times a^2$

(ii) Lateral surface area $= \frac{1}{2} \times$ perimeter × slant height $= \frac{1}{2} \times 4a \times l = 2al$

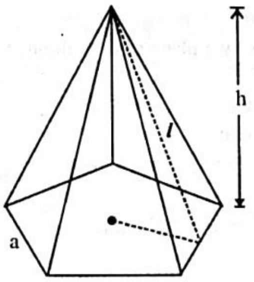
(iii) Total area of the pyramid $= 2al + a^2 = a(2l + a)$

(iv) Pentagonal Pyramid :

(i) Total area of the pyramid = Lateral surface Area + Area of the base

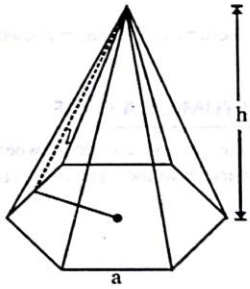
$$= \frac{1}{2} \times 5al + \sqrt{3}a^2$$

Where is the slant height



(ii) Volume $= \frac{1}{3} \times h \times$ area of base $= \frac{1}{3} \times h \times \sqrt{3}\, a^2 = \frac{ha^2}{\sqrt{3}}$

(v) Hexagonal pyramid :



(i) Total surface area $= \frac{1}{2} \times 6al + 2.5981a^2$

Where 'l' is the slant height

(ii) Volume $= \frac{1}{3} h \times$ area of base $= \frac{1}{3} h \times 2.5981a^2$

SPHERE :

Sphere is a set of points in the space which are equidistant from a fixed point. The fixed point is called the centre of the sphere and the distance is called the radius of the sphere. A foot ball, a cricket ball, the earth are some examples of sphere.

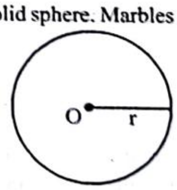
A line joining any two points on the surface of sphere and passing through the centre of the sphere is called its diameter.

(a) Solid Sphere:

A solid sphere is the region in space bounded by a sphere. The centre of a sphere is also a part of solid sphere. Marbles and leadshots are some examples of solid spheres.

Surface Area $= 4\pi r^2$ sq. units

Volume $= \frac{4}{3}\pi r^3$ cubic units



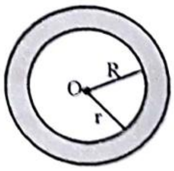
(b) Holow Sphere/Spherical Shell :

From a solid sphere, a smaller sphere having the same centre of the solid sphere, is cut off, then hollow sphere is obtained.

External Surface Area $= 4\pi R^2$ sq. units

Internal Surface Area $= 4\pi r^2$ sq. units

Volume $= \frac{4}{3}\pi\left(R^3 - r^3\right)$ cubic units



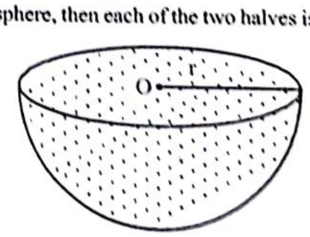
(c) Solid Hemisphere :

If a solid sphere is cut into two halves by a plane passing through the centre of sphere, then each of the two halves is called a solid hemisphere.

Curved Surface Area $= 2\pi r^2$ sq. units

Total Surface Area $= 3\pi r^2$ sq. units

Volume $= \frac{2}{3}\pi r^3$ cubic units



(d) Hemispherical Shell:

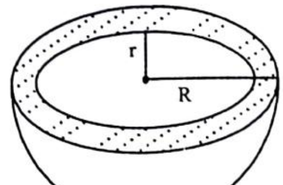
If a spherical shell is cut into two halves by a plane passing through the centre of spherical shell, then each of the two halves is called a hemisphere.

Internal Curved Surface Area = $2\pi r^2$ sq. units

External Curved Surface Area = $2\pi R^2$ sq. units

Total Surface Area = Internal Surface Area + Ext. Surface Area + Area of ring

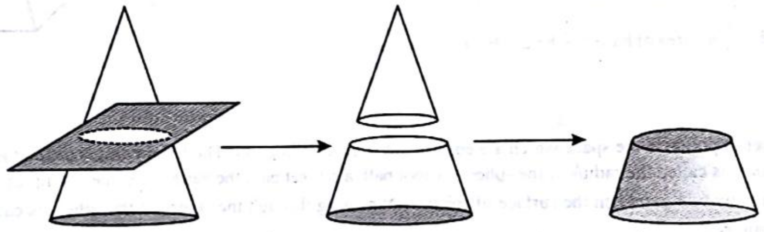
$= 2\pi r^2 + 2\pi R^2 + \pi(R^2 - r^2) = \pi r^2 + 3\pi R^2 = \pi(r^2 + 3R^2)$ units



Volume of the material used to form hemispherical shell = $\frac{2}{3}\pi\left(R^3 - r^3\right)$ cubic units

FRUSTUM OF A CONE :

The part of a cone between two planes parallel to the base of the cone. Especially the section between the base and a plane parallel to the base of the cone is called a frustum of a cone.



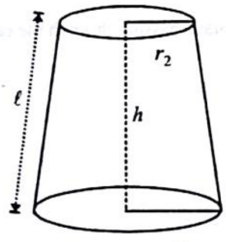
A Cone sliced by a plane parallel to base | The two parts separated | Frustum of a cone

Let h be the height, ℓ the slant height and r_1 and r_2 the radii of the ends ($r_1 > r_2$) of the frustum of a cone. Then we can directly find the volume, the curved surface area and the total surface area of frustum by using the formulae given below :

(i) Volume of the frustum of cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

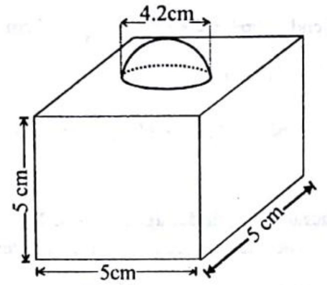
(ii) Curved surface area of the frustum of cone = $\pi(r_1 + r_2)\ell$ where $\ell = \sqrt{h^2 + (r_1 - r_2)^2}$

(iii) Total surface area of the frustum of the cone = $\pi\ell(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$, where $\ell = \sqrt{h^2 + (r_1 - r_2)^2}$



MISCELLANEOUS SOLVED EXAMPLES

1. The decorative block shown in figure is made of two solids-a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. (Take $\pi = 22/7$)



Sol. Side of cube $\ell = 5\text{cm}$.

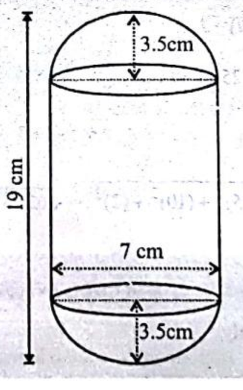
$\therefore$ Total surface area of cube $= 6\ell^2 = 6(5)^2 = 150 \text{ cm}^2$

Radius of hemisphere, $r = \dfrac{4.2}{2} = 2.1\text{cm}$.

Surface area of the block = TSA of cube + CSA of hemisphere – Base area of hemisphere

$= 150 + 2\pi r^2 - \pi r^2 = 150 + \pi r^2 = 150 + \dfrac{22}{7} \times 2.1 \times 2.1 = 150 + \dfrac{22}{7} \times 4.41 = 150 + 13.86 = 163.86 \text{ cm}^2$

2. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm. and the diameter of the cylinder is 7 cm. Find the volume and surface area of the solid.



Sol. Diameter of cylinder = 7 cm.

$\therefore$ Radius of cylinder $(r) = 3.5$ cm.

$\therefore$ Radius of each hemisphere end = 3.5 cm.

Height of solid = 19 cm.

$\therefore$ Height of cylinder $(h) = 19 - (3.5 + 3.5) = 19 - 7 = 12$ cm.

Volume of cylinder $= \pi r^2 h = \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \times 12 = 462 \text{ cm}^3$

Volume of each hemispherical end $= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2} = \frac{539}{6}\text{ cm}^3$

Total volume of solid $= 462 + \frac{539}{6} + \frac{539}{6} = \frac{2772+539+539}{6} = \frac{3850}{6} = 641.67\text{ cm}^3$

Curved surface area of cylinder $= 2\pi rh = 2\times\frac{22}{7}\times\frac{7}{2}\times 12 = 264\text{ cm}^2$

Curved surface area of each hemispherical end $= 2\pi r^2 = 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = 77\text{ cm}^2$

$\therefore$ Total surface area of solid $= 264 + 77 + 77 = 418\text{ cm}^2$

3. A closed cylindrical container, the radius of which is 7 cm and height 10 cm is to be made out of a metal sheet. Find
(i) the area of metal sheet required.
(ii) the volume of the cylinder made.
(iii) the cost of painting the lateral surface of the cylinder at the rate of ₹ 4 per cm².

Sol. (i) The area of the metal sheet required= The total surface area of the cylinder $= 2\pi r(r+h)$

$= 2\times\frac{22}{7}\times 7(7+10) = 44(17) = 748\text{ cm}^2.$

(ii) Volume $= \pi r^2 h = \frac{22}{7}\times 7\times 7\times 10 = 22\times 70 = 1540\text{ cm}^3$

(iii) To find the cost of painting the lateral surface, we need to find the curved (lateral) surface area.

$\therefore$ LSA $= 2\pi rh = 2\times\frac{22}{7}\times 7\times 10 = 440\text{ cm}^2$

Cost of painting $= 440\times 4 =$ ₹ 1760.

4. Find the surface area, volume and length of diagonal of a cuboid whose dimensions are 25 m, 10 m and 2 m.

Sol. Length $(\ell) = 25\ m$, breadth $(b) = 10\ m$, height $(h) = 2$ m
Surface area $= 2(\ell b + bh + \ell h)$
Surface area $= 2(25\times 10 + 10\times 2 + 25\times 2) = 2(250 + 20 + 50)$
Surface area $= 640\ m^2$
Volume of cuboid $= \ell\times b\times h = 25\times 10\times 2$
Volume of cuboid $= 500\ m^3$

Length of diagonal 'd' $= \sqrt{\ell^2+b^2+h^2} = \sqrt{(25)^2+(10)^2+(2)^2} = \sqrt{625+100+4} = \sqrt{729} = 27$ m

5. Volume of two cubes are in the ratio 1 : 27, find the ratio of their surface areas.

Sol. Let sides of two cubes be a_1 and a_2 respectively.

Volume of first cube $(V_1) = a_1^3$

Volume of second cube $(V_2) = a_2^3$

$$\therefore \quad \frac{V_1}{V_2} = \frac{a_1^3}{a_2^3} = \left(\frac{a_1}{a_2}\right)^3 = \frac{1}{27}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3} \qquad \ldots(1)$$

Now, surface area of first cube $(S_1) = 6a_1^2$, surface area of second cube $(S_2) = 6a_2^2$

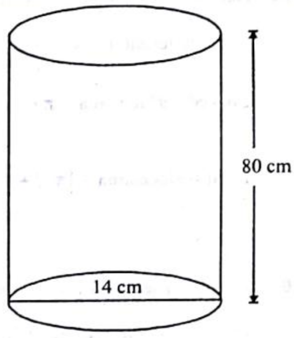
Now, $\dfrac{S_1}{S_2} = \dfrac{6a_1^2}{6a_2^2} = \left(\dfrac{a_1}{a_2}\right)^2$

$\dfrac{S_1}{S_2} = \left(\dfrac{1}{3}\right)^2$ (from equation (1))

$\dfrac{S_1}{S_2} = \dfrac{1}{9}$

$\therefore$ Ratio of surface area = 1 : 9.

6. The height of a cylinder is 80 cm and diameter of the base is 14 cm. Find its curved surface area, total surface area and volume.



Sol. Height $(h) = 80$ cm, Diameter of base = 14 cm

$\therefore$ radius of base $(r) = \dfrac{14}{2} = 7$ cm

$\therefore$ curved surface area of cylinder $= 2\pi rh$

$= 2 \times \dfrac{22}{7} \times 7 \times 80 = 44 \times 80 = 3520$ sq. cm

Total surface area (TSA) = C.S.A + Area of two ends

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r) = 2 \times \dfrac{22}{7} \times 7 \times (80 + 7) = 2 \times 22 \times 87$

T.S.A of cylinder = 3828 sq. cm

Volume (V) of cylinder $= \pi r^2 h = \dfrac{22}{7} \times (7)^2 \times 80 = 22 \times 7 \times 80 = 154 \times 80 = 12320$ cubic units.

7. By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder and the ratio of the height of the cylinder and the height of the cone is 4 : 3, find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively.

Then, $3r = 2R$ and $H : h = 4 : 3$ .... (1)

$\Rightarrow \dfrac{H}{h} = \dfrac{4}{3} \Rightarrow 3H = 4h$ .... (2)

Let n be the required number of cones which can be made from the materials of the cylinder. Then, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$$\pi R^2 H = \frac{n}{3}\pi r^2 h \Rightarrow 3R^2 H = nr^2 h$$

$$\Rightarrow n = \frac{3R^2 H}{r^2 h} = \frac{3 \times \frac{9r^2}{4} \times \frac{4h}{3}}{r^2 h} \qquad \left[\because \text{From (1) and (2), } R = \frac{3r}{2} \text{ and } H = \frac{4h}{3}\right]$$

$\Rightarrow n = \dfrac{3 \times 9 \times 4}{3 \times 4} = 9$. Hence, the required number of cones is 9.

8. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively, is melted into a cone of base diameter 8 cm. Find the height of the cone.

Sol. External radius of the hollow sphere $= \dfrac{8}{2} = 4$ cm, Internal radius of the hollow sphere $= \dfrac{4}{2} = 2$ cm

$\therefore$ Volume of the metal = Volume of external sphere – Volume of internal sphere

$= \dfrac{4}{3}\pi(4)^3 - \dfrac{4}{3}\pi(2)^3 = \dfrac{4}{3}\pi\left[4^3 - 2^3\right] = \dfrac{4}{3}\pi\ [64 - 8] = \dfrac{4}{3}\pi\ (56)\ \text{cm}^3$

Radius of the cone = $\frac{8}{2}$ cm = 4 cm

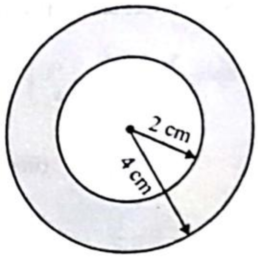
Let h be the height of the cone.

Then volume of the cone = $\frac{1}{3}\pi(4)^2 h$ cm^3

Since the metal of the spherical shell is to be converted into the conical solid,

$\therefore \frac{4}{3}\pi(56) = \frac{1}{3}\pi(16)h \Rightarrow h = \frac{4\times 56}{16} = 14$ cm.

Hence, the height of the cone is 14 cm.



9. Find the slant height, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Sol. Here, $r = 21$ cm and h = 28 cm.

$\therefore$ Slant height, $l = \sqrt{r^2+h^2} = \sqrt{(21)^2+(28)^2} = \sqrt{1225} = 35$ cm

Curved surface area = $\pi rl = \left(\frac{22}{7}\times 21\times 35\right)\text{cm}^2 = 2310\text{ cm}^2$

Total surface area = $\left(\pi rl + \pi r^2\right) = \left(2310 + \frac{22}{7}\times 21\times 21\right)\text{cm}^2 = 3696\text{ cm}^2$

10. A hemispherical tank of radius $1\frac{3}{4}$ m is full of water. It is connected with a pipe which empties it at the rate of 7 liters per second. How much time will it take to empty the tank completely ?

Sol. Radius of the hemisphere = $\frac{7}{4}\text{m} = \frac{7}{4}\times 100\text{cm} = 175\text{cm}$

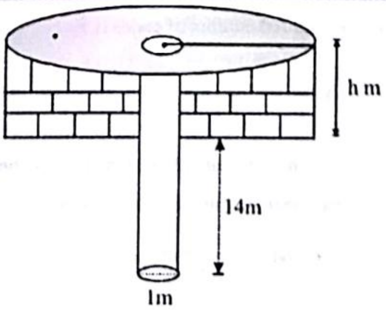
$\therefore$ Volume of the hemisphere = $\frac{2}{3}\times\pi\times 175\times 175\times 175\text{cm}^3$

The cylindrical pipe empties it at the rate of 7 litres i.e., 7000 cm^3 of water per second.

Hence, the required time to empty the tank = $\left(\frac{2}{3}\times\frac{22}{7}\times 175\times 175\times 175 \div 7000\right)$ sec

$$= \frac{2}{3}\times\frac{22}{7}\times\frac{175\times 175\times 175}{7000\times 60}\text{min} = \frac{11\times 25\times 7}{3\times 2\times 12}\text{min} = \frac{1925}{72}\text{min} \approx 26.74 \text{ min nearly}.$$

11. A well of diameter 2*m* is dug 14*m* deep. The earth taken out of it is spread evenly all around it to a width of 5m to form an embankment. Find the height of the embankment.



Sol. Let h m be the required height of the embankment.

The shape of the embankment will be like the shape of a cylinder of internal radius $1m$ and external radius $(5+1)m = 6m$ (fig.)

The volume of the embankment will be equal to the volume of the earth dug out from the well.

Now, the volume of the earth = volume of the cylindrical well

$= \pi \times 1^2 \times 14m^3 = 14\pi m^3$

Also, the volume of the embankment $= \pi(6^2 - 1^2)h\, m^3 = 35\pi h\, m^3$

Hence, we have $35\pi h = 14\pi \Rightarrow h = \frac{14}{35} = \frac{2}{5} = 0.4$

Hence, the required height of the embankment = 0.4 m.

12. Water in a canal, 30 dm wide and 12 dm deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes, if 8 cm. of standing water is required for irrigation.

Sol. Speed of water in the canal = 10 km/h = 10000 m/60 min = $\frac{500}{3}$ m/min.

$\therefore$ The volume of the water flowing out of the canal in 1 minute $= \left(\frac{500}{3} \times \frac{30}{10} \times \frac{12}{10}\right) m^3 = 600m^3$

$\therefore$ In 30 min, the amount of water flowing out of the canal $= (600 \times 30)\, m^3 = 18000\, m^3$

If the required area of the irrigated land is $x\, m^2$, then the volume of water to be needed to irrigate the land $= \left(x \times \frac{8}{100}\right) m^3 = \frac{2x}{25} m^3$

Hence, $\frac{2x}{25} = 18000 \Rightarrow x = 18000 \times \frac{25}{2} = 225000$

Hence, the required area is 225000 m^2

13. The earth dug out of a well is spread evenly all around it to form an embankment which is 2 m wide. Find the height of the embankment if the diameter of the well is 2m and its depth is 20m.

Sol. Given, diameter of well = $2m$ and depth of well = $20m$

$\therefore$ volume of earth dug out $= \pi\left(\frac{2}{2}\right)^2 \times 20 = 20\pi$

Given that the width of the embankment is $2m$.

Let the height of the embankment be h.

Then volume of embankment = external volume − internal volume $= \pi(3)^2 h - \pi(1)^2 h = 8\pi h$

But volume of embankment = volume of earth taken out

$\therefore \quad 8\pi h = 20\pi \qquad \therefore \quad h = \frac{20}{8} = \frac{5}{2} = 2.5m$

14. A piece of copper of volume 1m³ is melted down and recast into a bar which is 36m long and has a square cross-section. An exact cube is cut off from one end of the bar. Find the cost of this cube, if the original piece of copper cost ₹ 10,800.

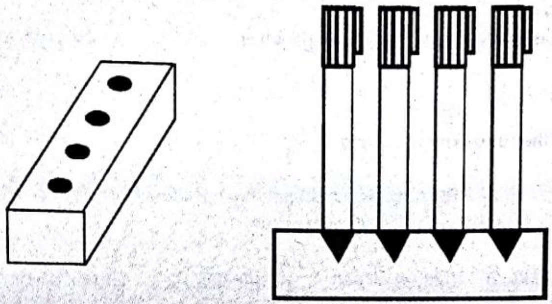
Sol. Given, volume of bar = $1m^2$ and length of bar = $36m$

$\therefore$ area of cross-section $= \frac{volume}{length} = \frac{1}{36} m^2 \qquad \therefore$ side of square cross-section $= \frac{1}{6} m$

$\therefore$ volume of cube cut off from bar $= \left(\frac{1}{6}\right)^3 = \frac{1}{216} m^3$

Cost of $1m^3$ of copper = ₹ 10,800 (Given) $\qquad \therefore$ cost of cube $= \frac{1}{216} \times 10800 =$ ₹ 50

15. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm. by 10 cm. by 3.5 cm. The radius of each of the depressions is 0.5 cm. and the depth is 1.4cm. Find the volume of wood in the entire stand.



Sol. Radius of the depression (conical cavity) = 0.5 cm.
and depth (i.e., vertical height) = 14 cm.

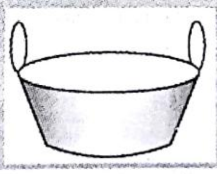
$\therefore$ Volume of wood taken out to make one conical cavity $= \frac{1}{3}\pi(0.5)^2(1.4)\text{ cm}^3 = \frac{11}{30}\text{ cm}^3$

Volume of wood taken out to make 4 such conical cavities $= 4 \times \frac{11}{30}\text{ cm}^3 = \frac{22}{15}\text{cm}^3$

Volume of wood in the pen stand = Volume of the cuboidal pen stand – volume of wood taken out to make 4 conical cavities

$$= (15 \times 10 \times 3.5) - \left(\frac{22}{15}\right)\text{cm}^3 = \left(525 - \frac{22}{15}\right)\text{cm}^3 = 523.53\text{ cm}^3.$$

16. Hanumappa and his wife Gangamma are busy making jaggery out of sugarcane juice. They have processed the sugarcane juice to make the molasses, which is poured into moulds in the shape of a frustum of a cone having the diameters of its two circular faces as 30 cm and 35 cm and the vertical height of the mould is 14 cm (see Fig.). If each cm^3 of molasses has mass about 1.2 g, find the mass of the molasses that can be poured into each mould. (Take $\pi = 22/7$)

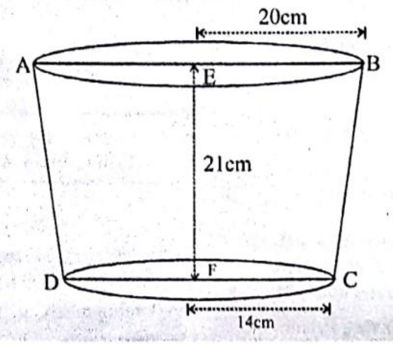


Sol. Since the mould is in the shape of a frustum of a cone, the quantity (volume) of molasses that can be poured into it is $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1r_2)$, where r_1 is the radius of the larger base and r_2 is the radius of the smaller base.

$$\frac{1}{3} \times \frac{22}{7} \times 14\left[\left(\frac{35}{2}\right)^2 + \left(\frac{30}{2}\right)^2 + \left(\frac{35}{2} \times \frac{30}{2}\right)\right]\text{cm}^3 = 11641.7\text{cm}^3$$

It is given that 1 cm^3 of molasses has mass 1.2g.
So, the mass of the molasses that can be poured into each mould = (11641.7 × 1.2) g = 13970.04 g = 13.97 kg = 14 kg (approx.)

17. A bucket is 40 cm. in diameter at the top and 28 cm. in diameter at the bottom. Find the capacity of the bucket in litres, if it is 21 cm. deep. Also, find the cost of tin sheet used in making the bucket, if the cost of tin is ₹ 1.50 per sq. dm.



Sol. Given : $r_1 = 20$ cm, $r_2 = 14$ cm. and $h = 21$ cm.

Now, the required capacity (i.e., volume) of the bucket $= \frac{\pi h^3}{3}(r_1^2 + r_1r_2 + r_2^2)$

$$= \frac{22\times(21)^3}{7\times3}(20^2+20\times14+14^2)\text{ cm}^3 = 22\times876\text{ cm}^3 = 19272\text{ cm}^3 = \frac{19272}{1000}\text{litres} = 19.272\text{ litres}$$

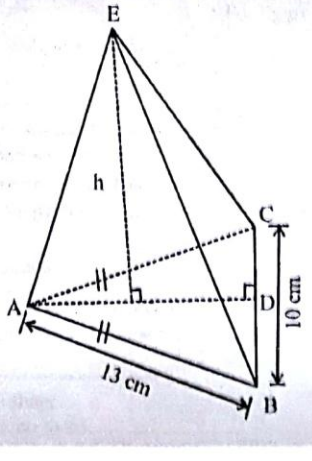
Now, $\ell = \sqrt{(r_1-r_2)^2+h^2} = \sqrt{(20-14)^2+21^2\text{cm}} = \sqrt{6^2+21^2}\text{cm} = \sqrt{36+441}\text{cm} = \sqrt{477}\text{cm} \simeq 21.84$ cm.

$\therefore$ Total surface area of the bucket (which is open at the top) $= \pi\ell(r_1+r_2)+\pi r_2^2$

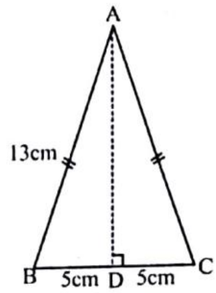
$$= \pi[(r_1+r_2)\ell + r_2^2] = \frac{22}{7}[(20+14)\times21.84+14^2] = 2949.76\text{ cm}^2$$

$\therefore$ Required cost of the tin sheet at the rate of ₹ 1.50 per dm^2 i.e., per 100 cm^2 = ₹ $\frac{1.50\times2949.76}{100}$ = ₹ 44.25

18. The diagram shows a right pyramid that has an isosceles triangular base. If the volume of the pyramid is 330 cm³, calculate its height, *h*.



Sol. $AD = \sqrt{13^2 - 5^2} = \sqrt{144} = 12\text{ cm}$



Area of base $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 10 \times 12 = 60\text{ cm}^2$

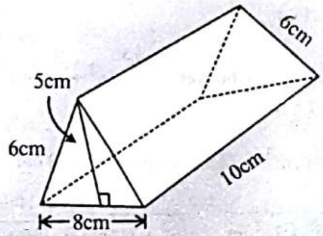
Volume of pyramid $= \frac{1}{3}$ (Area of base)$\times$ Height

$330 = \frac{1}{3} \times 60 \times h$

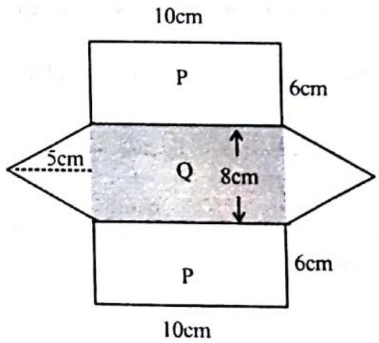
$20h = 330$

$\therefore h = \frac{330}{20} = 16.5\text{ cm}$

19. Calculate the surface area of the following Prism.



Sol. The given prism has 2 triangles and 3 rectangles on its surfaces.



Area of the two triangles $= 2\left[\frac{1}{2} \times 8 \times 5\right]$

Area of rectangle type $P = 2\,(10\times 6)$

Area of rectangle type $Q = 8 \times 10$

$\therefore$ Total surface area = area of two triangles + area of three rectangles

$= 2\left(\frac{1}{2} \times 8 \times 5\right) + 2(10 \times 6) + (8 \times 10) = 40 + 120 + 80 = 240\text{ cm}^2.$

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Volume of a frustum of a cone ....
2. Curved surface area of a frustum of a cone ....
3. Total surface area of frustum of a cone ....
4. The volume of a cube with diagonal d is ....
5. If f, e and v represent the number of rectangular faces, number of edges and number of vertices respectively of a cuboid, then the values of f, e, and v respectively are ....
6. A cube is a special type of ....
7. A road roller of length $3l$ metres and radius $\frac{l}{3}$ metres can cover a field in 100 revolutions, moving once over. The area of the field in terms of l is .... m^3.
8. If the heights of two cylinders are equal and their radii are in the ratio of 7 : 5, then the ratio of their volumes is ....
9. The volume of a solid is the measurement of the portion of the .... occupied by it.
10. If the volume of a cube is 64 cm^3, then its surface area is ....
11. A sphere and the base of a cylinder have equal radii. The diameter of the sphere is equal to the height of the cylinder. The ratio of the curved surface area of the cylinder and surface area of the sphere is ....
12. If the ratio of the base radii of two cones having the same curved surface areas is 6 : 7 then the ratio of their slant heights is ....
13. If the volume and the surface area of a solid sphere are numerically equal, then its radius is ....
14. The length of the diagonal of a cube that can be inscribed in a sphere of radius 7.5 cm is ....
15. Total number of faces in a prism which has 12 edges is.........
16. If the number of lateral surfaces of a right prism is equal to n, then the number of edges of the base of the prism is ....
17. W, P, H and A are whole surface area, perimeter of base, height and area of the base of a prism respectively. The relation between W, P, H and A is ....

T/F True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The face of a solid is two-dimensional in shape.
2. If a solid has a curved surface then it has no faces.
3. Volume of the solid is measured in cubic units.
4. Area is the length of the boundary of a closed figure.
5. Area is the total surface covered by a closed figure.
6. The volume of sphere of diameter is $\frac{\pi d^3}{6}$.
7. The total surface area of a solid cylinder of radius r and height h is $2\pi r(h+r)$.
8. If a right circular cone and a cylinder have equal circles as their base and have equal heights, then the ratio of their volumes is 2 : 3
9. If the base area and the volume of a cone are numerically equal, then its height is 3 units.
10. If the curved surface of a right circular cylinder is 1760 cm^2 and its radius is 21 cm, then its height is $\frac{80}{3}$ cm.
11. If the total surface area of a cube is $\frac{50}{3}m^2$, then its side is (5/3) m.
12. If s is the perimeter of the base of a prism, n is the number of sides of the base, S is the total length of the edges and h is the height, then $S = nh + 2s$
13. Surface area of square pyramid is $S = s^2 + 2sl$.

MTF Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. For figure shown, match the column

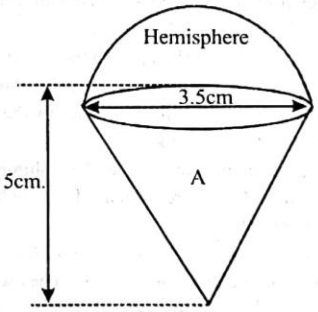
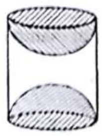


Fig. : Top ((Lattu)

Column I	Column II
(A) curved area of hemisphere	(p) 3.25
(B) height of cone	(q) 77/4
(C) slant height of cone	(r) 3.7
(D) surface area of top	(s) 39.6

2. For a wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, match the column.



Column I	Column II
(A) volume of cylinder	(p) 616/3
(B) volume of scoops	(q) 374
(C) total surface area	(r) $122.5\,\pi$
(D) volume of the article	(s) $171.5/3\,\pi$

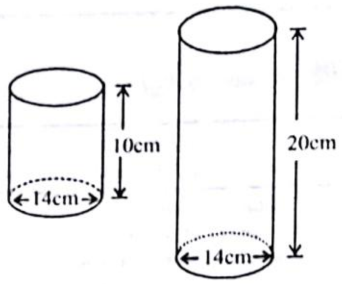
3. From a solid cylinder of height 2.4 cm. and diameter 1.4cm., a conical cavity of the same height and some diameter is hollowed out then match the column.

Column I	Column II
(A) Area of bottom of cylinder	(p) 10.56
(B) outer curved surface area	(q) 1.54
(C) curved area of conical cavity	(r) 5.5
(D) total surface area	(s) 17.6

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

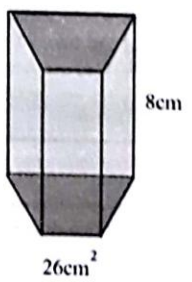
1. An underground water tank is in the form of a cuboid of edges 48 *m*, 36m and 28 *m*. Find the volume of the tank.
2. How many 3 metres cubes can be cut from a cuboid measuring 18m× 12 m × 9m?
3. The model of a building is constructed with scale factor 1 : 30. If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.
4. If a rectangular paper of length 6 cm. and width 3 cm is rolled to form a cylinder with height equal to the width of the paper, then find its base radius.
5. Find the percentage increase in the volume of a cuboid when the three edges are increased by 100%, 150% and 250% respectively.
6. A cylindrical container, used for holding petrol, had a diameter of 16 m and a height of 3 m. The owner wishes to increase the volume. However, he wishes to do it such that if X m are added to either the radius or the height, the increase in volume is the same. Find the value of *X*.
7. Two cylindrical cans have bases of the same size. The diameter of each is 14 cm. One of the cans is 10 cm high and the other is 20 cm high. Find the ratio of their volumes.



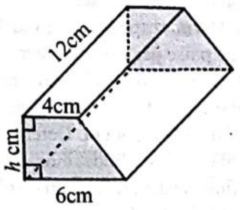
8. How many metres of cloth 5 *m* wide will be required to make a conical tent, the radius of whose base is 7 *m* and whose height is 24 *m*?

$\left(Take\ \pi = \frac{22}{7}\right)$

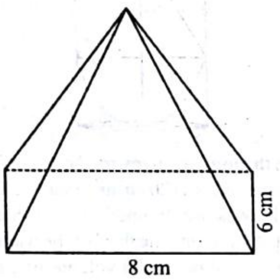
9. A solid metal sphere is cut through its centre into 2 equal parts. Find the total surface of each part if the radius of the sphere is 3.5 cm.
10. A joker's cap is in the form of a cone of radius 7 cm and height 24 cm. Find the area of the cardboard required to make the cap.
11. A reservoir is in the form of a rectangular parallelopiped. Its length is 20m. If 18 kl of water is removed from the reservoir, the water level goes down by 15 cm. Find the width of the reservoir (1 kl = 1 m^3)
12. If the ratio of the radii of two cylinders is 2 : 1 and the ratio of their heights is 1 : 2, compare their volumes.
13. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.
14. A cylindrical pipe has an inner diameter of 7 cm. Water flows through it at the rate of 192.5 litres per minute. Find the rate of flow in km per hour.
15. A cone of height 15 cm. and diameter 7 cm. is mounted on a hemisphere of the same radius. Find the volume of the solid formed by the cone and the hemisphere.
16. Find the volumes of these right prisms



17. The cross section of a prism 12 cm long, is a trapezium with the measurements shown. If the volume of the prism is 300 cm^3, calculate the value of h.



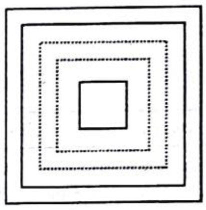
18. The diagram shows a right rectangular pyramid. If the volume of the pyramid is 80 cm^3, calculate its height, in cm.



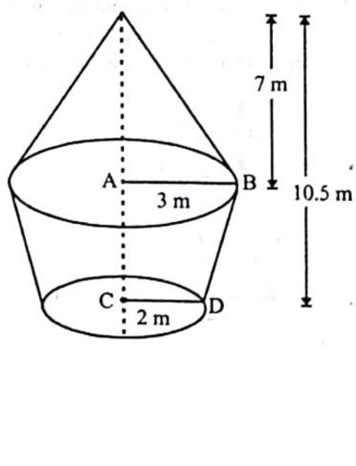
SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

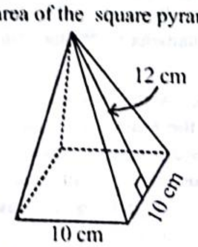
1. A circus tent is made of canvas and is in the form of a right circular cylinder and a right circular cone above it. The diameter and height of the cylindrical part of the tent are 126*m* and 5*m* respectively. The total height of the tent is 21*m*. Find the total cost of tent if the canvas used costs Rs. 12 per square metre.
2. Cubes A, B, C having edges of 18 cm, 24 cm and 30 cm respectively are melted and moulded into a new cube D. Find the edge of the bigger cube D.
3. A metallic sheet is of rectangular shape with measurements 48 cm × 36 cm. From each one of its corners a square of 8 cm is cut off. An open box is made of the remaining sheet. What is the volume of the box?
4. The radius of the base and the height of a right circular cone are 7 cm and 24 cm respectively. Find the volume and total surface area of the cone.
5. If 1 cubic cm of cast iron weighs 21 gms. then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.
6. A right circular cone and a right circular cylinder have equal heights and equal bases. Their respective curved surfaces are in the ratio 5 : 8. Find the ratio of their base radius to the height.
7. Keeping the volume of a wire the same as before, if we decrease its diameter by 5% then find the percent change in its length.
8. The figure below shows a set of concentric squares. If the diagonal of the innermost square is 2 units, and if the distance between the corresponding corners of any two successive squares is 1 unit, find the difference between the areas of the eighth and the seventh square, counting from the innermost square.



9. A cylindrical container, used for holding petrol, had a diameter of 16 m and a height of 3 m. The owner wishes to increase the volume. However, he wishes to do it such that if X m are added to either the radius or the height, the increase in volume is the same. Find the value of X.
10. If the radius of the sphere is increased by 100%, then by what is the percentage increase in volume of the corresponding sphere.
11. The diameter of a copper sphere is 6 cm. The sphere is melted and recast into a wire. If the length of the wire be 36 cm, find its radius.
12. The diameters of two cones are equal. If their slant height are in the ratio 5 : 4, find the ratio of their curved surface areas.
13. Marbles of diameter 3 cm are dropped into a cylindrical beaker containing some water and are fully submerged. The diameter of the beaker is 12 cm. Find how many marbles have been dropped in it if the water rises by 10 cm.
14. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.
15. The lower portion of a haystack is an inverted cone frustum and upper part is a cone [See fig.] Find the total volume of the haystack $AB = 3$m and $CD = 2$m.



16. Find the surface area of the square pyramid shown below.

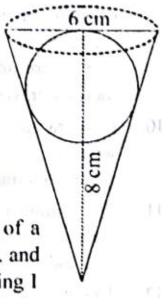


17. Find the volume of a pyramid of height 9 cm and with a rectangular base measuring 7 cm by 5 cm.

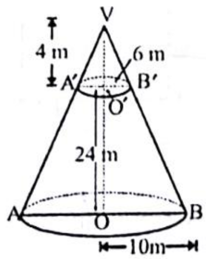
LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

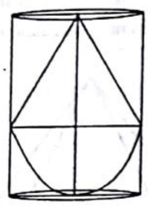
1. A solid right-circular cylinder has a base radius of 12 cm. and height of 16 cm. It is melted and made into eight spherical balls of equal size. Find the radius of each ball.
2. A conical vessel of radius 6 cm. and height 8cm. is filled with water. A sphere is lowered into the water (see figure) and its size is such that when it touches the sides of the conical vessel, it is just immersed. How much water will remain in the cone after the overflow ?



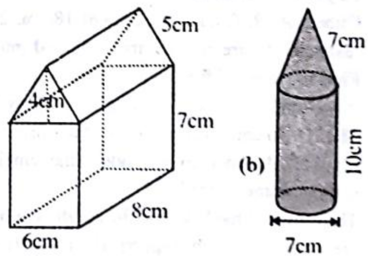
3. The internal and external diameters of a hollow hemispherical vessel are 21 cm. and 25.2 cm. respectively. The cost of painting 1 sq. cm. of the surface is 10 paise. Find the total cost of painting the entire vessel.
4. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 21 cm.
5. A toy is in the form of cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface of the toy. [Use $\pi = 22/7$]
6. A teak wood log is cut first in the form of a cuboid of length 2.3 m, width 0.75 m and of a certain thickness. Its volume is 1.104 m^3. How many rectangular planks of size 2.3 m × 0.75 m × 0.04 m can be cut from the cuboid?
7. A tent is made in the form of a conic frustum surmounted by a cone. The diameters of the base and the top of the frustum are 20 m and 6 m respectively and the height is 24 m. If the height of the tent is 28 m, find the quantity of canvas required.



8. A right circular cone of height h is cut by a plane parallel to the base at a distance h/3 from the base, then find the ratio of the volumes of the resulting cone and the frustrum.
9. The outer and inner diameters of a hemispherical bowl are 17 cm. and 15 cm. respectively. Find the cost of polishing it all over at 25 paise per cm^2. (Take $\pi = 22/7$).
10. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy (shown in figure), find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)



11. A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16.8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one third of the water in the original cone overflows. What is the volume of each of the solid cones submerged?
12. Length of a class-room is two times its height and its breadth is $1\frac{1}{2}$ times its height. The cost of white-washing the walls at the rate of ₹ 1.60 per m^2 is ₹ 179.20. Find the cost of tiling the floor at the rate of ₹ 6.75 per m^2.
13. Calculate the total surface area of each of the following combined solids. $\left(\pi = \frac{22}{7}\right)$



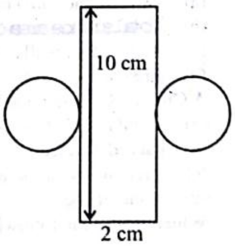
14. An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold.

2 EXERCISE

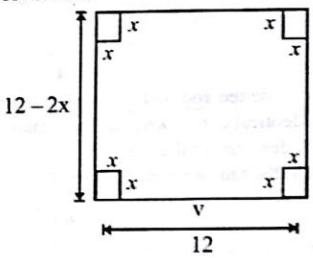
MCQ Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. Three identical cones with base radius r are placed on their bases so that each is touching the other two. The radius of the circle drawn through their vertices is –
   (a) smaller than r
   (b) equal to r
   (c) larger than r
   (d) depends on the height of the cones
2. The diameter of hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?
   (a) 50% (b) less than 50%
   (c) more than 50% (d) 100%
3. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and resolidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to
   (a) 3 (b) 3.5
   (c) 4 (d) 4.5
4. The diagram shows the parts of a right cylinder. The volume of the cylinder, in cm^3 is
   (a) $\frac{20}{\pi}$
   (b) $\frac{50}{\pi}$
   (c) $\frac{25}{\pi}$
   (d) 40π



5. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal. then the ratio of the slant height of the cone to the height of the cylinder is
   (a) 2 : 1 (b) 1 : 2
   (c) 1 : 3 (d) 3 : 1
6. If the perimeter of one face of a cube is 20 cm, then its surface area is
   (a) $120\,cm^2$ (b) $150\,cm^2$
   (c) $125\,cm^2$ (d) $400\,cm^2$
7. A cube of side 12 cm. is painted red on all the faces and then cut into smaller cubes, each of side 3 cm. What is the total number of smaller cubes having none of their faces painted?
   (a) 16 (b) 8
   (c) 12 (d) 24
8. A square tin sheet of side 12 inches is converted into a box with open top in the following steps – The sheet is placed horizontally. Then, equal sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?



   (a) 3 (b) 4
   (c) 1 (d) 2
9. If the diameter of the sphere is doubled, the surface area of the resultant sphere becomes x times that of the original one. Then x would be –
   (a) 2 (b) 3
   (c) 4 (d) 8
10. If h be the height and α the semi-vertical angle of a right circular cone, then its volume is given by –
   (a) $\frac{1}{3}\pi h^3 \tan^2\alpha$ (b) $\frac{1}{3}\pi h^2 \tan^2\alpha$
   (c) $\frac{1}{3}\pi h^2 \tan^3\alpha$ (d) $\frac{1}{3}\pi h^3 \tan^3\alpha$
11. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by –
   (a) 200% (b) 500%
   (c) 700% (d) 800%
12. If length, breadth and height of a cuboid is increased by x%, y% and z% respectively then its volume is increased by –
   (a) $\left[x+y+z+\frac{xy+xz+yz}{100}+\frac{xyz}{(100)^2}\right]\%$
   (b) $\left[x+y+z+\frac{xy+xz+yz}{100}\right]\%$
   (c) $\left[x+y+z+\frac{xyz}{(100)^2}\right]\%$
   (d) None of these

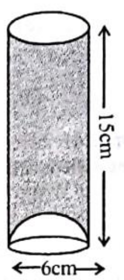
13. A 30°-60°-90° triangle has the smallest side equal to 10 cm. This triangle is first rotated about the smallest side and then about the second largest side. If the volumes of the cones generated a and b respectively, then –
(a) $a < b$ (b) $a = b$
(c) $a > b$ (d) $a = 2b$

14. There are two vessels – one is in the shape of a cylinder and the other in the shape of a right circular cone. Both the vessels have the same height and the same base radius. The cylindrical vessel and the conical vessel are filled with milk and water respectively and are both filled to half of their maximum heights. The cone is standing on its vertex. The contents of the conical vessel are emptied into the cylindrical vessel. What is the ratio of water to milk in the cylindrical vessel now –
(a) 1 : 1 (b) 1 : 12
(c) 1 : 9 (d) 1 : 4

15. A sphere is melted and half of the molten liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is –
(a) $\left(\frac{4}{3}\right)^{1/3}$ (b) $\left(\frac{8}{3}\right)^{1/3}$
(c) $(3)^{1/3}$ (d) 2

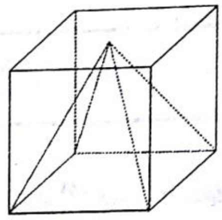
16. If a solid of one shape is converted to another, then the volume of the new solid
(a) remains same (b) increases
(c) decreases (d) can't say

17. In the adjoining figure, the bottom of the glass has a hemi-spherical raised portion. If the glass is filled with orange juice, the quantity of juice which a person will get is



(a) $135\pi\,cm^3$ (b) $117\pi\,cm^3$
(c) $99\pi\,cm^3$ (d) $36\pi\,cm^3$

18. A regular square pyramid is placed in a cube so that the base of the pyramid and that of the cube coincide. The vertex of the pyramid lies on the face of the cube opposite to the base, as shown in the below. An edge of the cube is 7 inches. How many square inches (approximately) are in the positive difference between the surface area of the cube and the surface area of the pyramid –



19. The base of a right prism is an equilateral triangle of edge 12 m. If the volume of the prism is $288\sqrt{3}\,m^3$, then its height is
(a) $6\,m$ (b) $8\,m$
(c) $10\,m$ (d) $12\,m$

20. The base of a right prism is a square of perimeter 20 cm and its height is 30 cm. The volume of the prism is
(a) $700\,cm^3$ (b) $750\,cm^3$
(c) $800\,cm^3$ (d) $850\,cm^3$

21. The base of a right pyramid is an equilateral triangle of perimeter 8 dm and the height of the pyramid is $30\sqrt{3}$ cm. The volume of the pyramid is
(a) $16000\,cm^3$ (b) $1600\,cm^3$
(c) $\frac{16000}{3}cm^3$ (d) $\frac{5}{4}cm^3$

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which one is/are correct ?
(a) Total surface area of cuboid is $2(lb + bh + hl)$
(b) Total surface area of a cube is $6a^2$
(c) Area of four walls = $2h(l + b)$
(d) Area of four walls = Height × Perimeter of the room

2. A Circular Cylinder can be separated into
(a) circular end at the bottom
(b) curved surface
(c) circular end at the top
(d) None of these

3. Which one of the following is correct ?
(a) Volume of triangular Prism $= \frac{\sqrt{3}}{4}a^2h$
(b) Total Surface area of triangular prism = lateral surface area + sum of areas of two ends $= 3ah + \frac{\sqrt{3}}{4}a^2$
(c) Volume of triangular prism $= \frac{\sqrt{3}}{4}ah$
(d) Total surface area of triangular prism $= ah + ah + ah + \frac{\sqrt{3}}{4}a^2$

4. The value of each of a set of coins varies as the square of its diameter, if its thickness remains constant and varies as the thickness, if the diameter remain constant. If the diameter remain constant. If the diameter of two coins are in the ratio 4 : 3, what should be the ratio of their thickness if the value of the first is four times that of the second?
   (a) 18:8 (b) 9:4
   (c) 8:18 (d) 4:9
5. Which one is / are correct ?
   (a) Total surface area of cylinder is $2\pi r^2 + 2\pi rh$.
   (b) Total surface area of a sphere is $4\pi r^2$.
   (c) Total surface area of cones is $\pi r^2 + \pi rl$.
   (d) Total surface area of a square Pyramid is $s^2 + 4sl$.
6. Which one of the following is/ are made up of combinations of two or more of the basic solids?
   (a) Buildings (b) top
   (c) Monuments (d) test tube
7. Among the following, which one is/ are correct?
   (a) The slant height is the longest side of a pyramid.
   (b) The section between the base and a plane parallel to the base of a solid is known as frustum.
   (c) All the surfaces of a cuboid are rectangular.
   (d) For a cylinder, the top, the bottom and the walls of the cylinder determine the total surface area.

PBQ Passage Based Questions

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

A test tube is in the form of a right circular cylinder, surmounted by a cone. The diameter of the cylinder is 24 m. The height of the cylindrical portion is 11m, while the vertex of the cone is 16 m above the ground.

1. The curved surface area of the cylindrical portion is
   (a) $(246\pi)m^2$ (b) $(264\pi)m^2$
   (c) $(426\pi)m^2$ (d) $(462\pi)m^2$
2. The slant height of the cone is
   (a) $31\,m$ (b) $25\,m$
   (c) $13\,m$ (d) $5\,m$
3. The area of the canvas required for the tent is
   (a) $1320\,m^2$ (b) $3120\,m^2$
   (c) $2130\,m^2$ (d) $1230\,m^2$

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect**.
(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion:** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2
   Reason: If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$
2. **Assertion:** If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.
   Reason: If r be the radius and h the slant height of the cone, then slant height = $\sqrt{h^2 + r^2}$
3. **Assertion:** If the radius of a cone is halved and volume is not changed, then height remains same.
   Reason: If the radius of a cone is halved and volume is not changed then height must become four times of the original height.
4. **Assertion:** If a ball in the shape of a sphere has a surface area of 221.76 cm^2, then its diameter is 8.4 cm.
   Reason: If the radius of the sphere be r, then surface area, $S = 4\pi r^2$, i. e. $r = \frac{1}{2}\sqrt{\frac{s}{\pi}}$
5. **Assertion:** No. of spherical balls that can be made out of a solid cube of iead whose edge is 44 cm, each ball being 4 cm. in diameter, is 2541
   Reason : Number of balls = $\frac{Volume\ of\ one\ ball}{volume\ of\ lead}$
6. **Assertion:** If the base area and height of a prism be 25 cm^2 and 6 cm respectively, then its volume is 150 cm^3.
   Reason: Volume of a pyramid = $\frac{Base\ area \times height}{3}$

MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and statements (p, q, r,s* and *t, u) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column I	Column II
(A) Solids	(p) Right circular cone
(B) Road rollers	(q) Sphere
(C) Ice-cream cone	(r) Cylinder
(D) Volleyball	(s) cuboid
	(t) Cube

2.

Column I	Column II
(A) Slant height	(p) 2× radius
(B) Diameter (d)	(q) $\dfrac{\pi dl}{2}$
(C) Volume of cone	(r) $\sqrt{h^2+\dfrac{d^2}{4}}$
(D) Volume of pyramid	(s) $\dfrac{\pi d^2 h}{12}$
	(t) $\left(\dfrac{S^6 h^3}{27}\right)^{1/3}$
	(u) $\dfrac{1}{3}\pi r^2 h$

HOTS Subjective Questions

DIRECTIONS : *Answer the following questions.*

1. Water flows at the rate of 5m per minute through a cylindrical pipe having its diameter as 5mm. How much time will it take to fill a conical vessel whose diameter of the base is 40 cm. and depth 24 cm.
2. The radii of the ends of a bucket of height 24 cm are 15 cm and 5 cm. Find its capacity and total surface area.
3. A pool has a uniform circular cross-section of radius 5m and uniform depth 1.4m. It is filled by a pipe which delivers water at the rate of 20 litres per second. Calculate in minutes, the time taken to fill the pool. If the pool is emptied in 42 minutes by another cylindrical pipe through which water flows at the rate of 2 *m/s*, calculate the radius of the pipe in cm.
4. A metallic right circular cone, 20 cm. high and vertical angle 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm., find the length of the wire.
5. (a) Each edge of cube is increased by 50%. Find the percentage increase in the surface area of the cube.
   (b) A solid cube is cut into two cuboids of equal volumes. Find the ratio of the total surface area of the given cube and that of one of the cuboids.
6. The interior of a building is in the form of a cylinder of base radius 12 m and height 3.5 surmounted by a cone of equal base and slant height 12.5m. Find the internal curved surface area and the capacity of the building.
7. Tennis balls, diameter 62 mm are placed in sixes in cylindrical card tubes (Fig. 14.90). Find the volume of the six balls and the internal volume of the tube. Find the volume of unfilled space in the tube and express this as a percentage of the volume of the tube.
8. Find the volume and total surface area of a tumbler in the form of a frustum of a cone, if the diameter of the ends are 6.50 cm and 3.50 and the perpendicular height of the tumbler is 7.80 cm.
9. The base of a right prism is a right angled triangle. The measure of the base of the right angled triangle is 3 *m* and its height 4 *m*. If the height of the prism is 7 *m*; then find
   (i) the number of edges of the prism.
   (ii) the volume of the prism
   (iii) the total surface area of the prism.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. $\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$
2. $\pi\ell(r_1+r_2)$
3. $\pi\ell(r_1+r_2)+\pi(r_1^2+r_2^2)$
4. $\frac{d^3}{3\sqrt{3}}$ cu units.
5. 6, 12 and 8
6. cuboid
7. $200\pi l^2\ cm^2$
8. 49 : 25
9. space
10. $96\ cm^2$
11. 1 : 1.
12. 7 : 6
13. 3 units
14. 15 cm
15. 6
16. n
17. W = P × H + 2A

TRUE / FALSE

1. True
2. True
3. True
4. False
5. True
6. True
7. True
8. False
9. True
10. False
11. True.
12. True
13. True

MATCH THE FOLLOWING :

1. (A) → q ; (B) → p ; (C) → r ; (D) → s
2. (A) → r ; (B) → s ; (C) → q ; (D) → p
3. (A) → q ; (B) → p ; (C) → r ; (D) → s

VERY SHORT ANSWER QUESTIONS :

1. Volume of the tank which is in the form of a cuboid $= l\times b\times h = 483884\ m^3$
2. Volume of cuboid $=(18\times12\times9)\ m^3$
Volume of each cube $=(3)^3 = 27\ m^3$
Let the required no. of cubes be n, then
$n(27) = 18\times12\times9$
$n = 72$.
3. Let the ratio of volume is $27\ m^3$, then the volume of model is
$$\frac{27}{(30)^3}=\frac{27}{30\times30\times30}\ m^3=\frac{1}{1000}\ m^3.$$
4. Circumference of the cylinder = length of the rectangular paper
base radius $=\frac{3}{\pi}$ cm
5. Old Volume = L B H,
New Volume $= 2L\times2.5\text{ B}\times3.5\ H = 17.5\ L\ B\ H$
$\Rightarrow$% increase in volume $=\frac{17.5LBH-LBH}{LBH}\times100$
$= 1650\%$.
6. Volume $=\pi R^2 H$
Initially $H = 3$, and $d = 16$ or $R = 8$
New H is $3+x$ and New R is $8+x$
$\Rightarrow$ New volume $=\pi\ .\ 8^2.\ (3+X)\ \Rightarrow X = 16/3 = 5.33$
7. 1 : 2
8. Radius of base, $r = 7$ cm Vertical height, $h = 24\ m$
Slant height, $l=\sqrt{h^2+r^2}=25\ m$
Curved surface area $=\pi rl = 550\ m^2$
Width of cloth $= 5\ m$
Length required to make conical tent $=\frac{550}{5}\ m = 110\ m$.
9. $115.5\ cm^2$.
10. Area of the cardboard required = curved surface area of the cap (or cone) $=\pi rl$
$\therefore$ Area of the cardboard required $= 550\ cm^2$
11. Volume of water removed from the reservoir $= 18\ m^3$
$\therefore\ 3x = 18 \Rightarrow x = 6m$
12. Ratio of the radii of two cylinders = 2 : 1
Ratio of height = 1 : 2
Let the radii be $2r$ and r, and the height h and $2h$.
$$\frac{\text{Volume of 1st cylinder}}{\text{Volume of 2nd cylinder}}=\frac{\pi r_1^2h_1}{\pi r_2^2h_1}=\frac{(2r)^2h}{r^2(2h)}=\frac{4r^2h}{2r^2h}=\frac{2}{1}$$
Ratio of volumes = 2 : 1
13. The ratio of the curved surfaces of two cones = 2 : 1
The ratio of their slant heights = 1 : 2
Curved surface of first cone $=\pi r_1\ell_1$
Curved surface of second cone $=\pi r_2\ell_2$
$$\frac{\pi r_1\ell_1}{\pi r_2\ell_2}=\frac{2}{1}\Rightarrow\frac{r_1}{r_2}\times\frac{\ell}{2\ell}=\frac{2}{1}\Rightarrow\frac{r_1}{r_2}=\frac{4}{2}$$
or $r_1 : r_2 = 4 : 1$
14. Speed = 3 km/h.
15. $282.333\ cm^3$
16. Volume of prism = Base area × Height $= 26\times8 = 208\ cm^3$
17. Volume of prism = Area of cross section × Length
$\therefore h=\frac{300}{60}=5$ cm

18. Volume of pyramid $= \frac{1}{3}$ (Area of base) (Height)

$h = \frac{80}{16} = 5$

SHORT ANSWER QUESTIONS :

1. Diameter = 126 m, $\therefore$ Radius (r) = 63m, Height of the cylinder (h) = 5m, Total height = 21m

$\therefore$ Height of the conical portion, $H = 21 - 5 = 16m$

Curved surface area of cylinder $= 2\pi rh = 1980\ sq.\ m$

Lateral surface area of cone $= \pi r\ell$

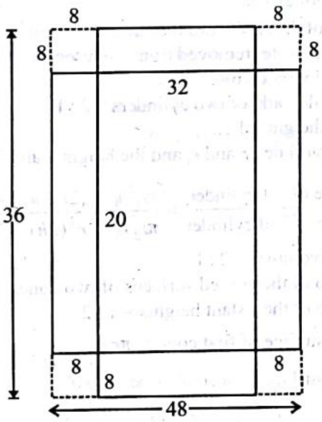
$\therefore$ Lateral surface area of cone $= \frac{22}{7} \times 63 \times \sqrt{(16)^2 + (63)^2}$

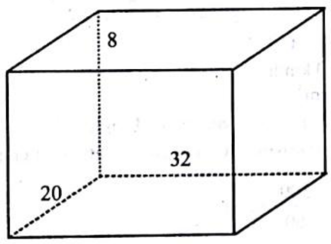
$= 12870$ sq. cm.

Required cost $= 12 \times (1980 + 12870) =$ ₹ 178200

2. Volume of $A = 18^3 = 5832\ cm^3$
Volume of $B = 24^3 = 13824\ cm^3$
Total volume A, B and $C = 46656\ cm^3$
Now, volume of new cube D = sum of volumes of A, B and C. Let a be the edge of D.
Then $a^3 = 46656 \Rightarrow a = 36$ cm.

3. Length of metallic sheet of rectangular shape = 48 cm.
Breadth of metallic sheet of rectangular shape = 36 cm
When a square of side 8 cm is cut off from each corner and the flaps turned up, we get an open box whose





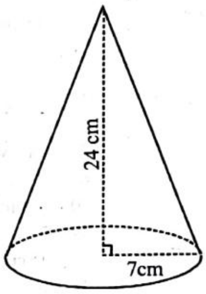
Length (l) = 32 cm, Breadth (b) = 36 − (8+8) = 20 cm
and Height = 8 cm
Volume of the box $= l \times b \times h = 5120\ cm^3$.

4. Here $r = 7$ cm and $h = 24$ cm
Volume of the cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24 = 1232\ cm^3$

Slant height (l) $= \sqrt{r^2 + h^2} = 25$ cm

Curved surface area $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550\ cm^2$.



$\therefore$ Total surface area $= \pi rl + \pi r^2 = 704\ cm^2$.

5. Weight of the pipe $= \left(\frac{8800}{7} \times \frac{21}{1000}\right)$ kg = 26.4 kg.

6. $r : h = 3 : 4$.

7. Volume of Wire = Volume of Cylinder $\Rightarrow$ Volume $= \pi r^2 h = \pi\ (d/2)^2\ h \Rightarrow$ (Old V) = (New V)
Hence, percentage change in length is 10.8%.

8. Diagonal of innermost square = 2
Diagonal of next (second from innermost) square $= 2 + 2 \times (1) = 4$
Diagonal of r_{th} square $= 2 + 2\ (r - 1)$
Thus, diagonal of 7th square = 14 and, diagonal of 8th square = 16, putting r = 7 and 8

Required difference in area $= \frac{(16)^2}{2} - \frac{(14)^2}{2}$

9. $X = 16/3 = 5.33$

10. Let radius of the sphere be r. An increase by 100% makes radius $= 2r$
Percent increase in volume

$$= \frac{V_2 - V_1}{V_1} \times 100\% = \frac{\left(\frac{32}{3} - \frac{4}{3}\right)\pi r^3}{\frac{4}{3}\pi r^3} \times 100 = 700\%$$

11. $r = 1$ cm.

12. Since diameters of two cones are equal therefore their radii are also equal. Let the radius of each cone be r.
Again since their slant heights are in the ratio 5 : 4, therefore let their slant heights be $5l$ and $4l$ respectively.

$\therefore$ Curved surface area of first cone $= \pi r(5l) = 5\pi rl$

and curved surface area of second cone $= \pi r(4l) = 4\pi rl$

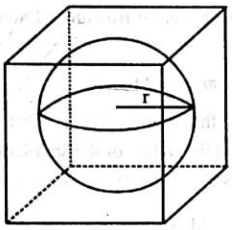
$\therefore$ Ratio of their curved surface areas $= \dfrac{5\pi rl}{4\pi rl} = \dfrac{5}{4} = 5:4$

13. 80

[**Hint.** No. of marbles

$$= \frac{\textit{Volume of water (raised)}}{\textit{Volume of one marble}} = \frac{\pi \times 6^2 \times 10}{\frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3} = 80.]$$

14. When the sphere exactly fit inside the cube, it is the largest sphere. Then.



Radius r of the largest sphere $= \dfrac{1}{2}a$,

where a is the side of the cube

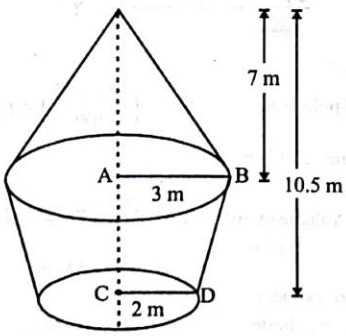
Now volume of cube $= a^3$ cubic units

Volume of sphere $= \dfrac{4}{3}\pi\left(\dfrac{a}{2}\right)^3$ cubic units. $= \dfrac{\pi a^3}{6}$

Required ratio $= 6 : \pi$

15. The total volume of the haystack = Volume of the cone + Volume of inverted cone frustum

$$= \frac{1}{3}\pi R^2 H + \frac{\pi h}{3}(R^2 + r^2 + Rr)$$



Here $R = AB = 3\ m$; $r = CD = 2\ m$

H = Height of cone = 7 m

h = height of the frustum = $(10.5 - 7)\ m = 3.5$ cm

$\therefore$ The total volume of the haystack

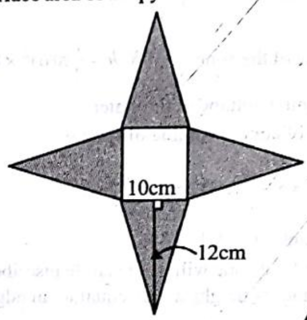
$$= \frac{1}{3}\pi(3)^2 \times 7 + \frac{1}{3}\pi(3.5)[3^2+2^2+(3\times2)] = 407\ m^3.$$

16. The given pyramid has one square and four equal triangles for its surface.

Area of the 4 triangles $= 4\left(\dfrac{1}{2}\times10\times12\right)$

Area of the square $= 10\times10$

Total surface area of the pyramid $= 340\ \text{cm}^2$

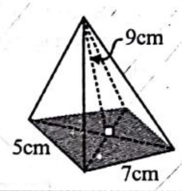


Net of the pyramid

17. Area of base $= 7\ \text{cm} \times 5\ \text{cm} = 35\ \text{cm}^2$

$\therefore$ Volume of pyramid

$= 105\ \text{cm}^3$



LONG ANSWER QUESTIONS :

1. Volume of cylinder $= \pi r^2 h = \dfrac{22}{7}\times12\times12\times16\text{cm}^3$

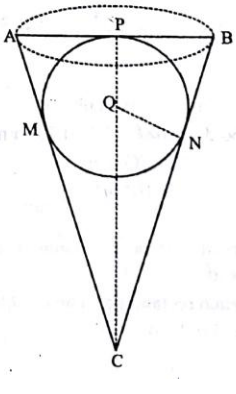
Let the radius of small ball be $= r$ cm.

$\therefore$ Volume of one small ball $= \dfrac{4}{3}\pi r^3$

Volume of eight balls = volume of cylinder

$r = 6$ cm.

2. The sphere is just immersed in the cone filled of water. Hence, uppermost point of the sphere and the centre of the base of the cone will be same, i.e. lie on point 'P'.



Now in ΔPBC, we have

$BC = 10$ cm.

We know that two tangents to a circle from the same point are equal.

$\therefore\ BP = BN = 6$ cm.

$\Rightarrow NC = 4$ cm.

Let the radius of the sphere (circle) $= ON = x$ cm.

$\therefore\ OC = (8 - x)$ cm.

Now in ΔONC, we get

$OC^2 = ON^2 + NC^2 \Rightarrow x = 3$ cm.

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3$ cu.cm.

Volume of the cone $= \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi(6)^2 \times 8$

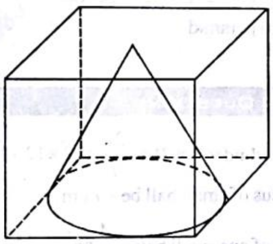
$\therefore$ Remaining volume of the water

= volume of cone – volume of sphere

$= \frac{1}{3}\pi(6)^2 \times 8 - \frac{4}{3}\pi(3)^3 = 188.57\text{ cm}^3$

3. Cost of painting = ₹ 184.33

4. The base of the cone will be the circle inscribed in a face of the cube and its height wil be equal to an edge of the cube (figure).



Edge of the cube = 21 cm.

$\therefore$ Radius of the circle $= r = \frac{21}{2} = 10.5$ cm.

and height of the cone $= h = 21$ cm.

Volume of the right circular cone $= \frac{1}{3}\pi r^2 h = 2425.5\text{ cm}^2$

5. 858 cm^2

6. Let x be the thickness of the cuboid

Volume of cuboid $= l \times b \times h = 2.3 \times 0.75 \times x\text{ m}^3$

Also Volume $= 1.104\text{ m}^3$ (Given)

$\therefore\ 2.3 \times 0.75 \times x\,m^3 = 1.104\,m^3$

$\Rightarrow x = 0.64\,m$

Length and breadth of each rectangular plank is the same as that of cuboid.

Thickness of each rectangular plank $= 0.04\,m$

Thickness of cuboid $= 0.64\,m$.

$\therefore$ Required no. of rectangular planks which can be cut from the cuboid $= \frac{\textit{Thickness of cuboid}}{\textit{Thickness of each plank}} = 16.$

7. Let h be the height of the frustum and r_1 and r_2 be the radii of its circular bases.

We have,

$h = 24$ m, $r_1 = 10$ m and $r_2 = 3$ m

$\ell^2 = (r_1 - r_2)^2 + h^2 = 25$ m

For cone $VA'B'$, we have

$l_2 = \sqrt{O'B'^2 + VO'^2} = 5$ m

$\therefore$ Quantity of canvas required

= Lateral surface area of frustum + Lateral surface area of cone $VA'B'$

$= \pi(r_1 + r_2)l + \pi r_2 l_2 = 340\,\pi\text{ m}^2$

8. The volume of the original cone is $V = \pi r^2 h/3$

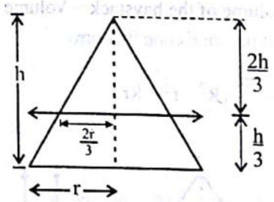
The height and the radius of the smaller cone are $2h/3$ and $2r/3$ respectively.

So its volume $= \frac{8V}{27}$

$\therefore$ Volume of the frustrum $= \frac{19V}{27}$.

Volume of resulting cone $= \frac{8V}{27}$

$\therefore$ Required ratio is 8 : 19.



9. Cost of polishing the bowl $=$ ₹ $\left(\frac{858 \times 25}{100}\right) =$ ₹ 214.50

10. 25.12 cm^3; 25.12 cm^3

[Hint. Volume of toy $= \left(\frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2^2 \times 2\right)\text{ cm}^3$

$= 8\pi\text{ cm}^3 = 8 \times 3.14\text{ cm}^3 = 25.12\text{ cm}^3$

Height of cylinder = (2+2) cm = 4cm.

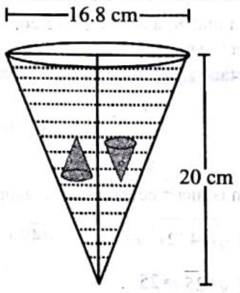
Volume of cylinder $= \pi \times 2^2 \times 4\text{ cm}^3 = 16\pi\text{ cm}^3$

$\therefore$ Difference of volume $= (16\pi - 8\pi)\text{ cm}^3 = 8\pi\text{ cm}^3$.]

11. Diameter of the cone = 16.8 cm

$\therefore$ Radius of the cone $= \frac{16.8}{2}$ cm $= 8.4$ cm.

Height of the cone = 20 cm



Volume of both equal solid cones
= Volume of water overflows

$= \frac{1}{3} \times$ Volume of total water in cone $= 492.80\ cm^3$.

$\therefore$ Volume of each solid cone $= 246.40\ cm^3$

12. Let the height of the classroom be h metres. Then,

Length = 2h metres and, Breadth $= \frac{3}{2}h$ metres

$\therefore$ Area of the four walls
$= 2\times$ Height $\times$ (Length + Breadth) $= 7h^2\ m^2$

$\therefore$ Cost of white-washing of the four walls = ₹ $11.20\,h^2$

But, the cost of white-washing is given as ₹ $179.20 \Rightarrow h = 4$

$\therefore$ Area of the floor of the room $= (8\times6)\ m^2 = 48\ m^2$

Cost of tiling of 1 m^2 of the floor = ₹ 6.75

$\therefore$ Cost of tilling the floor = ₹ 324

13. (a) 348 cm^2 (b) 335.5 cm^2

14. The total height of the bucket = 40 cm, which includes the height of the base. So, the height of the frustum of the cone = (40 – 6) cm = 34 cm. Therefore, the slant height of the frustum, $\ell = 35.44$ cm

The area of metallic sheet used = curved surface area of frustum of cone + area of circular base + curved surface area of cylinder $= 4860.9\ cm^2$

Now, the volume of water that the bucket can hold (also, known as the capacity of the bucket)

$= \frac{\pi\times h}{3}\times(r_1^2 + r_2^2 + r_1r_2)$

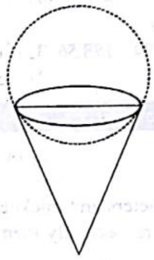
= 33.62 litres (approx.)

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (c) The centres of the bases of the cones form a triangle of side 2r. The circumference of the circle will be identical to a circle drawn through the vertices of the cones and thus, it will have a radius of $2/\sqrt{3}$ times r, which is greater than r.

2. (c). Though it is given that diameter of the cone is equal to the diameter of the spherical ball. But the ball will not fit into the cone because of its slant shape. Hence more than 50% of the portion of the ball will be outside the cone.



3. (b) Volume of the given ice cuboid $= 8\times 11\times 2 = 176$

Let the length of the required rod is ℓ.

$\therefore\ \pi\ell\frac{8^2}{4} = 176$ $\quad\therefore\ \ell = 3.5$ inches

4. (b)

5. (a) [Hint. $\pi rl = 2\pi rh \Rightarrow \frac{1}{h} = \frac{2}{1}$.]

6. (b) [Hint. Edge of cube $= \frac{20}{4}$ cm $= 5$ cm,

surfae area $= 6\times5^2\ cm^2 = 150\ cm^2$]

7. (b)

8. (d) $\therefore$ Volume of the box will be $V = (12-2x)^2 . x$

For V to be maximum $\frac{dV}{dx} = 0$

This will give x = 2, 6. x cannot be 6.

9. (c) 10 (a)

11. (c) When the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

12. (a) % change in volume

$$= \frac{100^2(x+y+z)+100(xy+xz+yz)+xyz}{100^3}\times100$$

$$= \left[x+y+z+\frac{xy+xz+yz}{100}+\frac{xyz}{(100)^2}\right]\%$$

13. (c) The smaller sides are 10 cm. and $10\sqrt{3}$ cm. When rotated about the 10 cm side, the volume is $(1/3)\pi(10\sqrt{3})^2(10)\ cm^3$. In the second case the volume is $(1/3)\pi(10^2)(10\sqrt{3})\ cm^3$. The first volume is greater.

Note that in such cases, rotation about the smaller of the two sides gives the larger volume.

14. (b)

15. (b) As per the given conditions,

$11a^3 = 7\times\frac{4}{3}\times\pi\times r^3 \quad \therefore \frac{a}{r}=\left(\frac{8}{3}\right)^{1/3}$

16. (a)

17. (b) [Hint. Quantity of juice $=\left(\pi\times3^2\times15-\frac{2}{3}\pi\times3^3\right)\text{cm}^3$]

18. (d) The surface area of the pyramid is
$pyr = 7^2+4t = 7^2+4\times27.39183 = 158.56732$
The answer is :
$cube - pyr = 294 - 158.56732 = 135.43268 = 135.4$

19. (b) 20. (b) 21. (c)

MORE THAN ONE CORRECT :

1. (a, b, c, d) 2. (a, b, c)
3. (a, b, d)
4. (a, b) Let the diameters and thickness of the two coins be d_1t_1 and d_2t_2 respectively then

$\frac{v_1}{v_2}=\frac{d_1^2}{d_2^2}\times\frac{t_1}{t_2} \Rightarrow \frac{4}{1}=\frac{4^2}{3^2}\times\frac{t_1}{t_2}\Rightarrow\frac{9}{4}=\frac{t_1}{t_2}$

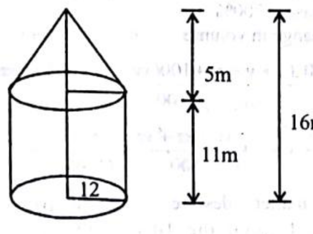
$\frac{t_1}{t_2}=\frac{9}{4}\times\frac{2}{2}=\frac{18}{8}$

5. (a, b, c) 6. (a, b, c, d)
7. (a, b, c, d)

PASSAGE BASED QUESTIONS :

1. (b) $R = \text{Radius} = \frac{24}{2} = 12$ cm.
$H = \text{Height} = 11$ m
Curved surface area of the cylindrical portion $= 2\pi RH$
$= 2\pi(12)(11) = (264\pi)\, m^2$

2. (c) h = Height of the cylindrical portion $= 16 - 11 = 5\, m$
Slant height, $l = \sqrt{h^2+R^2} = \sqrt{25+144} = 13\, m.$



3. (a) Area of canvas required for the tent = curved surface area of the cylindrical portion + curved surface area of the cone curved surface area of the cone $= \pi Rl$
$= \pi(12)(13) = (156\pi)\, m^2$

Hence, Area of canvas $= (264\pi + 156\pi)\, m^2$
$= (420\pi)\, m^2 = 420\times\frac{22}{7} = 1320\, m^2.$

ASSERTION & REASON :

1. (a) Assertion and Reason both are correct and reason is the correct explanation of the assertion .
Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r(h+r)$
$= 2\times\frac{22}{7}\times14(30+14) = 88(44)$
$= 3872\,\text{cm}^2$

2. (d) Assertion is incorrect here, but reason is correct. slant height $= \sqrt{(14/2)^2+(24)^2} = \sqrt{49+576}$
$= \sqrt{625} = 25$

3. (d) Assertion is incorrect and reason is true.
$\frac{V_1}{V_2}=\frac{(1/3)\pi r^2 h_1}{(1/3)\pi(r/2)^2 h_2}=\frac{4h_1}{h_2}$
as $V_1 = V_2$
$\therefore h_2 = 4h_1$

4. (a) Both assertion and reason are correct and reason is the correct explanation of the assertion.
5. (c) Assertion is correct but reason is not correct.
6. (b) Assertion and reason both are correct, but reason is not the correct explanation of the assertion. Volume of a prism = Base area ×height $= 25\times6 = 150\,\text{cm}^3$.

MULTIPLE MATCHING QUESTIONS :

1. (A) $\to$ p; q, r, s, t (B) $\to$ r; (C) $\to$ p; (D) $\to$ q
2. (A) $\to$ r; (B) $\to$ p; (C) $\to$ s,u; (D) $\to$ t

HOTS SUBJECTIVE QUESTIONS :

1. Diameter of the pipe $= 5\text{ mm} = \frac{5}{10}\text{ cm} = \frac{1}{2}\text{cm}.$
$\therefore$ Radius of the pipe $= \frac{1}{2}\times\frac{1}{2}\text{cm} = \frac{1}{4}\text{cm}$
In 1 minute, the length of the water column in the cylindrical pipe = 10m = 1000 cm.
$\therefore$ Volume of water that flows out of the pipe in 1 minute
$= \pi\times\frac{1}{4}\times\frac{1}{4}\times1000\text{cm}^3$

Also, volume of the cone $= \frac{1}{3}\times\pi\times20\times24\text{cm}^3$

Hence, the time needed to fill up this conical vessel
$= \left(\frac{20\times20\times24}{3}\times\frac{4\times4}{1000}\right) = \frac{4\times24\times16}{30}$ minutes
$= \frac{256}{5}$ minutes $= 51.2$ minutes

Hence, the required time is 51.2 minutes.

2. The bucket is the frustum of a cone of radius 15 cm.
To find the height of this cone, consider the similar triangles ABC and DEC.

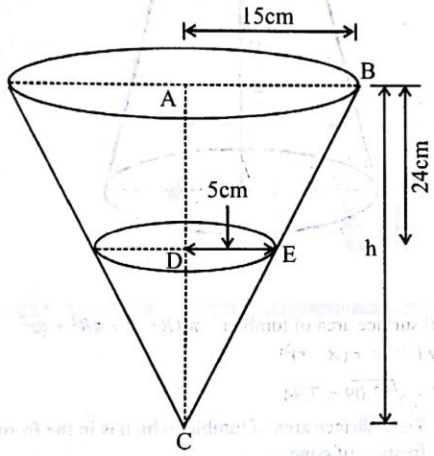
$$\frac{AB}{DE}=\frac{AC}{DC}$$

$$\Rightarrow \frac{15}{5}=\frac{h}{h-24} \quad \Rightarrow 3h-72=h$$

$\Rightarrow 2h=72 \Rightarrow h=36$

$\therefore\ DC=h-24=36-24=12$ cm.

$\therefore$ volume of bucket = volume of big cone − volume of small cone



$$=\frac{1}{3}\times\frac{22}{7}\times 25\times 12\times 26=\frac{57200}{7}=8171.43\text{ cm}^3$$

Slant height of big cone

$=\sqrt{r^2+h^2}=\sqrt{15^2+36^2}=\sqrt{225+1296}=\sqrt{1521}=39$cm.

Slant height of small cone

$=\sqrt{5^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$cm.

$\therefore$ curved surface area of bucket

= curved surface of big cone − curved surface of small cone

$=\pi\times 15\times 39-\pi\times 5\times 13=\pi\times 5\times 13[(3\times 3)-1]$

$$=\frac{22}{7}\times 65\times 8\text{ cm}^2=\frac{11440}{7}\text{cm}^2$$

Area of base of bucket $=\pi r^2=\frac{22}{7}\times 5^2=\frac{550}{7}\text{cm}^2$

$\therefore$ total surface area

$$=\frac{11470}{7}+\frac{550}{7}=\frac{11990}{7}\text{cm}^2=1712.86\text{cm}^2$$

3. Volume of the pool $=\pi(5)^2(1.4)$ cu cm. ...(1)

Volume of water poured by the pipe in 1 minute = 20 × 60 litres = 1.2 cu m

Time (in min.) taken to fill the pool

$$=\frac{\text{Volume of the pool}}{\text{Volume of water pouring into the pool in one minute}}$$

$=91\frac{2}{3}$ minutes

Let r cm be the radius of the second pipe, i.e., $\frac{r}{100}m$ is the radius of the second pipe.

Amount of water that comes out through this pipe in 2 minutes

$=\pi\left(\frac{r}{100}\right)^2(120)\times 42$ *cu m* ...(2)

Equating (1) and (2), we have

$r=\frac{50}{6}m$ or $8\frac{1}{3}m$

Hence, the radius of the pipe is $8\frac{1}{3}$m or 833.3 m.

4. Here, P is the mid-point of OQ, i.e., $OP=PQ$

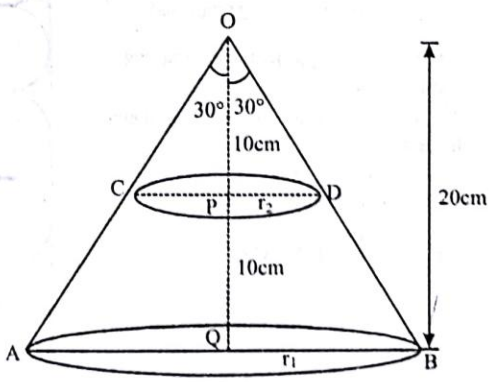
Let r_1 and r_2 ($r_1>r_2$) be the radii of the two circular ends of the frustum.

$\Rightarrow r_1=\frac{20}{\sqrt{3}}$cm. and $r_2=\frac{10}{\sqrt{3}}$cm.

The height (h) of the frustum is 10 cm.

So, the volume of the frustum

$=\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$ cu units $=\frac{7000}{9}\pi$ cu cm



Now let us suppose that x cm. of wire with diameter $\frac{1}{16}$cm is made of the volume of the frustum.

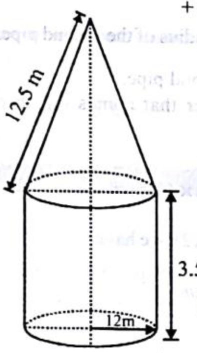
Then, $\pi\left(\frac{1}{32}\right)^2 x = \frac{7000}{9}\pi$

$\Rightarrow x = \frac{7000}{9} \times 32 \times 32\, cm = \frac{7168000}{9} cm = 7.9644$ km.

5. (a) There is 125% increase in the surface area of the cube.
   (b) 3 : 2

6. Capacity of the building = Volume of cylindrical part + volume of conical part

$= 2112\, m^3$

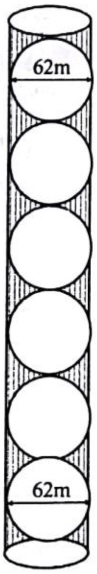


Hence internal curved surface area = 735.43 m^2 and capacity of the building 2112 m^3.

7. Diameter of the tennis balls = 62 *mm*.

$\because$ Radius of the balls and tube is half the diameter

$\therefore r = \frac{1}{2} \times 62 = 31\, mm$



Volume of one ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 31 \times 31 \times 31 = 124.788\, mm^3$

$\therefore$ Volume of 6 tennis balls = 6× 124.788

$= 748.728\, mm^3$

Height of tube (h) is 6 times the diameter of the ball

$\therefore$ h = 6 × 62 = 372 *mm*

$\therefore$ Volume of the tube

$= \pi r^2 h = \frac{22}{7} \times 31 \times 31 \times 372$

$= 1123.094\, mm^3$

$\therefore$ Volume of unfilled space (shaded area) in the tube

$= 1123.094 - 748.728 = 374.366\, mm^3$

Space as a percentage of the volume of the tube $= \frac{374.366}{1123.094} \times 100 = 33.33$

Hence volume of unfilled space is 33.3% of tube.

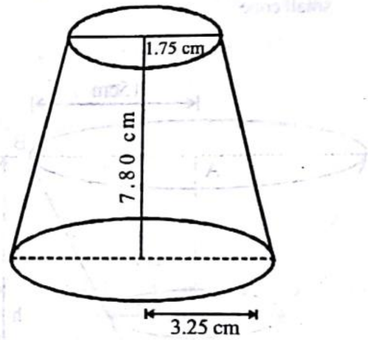
8. Here $R = \frac{1}{2} \times 6.50 = 3.25$ cm

$r = \frac{1}{2} \times 3.50 = 1.75$ cm

$h = 7.80$ cm.

Volume of tumbler which is in the form of a frustum of a cone

$= \frac{\pi h}{3}\left[R^2 + r^2 + Rr\right] = 157.8\, cm^3$



Total surface area of tumbler $= \pi l(R + r) + \pi R^2 + \pi r^2$

Now $l^2 = h^2 + (R - r)^2$

$\Rightarrow l = \sqrt{63.09} = 7.94$

$\therefore$ Total surface area of tumbler which is in the from of a frustum of cone

$= \frac{22}{7} \times 7.94(3.25 + 1.75) + \frac{22}{7}(3.25)^2 + \frac{22}{7}(1.75)^2$

$= \frac{22}{7} \times 55.325 = 167.6\, cm^2$

9. (i) The number of the edges = The number of sides of the base × 3 = 3× 3 = 9
   (ii) The volume of the prism = Area of the base × Height of the prism $= \frac{1}{2}(3\times4)\times7 = 42\, m^3$
   (iii) TSA = LSA + 2 (area of base) = ph + 2 (area of base) where, p = perimeter of the base = sum of lengths of the sides of the given triangle,

As, hypotenuse of the triangle $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ m

$\therefore$ Perimeter of the base = 3 + 4 + 5 = 12 m

$\Rightarrow$ LSA $= ph = 12 \times 7 = 84\, m^2$

TSA = LSA + 2 (area of base) $= 84 + 2\left(\frac{1}{2}\times 3 \times 4\right)$

$= 84 + 12 = 96\, m^2$

chapter 14

$$\overline{X} = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

STATISTICS

Introduction

Statistics is the science with deals with the collection, analysis and interpretation of numerical data. In Mahabharata times, king Rituparna while going along with King Nala for the Swaymbara of Damyanti, estimated accurately the number of leaves and fruits on the basis of sample of a tree. The administrative setup were based on registration of births and deaths in the region of Chandragupta Maurya (324-300 B.C.) is found in Kautilya's Arthashastra. Similarly in Ain-e-Akbari written by Abul Fazal detailed description of the administrative and statistical surveys conducted during Akbar's region (1556-1605 A.D.) is found. King Todarmal maintained a systematic record of land and agricultural statistics. Similarly many examples of use of statistics in administrative setup and in wars are found.

In earlier classes, you have studied the classification of ungrouped data into grouped frequency distributions and to represent the grouped data pictorially in the form of various graphs such as bar graph, histogram and frequency polygon. You have also learnt to find the central tendency namely mean, median and mode of ungrouped data. In this chapter you will study the concept of cumulative frequency; the cumulative frequency distribution; mean, mode and median of descrete and continuous frequency distribution; to draw cumulative frequency curve, called ogive and to find the median by using the ogive.

DISCRETE AND CONTINUOUS FREQUENCY DISTRIBUTION :

(i) **Discrete Frequency Distribution :** If each data is given with their frequency, then this type of frequency distribution is called Discrete Frequency Distribution.

(ii) **Continuous Frequency Distribution :** If the data is given in the form of class interval with frequency, then this type of frequency distribution is called Continuous Frequency Distribution.

MEAN OF DISCRETE FREQUENCY DISTRIBUTION :

(i) **Direct Method:** For the discrete frequency distribution

x_i	x_1	x_2	x_3	....	x_n
f_i	f_1	f_2	f_3	....	f_n

$$\text{Mean } (\overline{x}) = \frac{f_1x_1 + f_2x_2 + \text{..............} + f_nx_n}{f_1 + f_2 + \text{..............} + f_n} = \sum_{i=1}^{n} \frac{f_ix_i}{\Sigma f_i}$$

ILLUSTRATION 14.1

Find the arithmetic mean of the following frequency distribution :

x	4	7	10	13	16	19
f	7	10	15	20	25	30

SOLUTION :

x_i	f_i	f_ix_i
4	7	28
7	10	70
10	15	150
13	20	260
16	25	400
19	30	570
	$\Sigma f_i = 107$	$\Sigma f_ix_i = 1478$

$$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1478}{107} = 13.81$$

ILLUSTRATION 14.2

Find the value of k if mean of the following data is 14

x_i	5	10	15	20	25
f_i	7	k	8	4	5

SOLUTION :

x_i	f_i	f_ix_i
5	7	35
10	k	$10k$
15	8	120
20	4	80
25	5	125
Total	$\sum_{i=1}^{n} f_ix_i = 24 + k$	$\sum_{i=1}^{n} f_ix_i = 360 + 10k$

Now, $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \Rightarrow 14 = \dfrac{360+10k}{24+k}$

$\Rightarrow 336 + 14k = 360 + 10k$

$\Rightarrow 14k - 10k = 360 - 336$

$\Rightarrow 4k = 24 \Rightarrow k = 6$

(ii) Assumed Mean Method :

$\bar{x} = A + \frac{1}{N}\sum_{i=1}^{n} f_i d_i$, where A is assumed mean, $d_i = x_i - A$ and $N = \sum_{i=1}^{n} f_i$

NOTE: Generally the middle data is considered as assumed mean.

If there are two middle data, then you can take those middle data as assumed mean, whose frequency is greater.

ILLUSTRATION 14.3

Compute the mean height by Assumed mean method of plants from the following frequency distribution :

Height in cm	61	64	67	70	73
No. of plants	5	18	42	27	8

SOLUTION :

Height (x_i)	No. of plants (f_i)	$d_i = x_i - A$	$f_i d_i$
61	5	–6	–30
64	18	–3	–54
67 (A)	42	0	0
70	27	3	81
73	8	6	48
	$N = \sum_{i=1}^{n} f_i = 100$		$\sum_{i=1}^{n} f_i d_i = 45$

Let assumed mean = 67 (generally middle data)

$\Rightarrow \bar{x} = A + \frac{1}{N}\sum_{i=1}^{n} f_i d_i \Rightarrow \bar{x} = 67 + \frac{45}{100} = 67 + 0.45 = 67.45$

(iii) Step Deviation (or Short cut) Method :

This method is used if the deviation d_i's are divisible by any common number C.

$\bar{x} = A + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) C$ where $u_i = \dfrac{x_i - A}{C}$

ILLUSTRATION 14.4

Find the arithmetic mean for the following frequency distribution :

x_i	5	10	15	20	25	30	35	40	45	50
f_i	20	43	75	67	72	45	39	9	8	6

SOLUTION :

First of all we construct the calculation table by taking 25 as assumed mean (A).

x_i	Frequency (f_i)	$u_i = \frac{x_i - 25}{5}$	$f_i u_i$
5	20	−4	−80
10	43	−3	−129
15	75	−2	−150
20	67	−1	−67
25 (A)	72	0	0
30	45	1	45
35	39	2	78
40	9	3	27
45	8	4	32
50	6	5	30
Sum	$N = \Sigma f_i = 384$		$\Sigma f_i u_i = -214$

Thus Arithmetic mean $(\bar{x}) = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h = 25 + \left(\frac{-214}{384}\right) \times 5 = 25 - 2.786 = 22.214$

ILLUSTRATION 14.5

The measurements (in milli-metres) of the diameter of the heads of 107 screws are given below :

Diameter in mm (x_i)	34	37	40	43	46
No. of screws (f_i)	17	19	23	21	27

Calculate mean diameter by step deviation method of the heads of the screws.

SOLUTION :

Let us suppose assume mean, $A = 40$

As $x_i - 40$, where $x_i = 34, 37, 40, 43, 46$, is divisible by 3, therefore $C = 3$

Let us reconstruct the table :

x_i	f_i	$u_i = \frac{x_i - 40}{3}$	$f_i u_i$
34	17	−2	−34
37	19	−1	−19
40(A)	23	0	0
43	21	1	21
46	27	2	54
	$\Sigma f_i = 107$		$\Sigma f_i u_i = 22$

$$\Rightarrow \bar{x} = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times C \quad \Rightarrow \bar{x} = 40 + \frac{22 \times 3}{107} = 40 + 0.6 = 40.6mm \text{ nearly.}$$

MEAN OF CONTINUOUS FREQUENCY DISTRIBUTION :

(i) Direct Method:

For the discrete frequency distribution

x_i	x_1	x_2	x_3	....	x_n
f_i	f_1	f_2	f_3	....	f_n

$$\text{Mean }(\overline{x}) = \frac{f_1x_1 + f_2x_2 + f_3x_3 + ...f_nx_n}{f_1 + f_2 + f_3 + ... + f_n}$$

ILLUSTRATION 14.6

The following table shows the marks secured by 100 students in an examination.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students	15	20	35	20	10

Find mean marks obtained by a student.

SOLUTION :

Marks	Mid value	Frequency (f_i)	f_ix_i
0 – 10	5	15	75
10 – 20	15	20	300
10 – 30	25	35	675
30 – 40	35	20	700
40 – 50	45	10	450
Total	-------	$\Sigma f_i = 100$	$\Sigma f_ix_i = 2400$

$$\overline{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{2400}{100} = 24$$

(ii) Assumed Mean Method :

$$\text{Mean }(\overline{x}) = A + \frac{\Sigma f_id_i}{\Sigma f_i}$$

Here, $d_i = x_i - A$

x_i = mid- point of ith class interval

A = Assumed mean, which is generally the mid-value of middle most class Interval.

If there are two middle class interval, then you can take mid-point of that middle class interval whose frequency is more as assumed mean.

ILLUSTRATION 14.7

Find the mean of the following by Assumed Mean Method.

Class Interval	Frequency
0–4	6
4–8	3
8–12	6
12–16	16
16–20	3
20–24	14
24–28	10
28–32	8

SOLUTION :

Class Interval	Mid-point value (x_i)	Frequency (f_i)	Deviation (d_i) = $x_i - A$	$f_i d_i$
0–4	2	6	–12	–72
4–8	6	3	–8	–24
8–12	10	6	–4	–24
12–16	14 (A)	16	0	0
16–20	18	3	4	12
20–24	22	14	8	112
24–28	26	10	12	120
28–32	30	8	16	128
		$\Sigma f_i = 66$		$\Sigma f_i d_i = 252$

$$Mean = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 14 + \frac{252}{66} = 14 + 3.818 = \mathbf{17.818}$$

(iii) Step Deviation or Short cut Method:

Mean $(\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$, Here, A = Assumed mean, $u_i = \frac{x_i - A}{h}$, x_i = Mid-point of ith Class Interval, h = Class Size

ILLUSTRATION – 14.8

Calculate the mean for the following frequency distribution (By step deviation method).

Class interval	0 – 80	80 – 160	160 – 240	240 – 320	320 – 400
frequency	22	35	44	25	24

SOLUTION :

Class Interval	Mid - value (x_i)	f_i	$u_i = (x_i - A)/h$	$f_i u_i$
0 – 80	40	22	–2	–44
80 – 160	120	35	–1	–35
160 – 240	200 = (A)	44	0	0
240 – 320	280	25	1	25
320 – 400	360	24	2	48
Total		$\Sigma f_i = 150$		$\Sigma f_i u_i = -6$

$$\text{Mean } \bar{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 200 + \frac{-6}{150} \times 80$$

$$= 200 - \frac{2 \times 8}{5} = 200 - \frac{16}{5} = \frac{1000 - 16}{5} = \frac{984}{5} = 196.8$$

MODE OF DISCRETE FREQUENCY DISTRIBUTION :

If x_i is the observation whose frequency is maximum then mode = x_i
If there are more than one observations whose frequency is maximum, then all those data are mode.

x_i	x_1	x_2	x_3	....	x_n
f_i	f_1	f_2	f_3	....	f_n

MODE OF CONTINUOUS FREQUENCY DISTRIBUTION :

Mode = $\ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$, where

ℓ = lower limit of the modal class*

h = size of the class interval

f_1 = frequency of the modal class

f_0 = fequency of the class preceding the modal class

f_2 = fequency of the class succeding the modal class.

***NOTE :** Modal class is the class having maximum frequency.

ILLUSTRATION 14.9

The given distribution shows the number of runs scored by some top batsman of the world in one-day international cricket matches. Find the mode of the data.

Runs scored	No. of batsman
3000 – 4000	4
$\boxed{4000-5000}$	$\boxed{18}$
5000 – 6000	9
6000 – 7000	7
7000 – 8000	6
8000 – 9000	3
9000 – 10000	1
10000 – 11000	1

SOLUTION :

Here, maximum frequency is 18. So the modal class is 4000-5000.

So, $\ell = 4000$, $h = 1000$, $f_1 = 18$, $f_0 = 4$, $f_2 = 9$

$$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 4000 + \left(\frac{18-4}{2\times18-4-9}\right)\times 1000 = 4000 + \frac{14}{23}\times 1000 = 4000 + 608.7 = 4608.7$$

if n (the number of observations) is odd and if n is even, then the median is $\frac{1}{2}\left[\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right]$ observations.

MEDIAN OF THE DISCRETE FREQUENCY DISTRIBUTION :

To find the median of the discrete frequency distribution, first arrange the observations in ascending order and make the cumulative frequency table by seeing the cumulative frequency table, check which one observation is the middle most. The middle most observation is the median. If there are two middle most observations, the median is the average of these two observation.

MEDIAN OF CONTINUOUS FREQUENCY DISTRIBUTION :

Algorithm to find the Median :

Step I : Make cumulative frequency table.

Step II : Choose the median class. Median class is the class whose cumulative frequency is greater than and nearest to $\frac{n}{2}$, where n is the sum of all frequencies.

Step III : Use this formula

$$\text{Median} = \ell + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

Where, ℓ = lower limit of median class
n = sum of all frequencies (or sum of all observations)
cf = cumulative frequency of class preceding the median class
f = frequency of the median class
h = class size

ILLUSTRATION – 14.10

The following table gives the distribution of the life time of 400 neon lamps.

Life time (in hours)	No. of Lamps
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

SOLUTION :

Class - Intervals	Frequency	Cumulative frequency
1500 – 2000	14	14
2000 – 2500	56	70
2500 – 3000	60	130
$\boxed{3000-3500}$	$\boxed{86}$	$\boxed{216}$
3500 – 4000	74	290
4000 – 4500	62	352
4500 – 5000	48	400

Here n = 400

$\therefore \frac{n}{2} = \frac{400}{2} = 200$

$\therefore$ Median class is 3000 – 3500.

So, $\ell = 3000, f = 86, h = 500, cf = 130$

$$\therefore \text{ Median } = \ell + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h = 3000 + \left[\frac{\frac{400}{2} - 130}{86}\right] \times 500 = 3000 + \frac{70}{86} \times 500 = 3406.98$$

THE RELATIONSHIP BETWEEN MEAN, MEDIAN AND MODE:

3 Median = Mode + 2 Mean

CUMULATIVE FREQUENCY CURVE (OGIVE) :

Cumulative frequency curve or an ogive is the graphical representation of a cumulative frequency distribution. There are two methods of constructing an ogive

(a) Less than method (b) More than method

(a) **Less Than Method:** Following are the steps for construction of less than ogive:

(i) Construct a cumulative frequency table of less than type.
(ii) Mark upper class limits along X-axis.
(iii) Mark the corresponding cummulative frequency along Y-axis.
(iv) Plot the points and join them by a smooth free hand curve

The curve we get is the **Less Than Type Ogive.**

ILLUSTRATION—14.11

Draw less than type ogive for the following data.

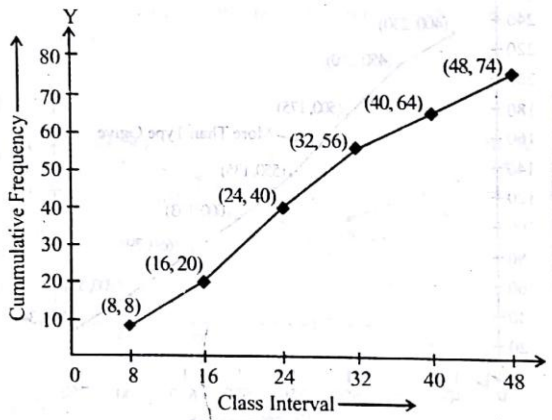
Class Interval	0-8	8-16	16-24	24-32	32-40	40-48
Frequency	8	12	20	16	8	10

SOLUTION:

First we prepare a Less Than Type frequency distribution table.

Class Interval	Cumulative Frequency
Less than 8	8
Less than 16	8 + 12 = 20
Less than 24	20 + 20 = 40
Less than 32	40 + 16 = 56
Less than 40	56 + 8 = 64
Less than 48	64 + 10 = 74

Now, mark the upper class limit on X-axis and frequency along Y-axis. plot the points (8, 8), (16, 20), (24, 40), (32, 56), (40, 64) and (48, 74). Join the points by smooth free hand curve to get the required less than type ogive.



(b) **More Than Method :**

Following are the steps for constructing more than type ogive.

(i) Construct a more than type frequency distribution table

(ii) Mark the lower class limit on X- axis.

(iii) Mark the corresponding cumulative frequencies on Y- axis.

(iv) Plot the points and join them by smooth free hand curve.

The obtained curve is the **More Than Type Ogive.**

ILLUSTRATION—14.12

The frequency distribution of scores obtained by 230 candidates in a medical entrance test is as follows:

Scores	400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-800
No. of Candidate	20	35	40	32	24	27	18	34

Draw cumulative frequency curve by more than method.

SOLUTION:

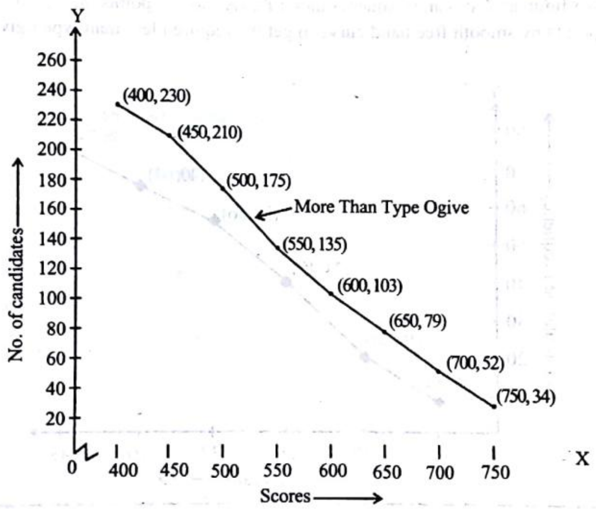
First convert the given frequency distribution table to More Than Type frequency distribution table.

Scores	No. of Candidates
More than or equal to 400	230
More than or equal to 450	230 - 20 = 210
More than or equal to 500	210 - 35 = 175
More than or equal to 550	175 - 40 = 135
More than or equal to 600	135 - 32 = 103
More than or equal to 650	103 - 24 = 79
More than or equal to 700	79 - 27 = 52
More than or equal to 750	52 - 18 = 34

Now mark the lower limits along X-axis and cumulative frequencies along Y-axis, and plot the points (400, 230), (450, 210), (500, 175), (550, 135), (600, 103), (650, 79), (700, 52), (750, 34).

Join the points listed above by smooth free hand curve to obtain the more than type ogive.

Scale:



MEDIAN BY GRAPH :

We can find the median graphically in two ways :

(a) By drawing more than and less than Ogives.

(b) By drawing only less than Ogive.

(a) Median by Drawing Both More Than and Less Than Ogives:

If we draw the two types of curves i.e. Less Than type Ogive and More Than type Ogive on same pair of axes, then these two curves intersects each other at a point. From this point of intersection, if we draw a perpendicular to X-axis, it will intersect the X-axis at some point. The X-coordinate of this point is the **median.**

(b) Median by Drawing Only Less than Ogive:

General, we use only Less Than type Ogive to calculate the median. Steps are as follows:

(i) First prepare a less than type cumulative frequency distribution table.

(ii) Draw less than cumulative frequency curve (ogive).

(iii) If total number of observations is N, then locate $\frac{N}{2}$ on cumulative frequency axis i.e. Y-axis.

(iv) From this point on Y-axis, draw a horizontal line parallel to X-axis which meet the ogive at some point.

(v) Through this point on the curve, draw a perpendicular to X-axis.

(vi) The perpendicular meets the X-axis at some point. X-coordinate of this point is the required median.

ILLUSTRATION-14.13

For the following frequency distribution, determine the median by drawing ogives.

Marks Obtained	50-60	60-70	70-80	80-90	90-100
No. of Students (Cumulative frequency)	4	8	12	6	6

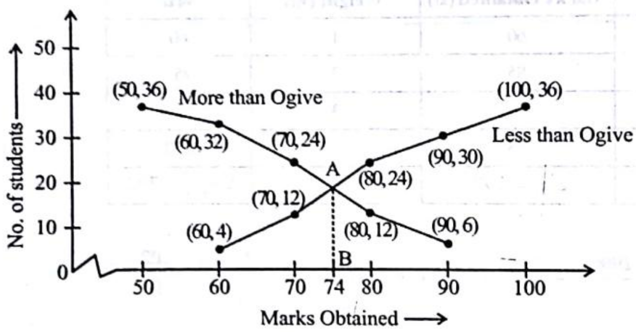
SOLUTION:

Marks Obtained	No. of Students (Cumulative frequency)
More than or equal to 50	36
More than or equal to 60	$36-4=32$
More than or equal to 70	$32-8=24$
More than or equal to 80	$24-12=12$
More than or equal to 90	$12-6=6$

Marks Obtained	No. of Students (Cumulative frequency)
Less than 60	4
Less than 70	12
Less than 80	24
Less than 90	30
Less than 100	36

Median by drawing both the Ogives :

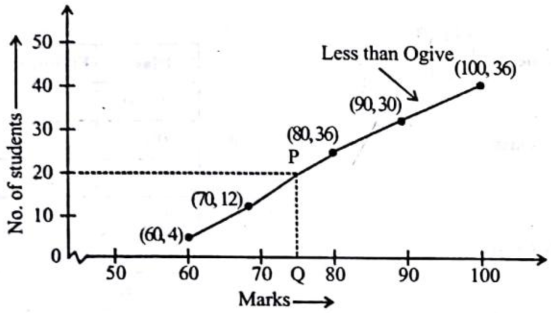
On same pair of axes plot (60, 4), (70, 12), (80, 24), (90, 30) and (100, 36) for less than type ogive and plot (50, 36) (60,32), (70, 24), (80, 12) & (90, 6) for more than type ogive.



Perpendicular from point of intersection A of two curves intersect X- axis at point B, which corresponds to 74 marks. Hence median = 74.

Median by drawing Less than Ogive only :

Plot the points (60, 4), (70, 12), (80, 24), (90, 30) and (100, 36) for less than ogive.



Locate $\frac{N}{2}=\frac{36}{2}=18$ on Y-axis and draw horizontal line through 18 on Y-axis which intersects curve at point P. Draw perpendicular from P on X-axis which intersects X-axis at Q. Value corresponding to point Q is the median. Value of point Q = 74. Hence Median = 74.

MISCELLANEOUS SOLVED EXAMPLES

1. The following table contains the marks obtained by a student of class XI and the approved weightage for every subject prescribed by the selection committee of a professional college.

S.No.	Subject	Weightage	Marks Obtained
1	English	1	60
2	Mathematics	3	85
3	Physics	3	79
4	Chemistry	2	75

Compare the arithmetic mean and weighted mean of the marks obtained.

Sol. Calculation of mean and weighted mean

S.No.	Subject	Marks Obtained (x_i)	Weight (w_i)	$w_i x_i$
1	English	60	1	60
2	Mathamatics	85	3	255
3	Physics	79	3	237
4	Chemistry	75	2	150
	Total	$\Sigma x_i = 299$	9	$\Sigma w_i x_i = 702$

Thus Mean $= \frac{\sum x_i}{n} = \frac{299}{4} = 74.75$ Marks and weighted mean $= \frac{\sum w_i x_i}{\sum w_i} = \frac{702}{9} = 78$ marks.

2. Calculate the median for the following distribution class :

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	5	10	20	7	8	5

Sol. (i) first we find $\left(\frac{N}{2}\right)^{th}$ value i.e. $\left(\frac{55}{2}\right)^{th} = 27.5^{th}$

which lies in 20 – 30.

$\therefore$ 20 – 30 class is median class.

Here $\ell = 20$

$\frac{N}{2} = 27.5$, c.f. $= 15, f = 20, h = 10$

$\therefore$ median $= 20 + \frac{275-15}{20} \times 10$

Median $= 26.25$

Class	Frequency	c.f.
0 – 10	5	5
10 – 20	10	15
20 – 30	20	35
30 – 40	7	42
40 – 50	8	50
50 – 60	5	55

3. A family requires the commodities listed in the table below for regular use. The importance (weights) attached to each commodity is also given. Find the difference in the mean and the weighted mean price per kg.

Commodity	Weight (kg)	Price/kg (Rs)
Rice	1	10.50
Wheat	3	2.75
Pulses	2	8.50
Vegetable	1	5.00
Oils	1	26.00

Sol. For calculating weighted mean and mean price per kg, we prepare the following table :

Commodity	Weight (w) (kg)	Price/kg (in Rs) (x)	Product (wx)
Rice	1	10.50	10.50
Wheat	3	2.75	8.25
Pulses	2	8.50	17.00
Vegetable	1	5.00	5.00
Oils	1	26.00	26.00
Total	8	52.75	66.75

$$\text{Mean} = \frac{52 \cdot 75}{5} = 10.55$$

$$\text{Weighted mean} = \frac{66 \cdot 75}{8} = 8.34$$

$\therefore$ Difference $= 10.55 - 8.34 = 2.21$

$\therefore$ Difference in price per kg = Rs 2.21

4. The mean of the marks secured by 25 students of section A of class X is 47, that of 35 students of section B is 51 and that of 30 students of section C is 53. Find the combined mean of the marks of students of three sections of class X.

Sol. Mean of the marks of 25 students of $XA = 47$

$\therefore$ Sum of the marks of 25 students $= 25 \times 47 = 1175$ ....(i)

Mean of the marks of 35 students of $XB = 51$

$\therefore$ Sum of the marks of 35 students $= 35 \times 51 = 1785$ ....(ii)

Mean of the marks of 30 students of $XC = 53$

$\therefore$ Sum of the marks of 30 students $= 30 \times 53 = 1590$ ....(iii)

Adding (i), (ii) and (iii)

Sum of the marks of $(25 + 35 + 30)$ i.e., 90 students $= 1175 + 1785 + 1590 = 4550$

Thus the combined mean of the marks of students of three sections $= \frac{4550}{90} = 50.56$

5. Find the mean of the following distribution :

Class interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	11	18	24	17	11	9

Sol.

Class interval	Frequency (f_i)	Midpoint (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
0 – 20	11	10	–2	–22
20 – 40	18	30	–1	–18
40 – 60	24 (A)	50	0	0
60 – 80	17	70	1	17
80 – 100	11	90	2	22
100 – 120	9	110	3	27
Total	90			26

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 50 + \frac{26}{90} \times 20 = 50 + \frac{52}{9} = 50 + 5.78 = 55.78.$$

6. The mid values of a distribution are 54, 64, 74, 84 and 95. Find the class interval and class limits.

Sol. The class interval is the difference of two consecutive class marks, therefore class interval (h) = 64 – 54 = 10.
Here the mid values are given and class interval is 10.
So the class limits are

For 1st class : $54 - \frac{10}{2}$ to $54 + \frac{10}{2}$ or 49 to 59

For 2nd class : $64 - \frac{10}{2}$ to $64 + \frac{10}{2}$ or 59 to 69

For 3rd class : $74 - \frac{10}{2}$ to $74 + \frac{10}{2}$ or 69 to 79

For 4th class : $84 - \frac{10}{2}$ to $84 + \frac{10}{2}$ or 79 to 89

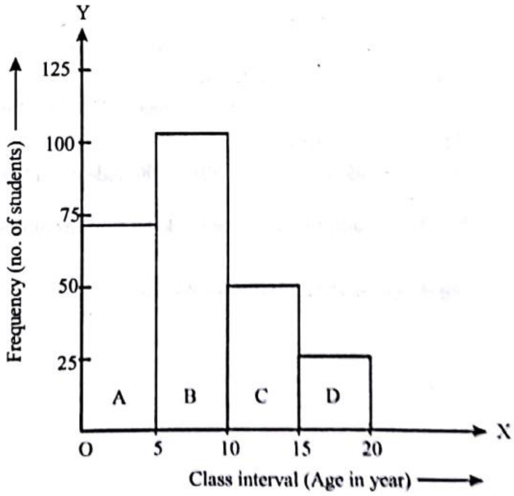
For 5th class : $94 - \frac{10}{2}$ to $94 + \frac{10}{2}$ or 89 to 99

Therefore, class limits are 49 – 59, 59 – 69, 69 – 79, 79 – 89 and 89 – 99.

7. Draw a histogram of the following frequency distribution :

Class (Age in year)	0-5	5-10	10-15	15-20
No. of students	72	103	50	25

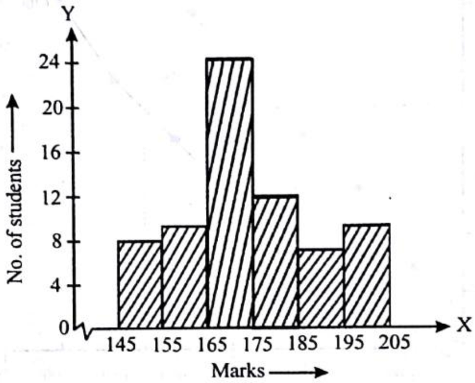
Sol. Here frequency distribution is grouped and continuous and class intervals are also equal. So mark the class intervals on the X-axis i.e., age in year (scale = 1 cm = 5 year). Mark frequency i.e., number of students (scale 1cm = 25 students) on the Y-axis. Now, since the number of students on class interval 0–5 is 72, so draw a line parallel to X-axis in front of frequency to construct a rectangle on class interval 0–5.
Repeating this procedure, construct rectangle B, C and D.



8. Construct a histogram from the following distribution of total marks obtained by 65 students of *X* class in the final examination.

Marks (mid-ponts)	150	160	170	180	190	200
Number of females	8	9	25	12	7	9

Sol. Lower and upper class limits :
Since the difference between the second and first mid-point is 160-150 = 10



$\therefore\ h = 10 \Rightarrow \frac{h}{2} = 5$

So, lower and upper limits of the first class are $150 - 5$ and $150 + 5$ i.e. 145 and 155 respectively.

$\therefore$ First class interval is $145 - 155$

Using the same procedure, we get the classes of other mid-points us under :

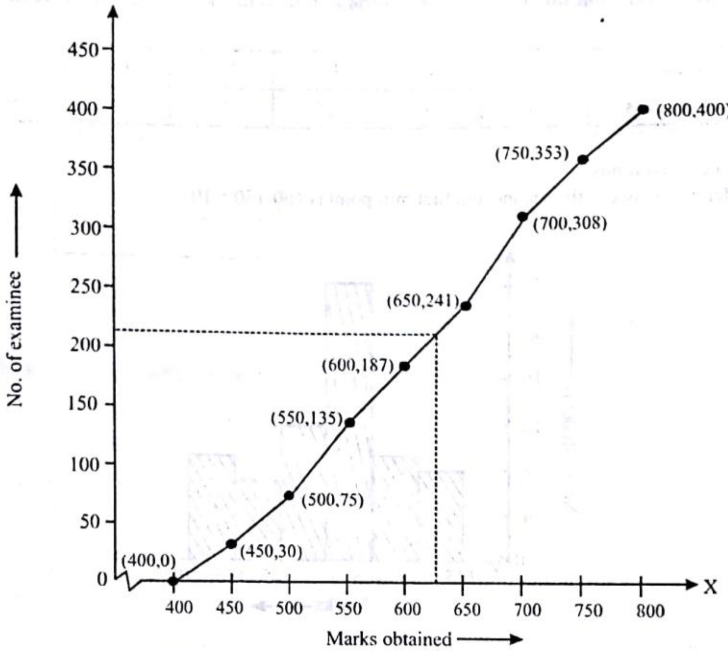
Class	145-155	155-165	165-175	175-185	185-195	195-205
Frequency	8	9	25	12	7	9

9. The marks obtained by 400 students in medical entrance exam are given in the following table :

400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-800
30	45	60	52	54	67	45	47

(a) Draw Ogive by less than method
(b) Draw Ogive by more than method.
(c) Find the number of examinees, who have obtained marks less than 625.
(d) Find the number of examinees, who have obtained 625 and more marks.

Sol. **(a)** Cumulative frequency table for less than Ogive method is as following :



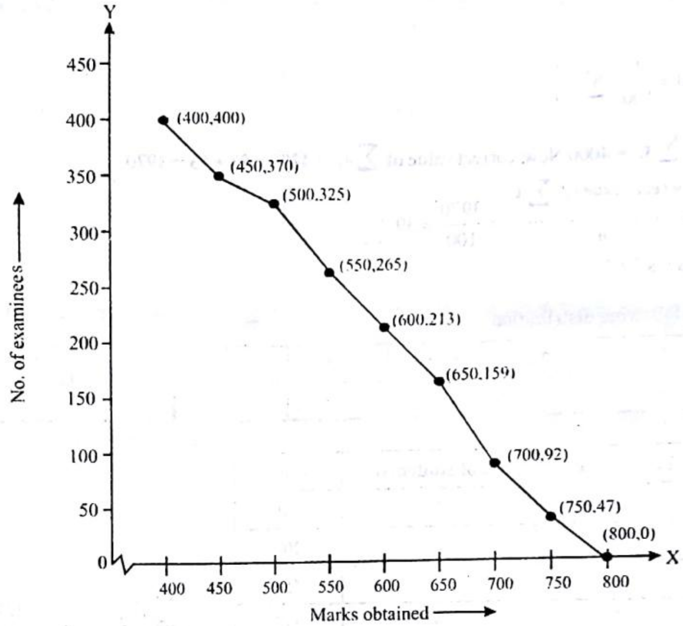
Marks Obtained	No. of Examinees
Less than 450	30
Less than 500	75
Less than 550	135
Less than 600	187
Less than 650	241
Less than 700	308
Less than 750	353
Less than 800	400

Adjoining figure is Ogive for the above cumulative frequency table by applying the given method and the assumed table.

(b) Cumulative frequency table for more than Ogive method is as following :

Marks Obtained	No. of Examinees
400 and more	400
450 and more	370
500 and more	325
550 and more	265
600 and more	213
650 and more	159
700 and more	92
750 and more	47

Following are the Ogive for the above cumulative frequency table.



(c) So, the number of examinees, scoring marks less than 625 are approximately 220.

(d) So, the number of examinees, scoring marks 625 and more will be approximately 190.

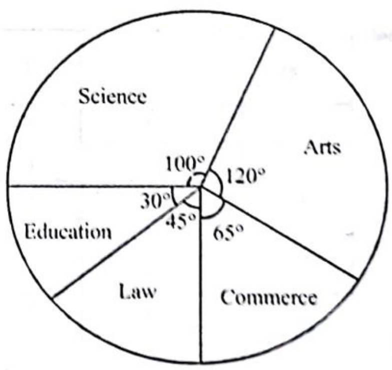
10. The number of students admitted in different faculties of a college are

Faculty	Science	Arts	Commerce	Law	Education	Total
No. of students	1000	1200	650	450	300	3600

Construct the pie chart for the following distribution.

Sol. The following table gives the share of each faculty as a component of 360°.

Faculty	No. of students	Share as a component of 360°
Science	1000	100°
Arts	1200	120°
Commerce	650	65°
Law	450	45°
Education	300	30°
	3600	



11. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

Sol. $n = 100,\ \bar{x} = 40$

$$\bar{x} = \frac{1}{n}\left(\sum x_i\right) \Rightarrow 40 = \frac{1}{100}\left(\sum x_i\right)$$

$\therefore$ Incorrect value of $\sum x_i = 4000$ Now, correct value of $\sum x_i = 4000 \$ 83 + 53 = 3970$

$$\therefore \text{ Correct mean} = \frac{\text{correct value of } \sum x_i}{n} = \frac{3970}{100} = 39.7$$

So, the correct mean is 39.7.

12. Find the mean for the following distribution :

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	6	8	13	7	3	2	1

Sol.

Items	Mid values x_i	No. of students f_i	$f_i x_i$
10-20	15	6	90
20-30	25	8	200
30-40	35	13	455
40-50	45	7	315
50-60	55	3	165
60-70	65	2	130
70-80	75	1	75
		$\Sigma f_i = 40$	$\Sigma f_i x_i = 1430$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1430}{40} = \frac{143}{4} = 35.75$$

13. Find the mean of following distribution with step-deviation method :

Class	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	5	6	8	12	6	3

Sol.

Class	x_i	f_i	$u_i = \frac{x_i - 27.5}{5}$	$f_i u_i$
10-15	12.5	5	$3	$15
15-20	17.5	6	$2	$12
20-25	22.5	8	$1	$8
25-30	27.5 (A)	12	0	0
30-35	32.5	6	1	6
35-40	37.5	3	2	6
		$\Sigma f_i = 40$		$\Sigma f_i u_i = \$\ 23$

$$\bar{x} = A + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) h \Rightarrow \bar{x} = 27.5 + 5 \times \left(\frac{-23}{40}\right) = 24.7$$

14. Find the mode of the following distribution :

Daily wages	31-36	37-42	43-48	49-54	55-60	61-66
No. of workers	6	12	20	15	9	4

Sol.

Daily wages	No. of workers	Daily wages	No. of workers
31-36	6	30.5-35.5	6
37-42	12	35.5-42.5	12
43-48	20	42.5-48.5	20
49-54	15	48.5-54.5	15
55-60	9	54.5-60.5	9
61-66	4	60.5-66.5	4

Modal class frequency is 42.5 – 48.5

$\ell = 42.5, f_1 = 20,\ f_0 = 12,\ f_2 = 15,\ h = 6\ \therefore$ Mode $= 42.5 + \dfrac{20-12}{2(20)-12-15} \times 6,\ \therefore$ Mode $= 46.2$

15. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the mean, median and mode of the data.

Monthly consumption (in units)	Number of consumers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

Sol.

Class interval	f_i	Cumulative frequency	x_i	$u_i = \dfrac{x_i - 275}{5}$	$f_i u_i$
65-85	4	4	75	–3	–12
85-105	5	9	95	–2	–10
105-125	13	22	115	–1	–13
125-145	20	42	135 (A)	0	0
145-165	14	56	155	1	14
165-185	8	64	175	2	16
185-205	4	68	195	3	12
	$\Sigma f_i = 68$				$\Sigma f_i u_i = 7$

Mean $= \ \bar{x} = A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h = 135 + \dfrac{7}{68} \times 20 = 137.05$

$\dfrac{n}{2} = \dfrac{68}{2} = 34 \Rightarrow$ 125 – 145 is the median class.

Median $= \ell + \left(\dfrac{\dfrac{n}{2} - c.f.}{f}\right) \times h = 125 + \left(\dfrac{34-22}{20}\right) \times 20 = 137$

Maximum frequency is 20 $\Rightarrow$ 125 – 145 is the modal class.

Mode $= \ell + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 125 + \left(\dfrac{20-13}{2 \times 20 - 13 - 14}\right) \times 20 = 125 + \dfrac{140}{13} = 135.76$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. The variance is the .... of the standard deviation.
2. Median divides the total frequency into .... equal parts.
3. The algebraic sum of the deviations from arithmetic mean is always ....
4. Percentile divides the number of items into .... equal parts.
5. The mean-deviation from the median is .... than that measured from any other value.
6. The class mark of a class is 25 and if the upper limit of that class is 40, then its lower limit is ....
7. The mid-value of 20-30 is ....
8. The sum of 12 observations is 600, then their mean is ....
9. Sizes of shoes are .... variables.
10. In the class interval 35-46, the lower limit is .... and upper limit is ....
11. The difference between the maximum and the minimum observations in data is called the .... of the data.
12. Consider the data : 2, 3, 2, 4, 5, 6, 4, 2, 3, 3, 7, 8, 2, 2. The frequency of 2 is ....
13. A class interval of a data has 15 as the lower limit and 25 as the size then the class mark is ....
14. 0-10, 10-20, 20-30 .... so on are the classes, the lower boundary of the class 20 - 30 is ....
15. The mid-point of a class interval is called its ....
16. Facts or figures, collected with a definite purpose, are called ....
17. Value of the middle-most observation (s) is called ....
18. The .... is the most frequently occurring observation.
19. 3 median = mode + ... mean

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The median for grouped data is formed by using the formula, $Median = l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$
2. Class mark = $\dfrac{\text{Upper class limit+Lower class limit}}{2}$
3. The median of grouped data with unequal class sizes cannot be calculated.
4. Classification brings out points of similarly and dissimilarity of data.
5. The modal value is the value of the variate which divides the total frequency into two equal parts.
6. The width of a rectangle in a histrogram represents frequency of the class.
7. The mean of x, y, z is y, then $x + z = 2y$
8. 2(Median – Mean) = Mode – Mean.
9. The ranges is the difference between the greatest and the least value of the variate.
10. Simple tabulation gives information about several independent characteristics.
11. Mean may or may not be the appropriate measure of central tendency.
12. If 16 observations are arranged in ascending order, then median is $\dfrac{(8\text{th observation} + 9\text{th observation})}{2}$
13. Median of 15, 28, 72, 56, 44, 32, 31, 43 and 51 is 42.
14. Mode of 2, 3, 4, 5, 0, 1, 3, 3, 4, 3 is 3.
15. Mean of 41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 49, 42, 52, 60 is 54.8

Match the Following

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1.

Column I	Column II
(A) the direct method	(p) $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} h$
(B) step deviation method	(q) $\ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
(C) mode	(r) $\ell + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$
(D) median	(s) $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

2. The table shows a frequency distribution of the life time of 400 radio tubes tested at a company.

Life time (hours)	Number of tubes	Life time (hours)	Number of tubes
300 – 399	14	800 – 899	62
400 – 499	46	900 – 999	48
500 – 599	58	1000 – 1099	22
600 – 699	76	1100 – 1199	6
700 – 799	68		
			Total 400

Column II gives data for description given in column I, match them correctly.

Column I	Column II
(A) upper limit of the fifth class	(p) 100
(B) lower limit of the eighth class	(q) 949.5
(C) class marks of the seventh class	(r) 1000
(D) class interval size	(s) 799

Very Short Answer Questions

DIRECTIONS : *Give answer in one word or one sentence.*

1. Find the mean and the median of the data 10, 15, 17, 19, 20 and 21.
2. Find the mean of first 726 natural numbers.
3. Find the range of the data 14, 16, 20, 12, 13, 4, 5, 7, 29, 32 and 6.
4. Calculate the mean of 96, 104, 121, 134, 142, 149, 153 and 161.
5. Calculate the mode for 17, 12, 19, 11, 20, 19, 10, 25
6. Find the mean of the observations 425, 430, 435, 440, 445, ... 495.
7. The mean of 10 observations is 15.5. By an error, one observation is registered as 13 instead of 34. Find the actual mean.
8. Given median = 99.6 and mean = 101.2, find the mode.
9. Find the sum of the deviations of the variate values 3, 4, 6, 7, 8, 14 from their mean.
10. In a school 85 boys and 35 girls appeared in a public examination. The mean marks of boys was found to be 40%, whereas the mean marks of girls was 60%. Determine the average marks percentage of the school.
11. The mean of 12 observations is 14. By an error one observation is registered as 24 instead of –24. Find the actual mean.
12. The mean weight of 20 students is 25 kg and the mean weight of another 10 students is 40 kg. Find the mean weight of the 30 students.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequency f_1 and f_2.

Class	Frequency
0 – 20	5
20 – 40	f_1
40 – 60	10
60 – 80	f_2
80 – 100	7
100 – 120	8

2. For a frequency distribution of marks in mathematics for 100 students the average was found to be 80 . Later on it was discovered that 48 was misread as 84. Find the correct mean.
3. The mean weight of 150 students in a class is 60 kg. The mean weight of the boys in the class is 70 kg. and that of girls is 55 kg. Find the number of boys and girls.
4. The mean of $x_1, x_2, x_3 .... x_n$ is $\bar{x}$. If $(a - 2b)$ is added to each of the observations, show that the sum of the new observations $= \bar{x} + (a - 2b)$
5. The mean of 50 numbers is 40. It was found that three numbers 23, 29 and 20 were taken as 28, 92 and 2. find the correct mean.
6. If the mean and the median of a unimodel data are 34.5 and 32.5, then find the mode of the data.
7. The weight (in kg) of 25 children of 9th class is given, find the mean weight of the children.

Weight (in kg)	40	41	42	43	44	45
Number of children	3	4	6	2	5	5

8. The mean of $x+2, x+4, x+5, x+9$ and $2x - 5$ is 21. find x and the mean of the first four numbers.
9. Given $\sum_1^n (x_i - 3n) = 84$ and $\sum_1^n (x_i + 2n) = 144$,

   find n and the mean.
10. The mean of the data is 15. If each observation is divided by 5 and 2 is added to each result, then find the mean of the observations so obtained.
11. The mean of n observations $x_1, x_2, x_3, ...x_n$ is $\bar{x}$. If each observation is multiplied by p, prove that the mean of the new observations is $p\bar{x}$.
12. The mean wages of 1000 workers in a factory was ₹ 600. the mean wages of 200 workers of the night shift was ₹ 1000. Find the mean wages of the remaining 800 workers of the day shift.
13. The average height of 30 students is 150 cm. It was detected later that one value of 165 cm was wrongly copied as 135 cm for the computation of mean. Find the correct mean.
14. A cricketer has a mean score of 58 runs in nine innings. Find out how many runs are to be scored in the tenth innings to raise the mean score to 61.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. The following expenditure gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data and interpret it.

Number of students per teacher	Number of States/U.T.
15 – 20	3
20 – 25	8
25 – 30	9
30 – 35	10
35 – 40	3
40 – 45	0
45 – 50	0
50 – 55	2

2. Find mode from the following data :

Class interval	1-3	3-5	5-7	7-9	9-11
Frequency	4	5	8	7	6

3. There are six numbers. Combinations of 3 numbers are selected and their mean was calculated. The resulting means were 2, 4, 6, ... 36, 38, 40. What is the average of the original six numbers?
4. If the mean of the following table is 30, find the missing frequencies.

Class interval	0 – 15	15 – 30	30 – 45	45 – 60	Total
Frequency	10	a	b	8	60

5. The following table presents the number of illiterate females in the age group in town.

Age Group	14-15	15-19	20-24	25-29	30-34
Number of females	300	980	800	580	290

Draw a histogram to represent the above data.

6. For the following frequencies distribution, draw a histogram and construct a frequency polygon with it.

Class	Frequency
0 – 10	8
10 – 20	10
20 – 30	6
30 – 40	7
40 – 50	9
50 – 60	8
60 – 70	8
70 – 80	6
80 – 90	3
90 – 100	4

7. Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 1.46.

Number of accidents (s)	0	1	2	3	4	5	Total
Frequency (f)	46	?	?	25	10	5	200

8. The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in class 20–40 and 60–80 are missing. Find the missing frequencies.

Class	0–20	20–40	40–60	60–80	80–100	Total
Frequency	17	f_1	32	f_2	19	120

9. Calculate the median for the following distribution class :

Class	Frequency
0 – 10	5
10 – 20	10
20 – 30	20
30 – 40	7
40 – 50	8
50 – 60	5

10. Find the mode of the following distribution :

Daily wages	31-36	37-42	43-48	49-54	55-60	61-66
No. of workers	6	12	20	15	9	4

11. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

12. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained :

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Find the median height and interpret the result.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains* **13** *multiple choice questions. Each question has 4 choices (a), (b), (c) and (d). Out of which ONLY ONE is correct.*

1. The mean of discrete observations $y_1, y_2, ..., y_n$ is given by

(a) $\dfrac{\sum_{i=1}^{n} y_i}{n}$ (b) $\dfrac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$

(c) $\dfrac{\sum_{i=1}^{n} y_i f_i}{n}$ (d) $\dfrac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

2. If the mean of the numbers $27+x, 31+x, 89+x, 107+x, 156+x$ is 82, then the mean of $130+x, 126+x, 68+x, 50+x, 1+x$ is

(a) 75 (b) 157
(c) 82 (d) 80

3. The number of observations in a group is 40. If the average of first 10 is 4.5 and that of the remaining 30 is 3.5, then the average of the whole group is

(a) $\frac{1}{5}$ (b) $\frac{15}{4}$
(c) 4 (d) 8

4. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class are 72, then the average marks of the girls

(a) 73 (b) 65
(c) 68 (d) 74

5. If the class-intervals are $10-19, 20-29, 30-39, ...$, then the upper limit of the first class-interval is

(a) 19.5 (b) 19
(c) 20 (d) None of these

6. The numbers 3, 5, 7 and 9 have their respective frequencies $x-2, x+2, x-3$ and $x+3$. If the arithmetic mean is 6.5 then the value of x is

(a) 3 (b) 4
(c) 5 (d) 6

7. If three sets of data had means of 15, 22.5 and 24 based on 6, 4, and 5 observations respectively, then the mean of these three sets combined is

(a) 20.0 (b) 20.5
(c) 22.5 (d) 24.0

8. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set

(a) Is increased by 2
(b) Is decreased by 2
(c) Is two times the original median
(d) Remains the same as that of the original set

9. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is

(a) 6 (b) 7
(c) 8 (d) 10

10. Consider the following statements

(1) Mode can be computed from histogram
(2) Median is not independent of change of scale
(3) Variance is independent of change of origin and scale.

Which of these is/are correct ?

(a) (1), (2) and (3) (b) only (2)
(c) Only (1) and (2) (d) Only (1)

11. The average marks, in a class of 30 students, are found to be 45. On checking two mistakes were found. After correction, if one student got 45 marks more and another students got 15 marks less, then the correct average marks are

(a) 45 (b) 44
(c) 47 (d) 46

12. If the arithmetic mean of n numbers of a series is $\bar{x}$ and the sum of the first $(n-1)$ numbers is k, then the nth number is

(a) $n+k$ (b) $n\bar{x}+k$
(c) $n\bar{x}-k$ (d) $n-k$

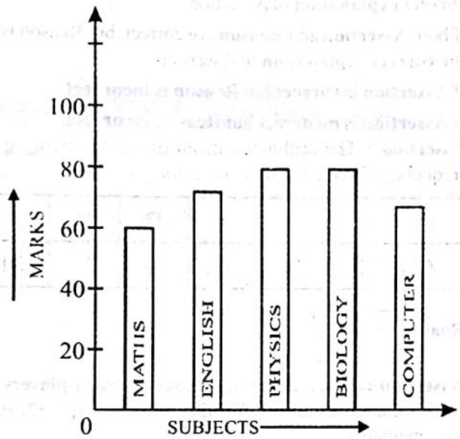
13. The mean of a set of 20 observation is 19.3. The mean is reduced by 0.5 when a new observation is added to the set. The new observation is

(a) 19.8 (b) 8.8
(c) 9.5 (d) 30.8

MTOC More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d). Out of which ONE OR MORE may be correct.*

Marks scored by Rishi in an examination (out of 100 marks) in different subjects are shown by the bar graph given below. Read the graph and answer the questions from 1 to 3.



1. In which subject Rishi is best?

(a) Physics (b) Biology
(c) English (d) Computers

2. The ratio of the highest marks to the lowest marks is

(a) 4 : 3 (b) 3 : 4
(c) 9 : 16 (d) 12 : 9

3. Percentage of marks obtained is 70 in which of the following subjects.

(a) Maths (b) English
(c) Biology (d) Computers

4. Which of the following is/are correct?

(a) Class mark $= \dfrac{\text{upper class limit} + \text{lower class limit}}{2}$

(b) Mode $= \ell + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

(c) 3 median = mode + 2 mean

(d) If n is odd, then median $= \left(\dfrac{n+1}{2}\right)^{th}$ term

where, n is the number of terms.

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

PASSAGE-I

1. The following table gives the weekly wages of workers in a factory :

Weekly wages	Mid-value (x_i)	No. of (f_i) workers	$f_i x_i$	Cumulative frequency
50 – 55	52.5	5	262.5	5
55 – 60	57.5	20	1150.0	25
60 – 65	62.5	10	625.0	35
65 – 70	67.5	10	675.0	45
70 – 75	72.5	9	652.0	54
75 – 80	77.5	6	465.0	60
80 – 85	82.5	12	990.0	72
85 – 90	87.5	8	700.0	80
		$\Sigma f_i = 80$	$\Sigma f_i x_i = 5520$	

(1) The mean is

(a) 70 (b) 68
(c) 71 (d) 69

(2) The modal class is

(a) 60-65 (b) 55-60
(c) 50-55 (d) none of these

(3) The number of workers getting weekly wages, below ₹ 80 is

(a) 50 (b) 70
(c) 60 (d) 80

PASSAGE-II

The marks of 20 students in a test were as follows :
5, 6, 8, 9, 10, 11, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19, 20

(1) The mean is

(a) 14 (b) 13
(c) 12 (d) 15

(2) The median is

(a) 13.5 (b) 12.5
(c) 14.5 (d) 15.5

(3) The mode is

(a) 20 (b) 10
(c) 15 (d) 25

PASSAGE-III

A professor keeps data on students tabulated by performance and sex of the student. The data is kept on a computer disk, but unfortunately some of it is lost because of a virus. Only the following could be recovered.

	Performance			Total
	Average	Good	Excellent	
Male			10	
Female				32
Total		30		

Panic buttons were pressed but to no avail. An expert committee was formed, which decided that the following facts were self-evident : Half the students were either excellent or good. 40% of the students were females. One third of the male students were average.

Female students = 32 which is 40% of total students. Hence total number of students = $\frac{32}{0.4} = 80$.

Hence males = (80 – 32)=48. It is further given, that half the students were either excellent or good and one third of the male students were average. Hence the table can be completed as under :

	Performance			Total
	Average	Good	Excellent	
Male	16	22	10	48
Female	24	8	–	32
Total	40	30	10	80

(1) How many students are both female and excellent?
(a) 0 (b) 8
(c) 16 (d) 32

(2) What proportion of good students are male?
(a) 0 (B) 0.73
(c) 0.4 (D) 1.0

(3) What proportion of female students are good?
(a) 0 (b) 0.25
(c) 0.5 (d) 1.0

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** The arithmetic mean of the following given frequency distribution table is 13.81.

x	4	7	10	13	16	19
f	7	10	15	20	25	30

Reason : $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

2. **Assertion :** If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason : Median $= \left(\frac{n+1}{2}\right)^{\text{th}}$ value, if n is odd.

3. **Assertion :** If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Reason : Median $= \frac{1}{2}(\text{mode} + 2\text{ mean})$

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r, s ...) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1. For the following marks distribution of 5 students in an examination, match column-I with the datas given in column- II.

Class interval	0–10	10–20	20–30	30–40
No. of students	1	3	0	1

Here,
x_k = lower limit of the modal class interval
f_k = frequency of the modal class
f_{k+1} = frequency of the class succeeding the modal class
h = width of the class interval
f_{k-1} = frequency of the class proceeding the modal class

Column-I	**Column-II**
(A) x_k	(p) 3
(B) f_k	(q) 10
(C) f_{k-1}	(r) 0
(D) h	(s) (0, 4)
	(t) 1
	(u) (4, 12)

2. For the given frequency distribution match the column-I with Column-II.

Class	35–35	35–40	40–45	45–50	50–55	55–60	60–65
Frequency	14	16	18	23	18	8	3

h = width of the class interval
f = frequency of the class interval to which median belongs
c = cumulative frequency
l_1 = lower limit of the median class interval

Column-I	Column-II
(A) f	(p) 45.4
(B) c	(q) 45
(C) l_1	(r) (40, 50)
(D) median	(s) 23
	(t) Positive number
	(u) 48

HOTS Subjective Questions

DIRECTIONS : *Answer the following questions.*

1. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

2. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

3. Given below is a frequency distribution with median 46. In this distribution, some of the frequencies are missing : Determine the missing frequencies.

Marks	No. of students
10-20	12
20-30	30
30-40	?
40-50	65
50-60	?
60-70	25
70-80	18
Total	**229**

4. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequency f_1 and f_2.

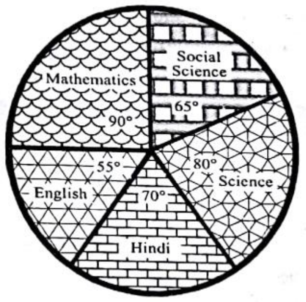
Class	Frequency
0 – 20	5
20 – 40	f_1
40 – 60	10
60 – 80	f_2
80 – 100	7
100 – 120	8

5. The number of students admitted in different faculties of a college given below :

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

Draw a pie chart to represent the above information.

6. The adjoining pie chart gives the marks scored in an examination by a student in English, Hindi, Science, Social Science and Mathematics. If the total marks obtained by the student were 540, answer the following questions.



(a) In which subject the student scored 105 marks?
(b) How many more marks were obtained by the student in Mathematics than in Hindi?
(c) Examine whether the sum of marks obtained in social Science and mathematics is more than that in Science and Hindi.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1.	square	2.	two	3.	zero	4.	hundred
5.	less	6.	10	7.	25	8.	50
9.	discrete	10.	35, 46	11.	range	12.	5
13.	27.5	14.	20	15.	class-mark		
16.	data	17.	median	18.	mode	19.	2

TRUE / FALSE

1.	True	2.	True	3.	False	4.	True
5.	False	6.	False	7.	True	8.	False
9.	True	10.	False	11.	True	12.	True
13.	True	14.	True	15.	True		

MATCH THE FOLLOWING :

1. (A) → s (B) → p (C) → q (D) → r
2. (A) → s (B) → r (C) → q (D) → p

VERY SHORT ANSWER QUESTIONS :

1. Mean = 17 Median = 18
2. 363.5 3. 28 4. 132.5 5. 19
6. 460 7. 17.6
8. 96.4 [Hint : 3(99.6) − 2(101.2) etc.]
9. $\text{Mean} = \dfrac{3+4+6+7+8+14}{6} = 7$

   Sum of deviation from mean

   $= (3-7)+(4-7)+(6-7)+(7-7)+(7-8)+(14-7) = 0$
10. $\bar{x}_c = \dfrac{n_1\bar{x}_1 + n_2.\bar{x}_2}{n_1+n_2} = \dfrac{5500}{120} = 45.83$
11. 10 12. 30 kg

SHORT ANSWER QUESTIONS :

1. $f_1 = 8, f_2 = 12$
2. Total marks for 100 students at the average of

   $80 = 100 \times 80 = 8000$

   Now, 48 was misread as 84 means $84 - 48 = 36$ was extra added to the total

   $\Rightarrow$ Correct total $= 8000 - 36 = 7964$

   $\Rightarrow$ Correct mean $= \dfrac{7964}{100} = 79.64$
3. Let the number of boys be x and the number of girls be y.

   $\Rightarrow \; x + y = 150$ ....(1)

   Weight of boys is $70x$ and of girls is $55y$.

   Total weight of 150 student $= 150 \times 60 = 9000$

   $70x + 55y = 9000$

   $14x + 11y = 1800$ ....(2)

   Solving (1) and (2)

   $x = 50$ and $y = 100$
4. Given $\dfrac{x_1 + x_2 + x_3 + \dots x_n}{n} = \bar{x}$

   $\Rightarrow x_1 + x_2 + \dots x_n = n\bar{x}$ ... (1)

   Now the sum of the new 'n' observations

   $= x_1 + (a-2b) + x_2 + (a-2b) + \dots x_n + (a-2b)$

   $= x_1 + x_2 + \dots x_n + n(a-2b)$

   $\Rightarrow n\bar{x} + n(a-2b) = n\{\bar{x} + (a-2b)\}$ ... (from 1)

   New mean $= \dfrac{n\{\bar{x}+(a-2b)\}}{n} = \bar{x} + (a-2b)$
5. Correct mean = 39
6. 28.5 7. 42.68 kg
8. $\dfrac{x+2+x+4+x+5+x+9+2x-5}{5} = 21$

   $\Rightarrow \dfrac{6x+15}{5} = 21 \Rightarrow 6x + 15 = 105$

   $\Rightarrow 6x = 90 \Rightarrow x = 15$

   The first four numbers are (15 + 2), (15 + 4), (15 + 5) and (15 + 9) = 17, 19, 20 and 24.

   $\text{Mean} = \dfrac{17+19+20+24}{2} = \dfrac{80}{2} = 40$
9. $\sum_{1}^{n}(x_1 - 3n) = x_1 + x_2 + \dots x_n - 3n = 84$

   $\Rightarrow S - 3n = 84 \dots (1) \; (S = x_1 + x_2 + \dots x_n)$

   $\sum_{1}^{n}(x_1 + 2n) = x_1 + x_2 + x_3 + \dots x_n + 2n = 144$

   $\Rightarrow S + 2n = 144$

   From (1) and (2), we get $S = 120$, $n = 12$

   $\text{Mean} = \dfrac{S}{n} = \dfrac{120}{12} = 10$
10. 5
11. According to question, $\dfrac{x_1 + x_2 + x_3 \dots\dots x_n}{n} = \bar{x}$ .... (1)

    New observations are $x_1 p, x_2 p, x_3 p, \dots\dots x_n p$

New mean $= \dfrac{x_1p + x_2p + x_3p.......+x_np}{n}$

$= \dfrac{p(x_1 + x_2 + x_3.......+x_n)}{n} = p.\bar{x}$ [From (1)]

12. Total wages of 1000 workers $= 600 \times 1000 = 600000$
Total wages of 200 night shift workers
$= 1000 \times 200 \Rightarrow 200000$
Total wages of 800 day shift workers
$= 600000 - 200000 = 400000$
Mean wages of the day shift workers $= \dfrac{400000}{800} =$ ₹ 500

13. Correct mean height = 151 cm

14. Mean score of 9 innings = 58 runs
$\therefore$ Total score of 9 innings $= 58 \times 9 = 522$
Required mean score of 10 innings = 61 runs
$\therefore$ Total score of 10 innings = 610 runs
Runs to be scored in 10th innings $= 610 - 522 = 88$ runs.

LONG ANSWER QUESTIONS :

1.

Class	f_i	x_i	$u_i = \dfrac{x_i - 37.5}{5}$	f_iu_i
15-20	3	17.5	−4	−12
20-25	8	22.5	−3	−24
25-30	9	27.5	−2	−18
30-35	10	32.5	−1	−10
35-40	3	37.5	0	0
40-45	0	42.5	1	0
45-50	0	47.5	2	0
50-55	2	49.5	3	6
			$\Sigma f_i = 35$	$\Sigma f_iu_i = -58$

$a = 37.5,\ \ h = 5$

$\bar{x} = a + \dfrac{\Sigma f_iu_i}{\Sigma f_i} \times h = 37.5 + \dfrac{(-58)}{35} \times 5 = 29.2$

Now, $\ell = 30, h = 5, f_1 = 10, f_0 = 9, f_2 = 3$

Mode $= \ell + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 30.62$

Mode = 30.62 and Mean = 29.2
Most states/U.T. have a student-teacher ratio of 30.62, on an average this ratio is 29.2

2. $\because$ Maximum frequency = 8
$\therefore$ Model class = 5 − 7
$\therefore$ $\ell = 5, f_0 = 5, f_1 = 8, f_2 = 7, h = 2$

$\therefore$ Mode $= \ell + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 6.5$

3. Let the six numbers be $x_1, x_2, x_3, x_4, x_5, x_6$.

Number of combinations of $3 = {}^6C_3 = \dfrac{6\times5\times4}{1\times2\times3} = 20$

The mean of these 20 combinations are 2, 4, 6, ..., 36, 38, 40.

Sum of all these means $= \dfrac{2+40}{2} \times 20 = 420$

In the above sum, each number from the original six is repeated 10 times.

$\Rightarrow \dfrac{10(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{3} = 420$

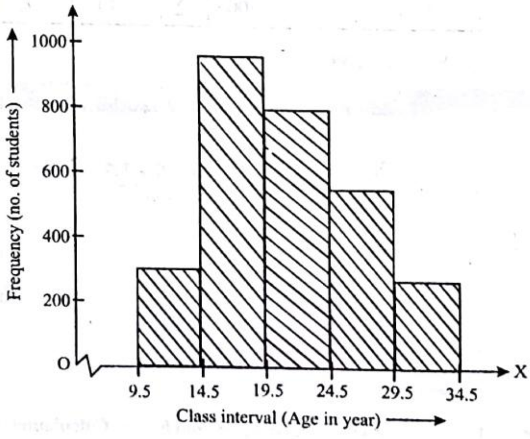
$\Rightarrow \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{3} = 42$

$\Rightarrow \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \dfrac{42}{2} = 21$

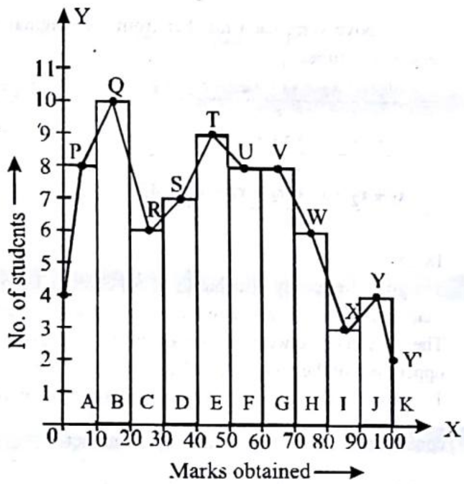
4. 18, 24

5. The given frequency distribution is not continuous. So, we shall first convert it into a continuous frequency distribution. The difference between the lower limit of a class and the upper limit of the preceeding class is 1 *i.e.*, *h*.
To convert the given frequency distribution into a continuous frequency distribution, we subtract $\dfrac{h}{2} = \dfrac{1}{2} = 0.5$ each lower limit and add $\dfrac{h}{2} = 0.5$ to each upper limit. The distribution so obtained is as given below:

Age Group	Number of females
9.5 – 14.5	300
14.5 – 19.5	980
19.5 – 24.5	800
24.5 – 29.5	580
29.5 – 34.5	290



6. Given frequency distribution is grouped and continuous. So we construct a histogram by using earlier method. 2Join the middle points of P, Q, R, S, T, U, V, W, X, Y of the upper horizontal lines of each rectangle A, B, C, D, E, F, G, H, I, J by straight line in successions.



7. Let missing frequencies be f_1 and f_2

x_i	f_i	f_ix_i
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
	$N = 86 + f_1 + f_2 = 200$	$\sum f_ix_i = 140 + f_1 + 2f_2$

We have, $N = 200$

$\Rightarrow 200 = 86 + f_1 + f_2 \Rightarrow f_1 + f_2 = 114 \Rightarrow$ Also, mean = 1.46

$\Rightarrow 1.46 = \dfrac{\sum f_ix_i}{N} \Rightarrow 1.46 = \dfrac{140 + f_1 + 2f_2}{200}$

$\Rightarrow 292 = 140 + f_1 + 2f_2$ ....(i)

$\Rightarrow f_1 + 2f_2 = 152$ ....(ii)

Solving equations (i) and (ii),

we get $f_1 = 76$ and $f_2 = 38$

8. Let the assumed mean be $A = 50$ and $h = 20$. Calculation of mean

Class	Frequency f_i	Mid-values x_i	$u_i = \dfrac{x_i - 50}{20}$	f_iu_i
0–20	17	10	–2	–34
20–40	f_1	30	–1	$-f_1$
40–60	32	50	0	0
60–80	f_2	70	1	f_2
80–100	19	90	2	38
	$N = \sum f_i = 68 + f + f$			$\sum f_iu_i = 4 - f_1 + f_2$

We have, $N = \sum f_i = 120$ (Given)

$\Rightarrow 68 + f_1 + f_2 = 120 \quad \Rightarrow f_1 + f_2 = 52$ ...(i)

Now, Mean = 50

$\Rightarrow A + h\left(\dfrac{1}{N}\sum f_iu_i\right) = 50 \Rightarrow 50 + 20 \times \left(\dfrac{4 - f_1 + f_2}{120}\right) = 50$

$\Rightarrow 50 + \dfrac{4 - f_1 + f_2}{6} = 50 \Rightarrow \dfrac{4 - f_1 + f_2}{6} = 0$

$\Rightarrow 4 - f_1 + f_2 = 0 \Rightarrow f_1 - f_2 = 4$... (ii)

Solving equations (i) and (ii), we get $f_1 = 28$ and $f_2 = 24$.

9. First we find $\left(\dfrac{N}{2}\right)^{th}$ value, where $N = 55$ i.e.

$\left(\dfrac{55}{2}\right)^{th} = 27.5^{th}$

which lies in 20 – 30 class.

$\therefore$ 20 – 30 class is the median class.

Class	f	c.f.
0 – 10	5	5
10 – 20	10	15
20 – 30	20	35
30 – 40	7	42
40 – 50	8	50
50 – 60	5	55

After finding the median class, we use the following formula for calculating the median.

Median $= l + \left(\dfrac{\frac{N}{2} - c.f}{f}\right) \times h,$

$\therefore$ Median $= 20 + \dfrac{27.5 - 15}{20} \times 10 = 26.25$

10. The class interval has to be made continuous and overlapping 37 – 36 = 1, 43 – 42 = 1 and so on. So, we subtract 0.5 in the lower limit and add 0.5 to the upper limit of each class interval and make the class continuous as 31 – 0.5 = 30.5, 36 + 0.5 = 36.5 and so on. So, classes are 30.5 to 36.5, 36.5 to 42.5, 42.5 to 48.5 etc. We draw the table with continuous class and find the modal class.

Daily wages	No. of workers
30.5-36.5	6
36.5-42.5	12
42.5-48.5	20
48.5-54.5	15
54.5-60.5	9
60.5-66.5	4

Modal class is 42.5 – 48.5 and mode lies in this class which is given by formula,

$$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Here, $\ell = 42.5, f_1 = 20, f_0 = 12, f_2 = 15, h = 6$

$$\therefore \text{Mode} = 42.5 + \frac{20-12}{2(20)-12-15} \times 6$$

$\therefore$ Mode = 46.2

11.

Monthly consumption	Number of consumers	x_i	$f_i x_i$
65 – 85	4	75	300
85 – 105	5	95	475
105 – 125	13	115	1495
125 – 145	20	135	2700
145 – 165	14	155	2170
165 – 185	8	175	1400
185 – 205	4	195	780
Total	68		9320

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{9320}{68} = 137.05$$

Here, maximum class frequency is $\left(\frac{N}{2}\right)^{th}$ 0, and the class corresponding to this frequency is 125 – 145.

So, the modal class is 125 – 145.

Now, $\ell = 125;\ h = 20,\ f_1 = 20,\ f_0 = 13,\ f_2 = 14.$

$$\therefore \quad \text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 135.75$$

Now, to find median we have to find the cumulative frequency (c.f.)

$\therefore$

Monthly consumption	Cumulative frequency	
	fi	c.f
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	64
185-205	4	68

Here $n = 68$, $\frac{n}{2} = \frac{68}{2} = 34$, Now 125 – 145 is the median class.

$$\therefore \text{Median} = \ell + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 125 + \left(\frac{34-22}{20}\right) \times 20 = 137$$

Median = 137 units, Mean = 137.05 units, Mode = 135.76 units. The three measures are approximately the same in this case.

12. To calculate the median height, we need to find the class intervals and their corresponding frequencies.

Frequency distribution table with the given cumulative frequencies is given below:

Class intervals	Frequency	Cumulative frequency
135 – 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51

Here, $n = 51$. So, $\frac{n}{2} = \frac{51}{2} = 25.5$. This observation lies in the class 145 - 150. Then, l (the lower limit) = 145,

c.f (the cumulative frequency of the class preceding 145 - 150) = 11,

f (the frequency of the median class 145 - 150) = 18,

h (the class size) = 5.

Using the formula, $\text{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h = 149.03$

This means that the height of 50% girls is less than the height 149.03 cm and 50% are taller than the height 149.03 cm.

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (a)
2. (a) Given,

$$82 = \frac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$$

$\Rightarrow \quad 82 \times 5 = 410 + 57 \Rightarrow 410 - 410 = 5x \Rightarrow x = 0$

$\therefore$ Required mean is,

$$\bar{x} = \frac{130 + x + 126 + x + 68 + x + 50 + x + 1 + x}{5}$$

$$\bar{x} = \frac{375 + 5x}{5} = \frac{375 + 0}{5} = \frac{375}{5} = 75$$

3. (b)

4. (b) Let the average marks of the girl students be x, then

$$72 = \frac{70 \times 75 + 30 \times x}{100}$$

(Number of girls $= 100 - 70 = 30$)

i.e., $\frac{7200-5250}{30} = x, \text{s}x \quad \therefore x = 65$

5. (a) **6.** (c) **7.** (a)

8. (d) Since n = 9, then median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ item.

Now, last four observations are increased by 2.

$\because$ The median is 5^{th} observation, which is remaining unchanged.

$\therefore$ There will be no change in median.

9. (c) Mode of the data is 8 as it is repeated maximum number of times.

10. (d) **11.** (d) **12.** (c) **13.** (b)

MORE THAN ONE CORRECT :

1. (a) and (b) **2.** (a) and (d)

3. (b) and (d) **4.** (a), (b), (c) and (d)

PASSAGE BASED QUESTIONS :

PASSAGE - I

(1) (d)

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{5520}{80} = ₹69$$

(2) (b)

Modal class : We know that class of maximum frequency is called the modal class. i.e., 55-60 is the modal class.

(3) (c)

Number of workers getting weekly wages below ₹80 according to table = 60 workers.

PASSAGE - II

Arrange in asscending order

(1) (b)

$$\text{Mean} = \frac{5+6+8+9+10+11+11+12+13+13+14+14+15+15+15+16+16+18+19+20}{2}$$

$$= \frac{260}{20} = 13$$

(2) (a)

Here, $n = 20$ which is an even number

$$\therefore \text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{ term} + \left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{\left(\frac{20}{2}\right)^{th}\text{ term} + \left(\frac{20}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{10^{th}\text{ term} + 11^{th}\text{ term}}{2}$$

$$= \frac{13+14}{2} = 13.5$$

(3) (c)

In the data, 15 occurs the maximum times i.e., 3 times

$\therefore$ Mode = 15

PASSAGE - III

(1) (a) There is no female excellent student in the class.

(2) (b) Proportion of good male students $= \frac{22}{30} = 0.73$

(3) (b) Proportion of good female students $= \frac{8}{30} = 0.26$

ASSERTION & REASON :

1. (a) Both assertion and reason are true, reason is the correct explanation of the assertion.

2. (d) Arranging the terms in ascending order,

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

median value $= \left(\frac{11+1}{2}\right)^{th} = 6^{th}$ value $= 27$

3. (c) Median $= \frac{1}{3}$(mode + 3 mean)

$$= \frac{1}{3}(60 + 2 \times 66) = 64$$

MULTIPLE MATCHING QUESTIONS :

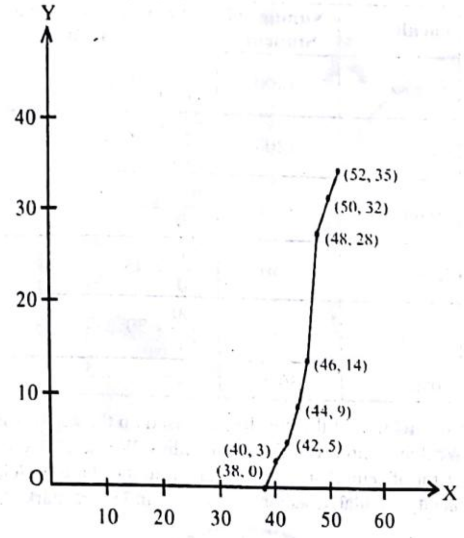
1. (A) → q, u; (B) → p, s; (C) → t, s; (D) → q, u

2. (A) → s, t; (B) → u, t, r; (C) → q, r, t; (D) → p, r, t

HOTS SUBJECTIVE QUESTIONS :

1. Use table with cumulative frequency for each less than the upper limit. We mark the upper limit of the class on X-axis and corresponding cumulative frequency at Y-axis.

We can draw the ogive by plotting the points : (38, 0), (40, 3), (42, 5), (44, 9), (40, 35), (46, 14), (48, 28), (50, 32) and (52, 35).



Here $\frac{n}{2} = 17.5$. Locate the point on the ogive whose ordinate is 17.5. The X-coordinate of this point will be the median. This is approximately 47.4.

To find the median by formula, we should convert less than type in continuous class interval as given below :

Weight (in kg)	Class frequency	Cumulative frequency
Less than 38	0	0
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Here $n = 35$. So $\frac{n}{2} = 17.5x$ Median class is 46-48.

$\ell = 46$, c.f. $= 28$, $f = 14$, $h = 2$

From formula, median $= \ell + \left(\frac{\left(\frac{n}{2} - cf\right)}{f}\right) \times h$

Median $= 46 + \left(\frac{17.5 - 14}{5}\right) \times 2 = 46 + \frac{3.5 \times 2}{5} = 47.4$

2.

Age (in years)	Number of patients(f_i)	Mid - values (x_i)	$f_i x_i$
5 - 15	6	10	60
15 - 25	11	20	220
25 – 35	21	30	630
35 – 45	23	40	920
45 - 55	14	50	700
55 - 65	5	60	300
Total	80		2830

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2830}{80} = 35.37$ years.

Here, maximum class frequency is 23, and the class corresponding to this frequency is 35 – 45. So, the modal class is 35 – 45.

Now, $\ell = 35, h = 10, f_1 = 23, f_0 = 21, f_2 = 14$

$\therefore$ Mode $= \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 35 + \left(\frac{20}{46 - 35}\right)$

$= 36.8$ year.

Mode = 36.8 years, Mean = 35.37 years. Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

3.

Marks	No. of students	c.f.
10-20	12	12
20-30	30	42
30-40	?	$42 + x$
40-50	65	$107 + x$
50-60	?	$107 + x + y$
60-70	25	$132 + x + y$
70-80	18	$150 + x + y$
Total	229	

Let the missing frequency corresponding to 30 – 40 and 50 – 60 class be x and y respectively.

Here, n = Total number of students = 229

$\therefore \frac{n}{2} = \frac{229}{2} = 114.5$, and Median = 46

$\therefore$ Median class = 40 – 50 and

$\ell = 40$, c.f. $= 42 + x$, $f = 65$, $h = 10$

We know,

Median $= \ell + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$

$\Rightarrow \quad 46 = 40 + \frac{114.5 - (42 + x)}{65} \times 10$

$\Rightarrow \quad 2x = 145 - 78 = 67 \quad$ or $x = \frac{67}{2} = 33.5$

$\therefore \; x = 34$ ($\because$ Number of students cannot be in fraction)

Now $\Sigma f_i = 229$

$\therefore \quad x + y + 150 = 229$

$x + y = 229 - 150 = 79 \quad ...(1)$

Putting the value of x in (1), we get

$34 + y = 79$

$y = 79 - 34 = 45$

$\therefore \quad x = 34, y = 45$

Hence, the missing frequency are 34 and 45 corresponding to class 30-40 and class 50-60 respectively.

4. We make the table with class, class frequency, mid value of class (class mark) and frequency × mid value as below :

Class	Frequency (f_i)	Mid-value (x_i)	$f_i x_i$
0 – 20	5	10	50
20 – 40	f_1	30	$30f_1$
40 – 60	10	50	500
60 – 80	f_2	70	$70f_2$
80 – 100	7	90	630
100 – 120	8	110	880
Total	$30+f_1+f_2$		$2060+30f_1+70f_2$

Given, Mean = 62.8 .... (1)

But we know, Mean

$$=\frac{\Sigma f_i x_i}{\Sigma f_i}=\frac{2060+30f_1+70f_2}{30+f_1+f_2} \quad \text{........ (2)}$$

∴ From equation (1) & (2), we have

$$\frac{2060+30f_1+70f_2}{30+f_1+f_2}=62.8$$

$\Rightarrow$ $2060+30f_1+70f_2=1884+62.8f_1+62.8f_2$

$\Rightarrow$ $176=32.8f_1-7.2f_2$ .... (3)

Also, given sum of all frequencies = 50

∴ $30+f_1+f_2=50 \Rightarrow f_1+f_2=20$ .... (4)

From equation (3) & (4) we have

$176=32.8[20-f_2]-7.2f_2$

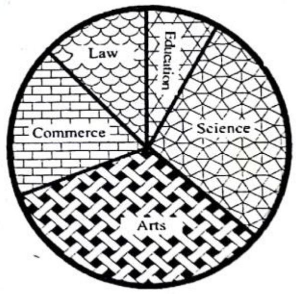
$\Rightarrow$ $f_2=\frac{480}{40}=12$, ∴ $f_1=8$

Hence, missing frequency $f_1=8$ and $f_2=12$

5. The following table gives the share of each faculty as a component of 360°

Faculty	Number of Students	Share as a component of 360°
Science	1000	$\frac{1000}{3600}\times 360°=\frac{1000}{10}=100°$
Arts	1200	$\frac{1200}{10}=120°$
Commerce	650	$\frac{650}{10}=65°$
Law	450	$\frac{450}{10}=45°$
Education	300	$\frac{300}{10}=30°$
Total	3600	360°

Construction of the pie chart is based on the above values. We draw a circle of convenient radius. We have first fixed a sector of central angle 100° depicting the share of Science faculty. Similarly, sectors of other faculties are marked.



6. (a) For the total marks 540, central angle = 360°

∴ For 105 marks, central angle

$=\frac{360°}{540}\times 105=70°$

Since, central angle of the sector representing Hindi is 70°, hence the student scored 105 marks in Hindi.

(b) The difference of central angles of the scores representing Mathematics and Hindi

$=90°-70°=20°$

∴ Corresponding difference of marks

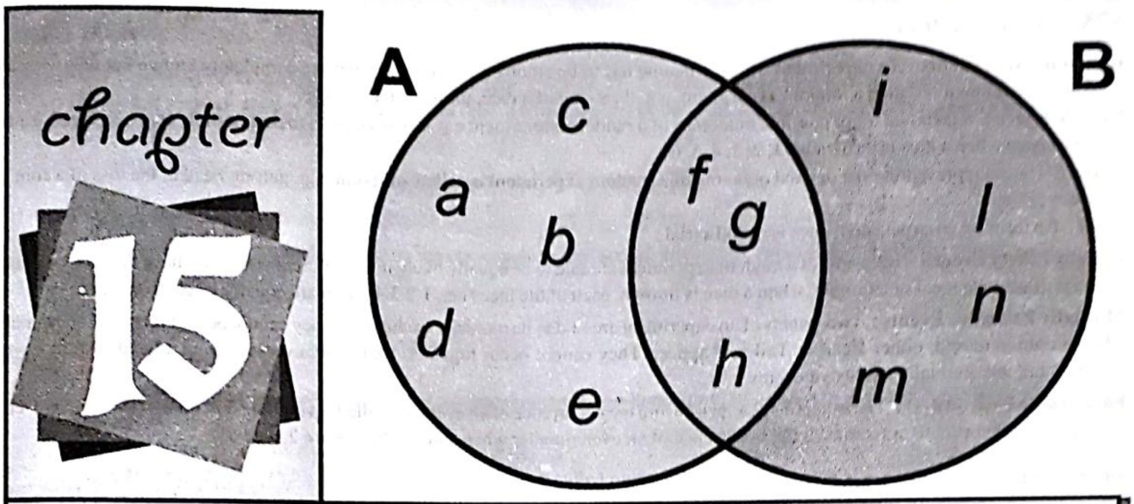
$=\frac{20°}{360°}\times 540=30$

Hence, the student scored 30 marks more in Mathematics than that in Hindi

(c) The sum of central angles of the sectors representing Social Science and Mathematics = 65° + 90° = 155°

Similarly, the sum of central angles of the sectors representing Science and Hindi = 80° + 70° = 150°

Since 155° > 150°, so the sum of marks obtained in Social science and mathematics is more than that in Science and Hindi.



PROBABILITY

Introduction

If you go to buy 10 kg of sugar at ₹30 per kg, you can easily find the exact price of your purchase is ₹300. On the other hand, the shopkeeper may have a good estimate of the number of kg of sugar that will be sold during the day, but it is impossible to predict the exact amount. The number of kg of sugar that customers will purchase during a day is random because the quantity can not be predicted exactly.

So, it is best for the shopkeeper to determine the sale, whose probability is highest.

To describe the situation, where we are not able to find the exact outcome, we generally use the chance, probably, most probably. For example:

(a) Probably India may win the cricket match against Pakistan.

(b) Chances of Neha passing the MBA examination is very low.

In all the cases, where it is not possible to find the exact outcome, we find the probability of occurrence of the outcome.

Probability, like many branches of mathematics, evolved out of practical considerations. It is used practically in every field. Probability theory can be thought of as the science of uncertainty. In this chapter you will learn how to find the probability of occurrence or non-occurrence of an outcome in simple situations.

SOME BASIC TERMS :

Random Experiments : An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is called a random experiment, e.g. throwing of a dice, tossing of a coin etc.

Sample Space : It is the set of all possible outcomes of a random experiment e.g. when a coin is tossed, sample space is (Head, Tail). Sample space when a dice is thrown is {1, 2, 3, 4, 5, 6}

Event : The set representing the desired outcome of a random experiment is called an event, e.g. getting head in the toss of a coin is an event {H}.

Trial : Performing an experiment once is called a trial.

Equally Likely Events: The results of a random experiment are said to be equally likely if the different outcomes have the same or equal chances of occurrence. For example, when a dice is thrown, each of the faces i.e., 1,2,3,4,5 or 6 are equally likely to appear.

Mutually Exclusive Events : Two events of an experiment are said to be mutually exclusive, if they cannot occur together. For example when a coin is tossed, either Head or Tail will appear. They cannot occur together. Hence the events getting a head and the event getting a tail are mutually exclusive events.

Favourable Outcome(s) : The outcome(s) which ensure the occurrence of an event are called favourable outcome(s) to that event. For example, the favourable outcomes to the occurrence of an even number when a die is thrown are 2, 4 or 6.

MEASUREMENT OF PROBABILITY :

If an event A can happen in 'm' ways and fail in 'n' ways, all these ways being equally likely to occur, then the probability of the happening of the event A,

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of all possible outcomes}}$$

$$\Rightarrow \quad P(A) = \frac{m}{m+n} \qquad \text{..........(i)}$$

and probability of its failing,

$$P(\overline{A}) \text{ or } P(\text{not } A) = \frac{n}{m+n} \qquad \text{..........(ii)}$$

from (i) and (ii)

$$P(A) + P(\overline{A}) = \frac{m}{m+n} + \frac{n}{m+n} \Rightarrow P(A) + P(\overline{A}) = \frac{m+n}{m+n} = 1$$

$$\therefore P(A) + P(\overline{A}) = 1 \Rightarrow P(A) = 1 - P(\overline{A}) \Rightarrow P(\overline{A}) \text{ or } P(\text{not}A) = 1 - P(A)$$

For example: On tossing a coin, there are two possibilities either head may come up or tail may come up. Therefore, total no. of possible outcomes = 2.

The number of favourable outcomes for getting a head = 1

$\therefore$ probability of getting a head $P(H) = \frac{1}{2}$

Similarly, probability of getting a tail $P(T) = \frac{1}{2}$

IMPOSSIBLE EVENT :

In throwing a die, there are only six possible outcomes 1, 2, 3, 4, 5 and 6. Let we are interested in getting a number 7 on throwing a die. Since no face of the die is marked with 7, so 7 cannot come under any circumstances. Hence getting a 7 is impossible, this type of event is called an impossible event.

$$P(7) = \frac{0}{6} = 0$$

Probability of an impossible event is always zero.

SURE EVENT :

Suppose, we want to find the probability of getting a number less than 7 in a single throw of a die having numbers 1 to 6 on its six faces. We are sure that we shall always get a number less than 7 whenever we throw a die. So the above event is a sure event

P (Getting a number less than 7) = $\frac{6}{6} = 1$

Probability of a sure event is always 1.

From above discussion and various examples, we can conclude that probability of any event lies in the range from 0 to 1.

Mathematically, $0 \le P(A) \le 1$

INDEPENDENT EVENTS :

Two or more events are said to be independent if the happening (or non-happening) of one event is not affected by the happening or non-happening of others.

For any two Independent events A and B, P (A and B) = P (A) × $P(B)$

Also, if P(A and B) = $P(A) \times P(B)$, then two events A and B are called independent events.

ALGEBRA OF EVENTS :

Let A and B be two events related to a random experiment. We define

(i) The event "A or B" denoted by "$A \cup B$", which occurs when A or B or both occur. Thus,

$P(A \cup B) =$ Probability that at least one of the events occur

(ii) The event "A and B", denoted by "$A \cap B$", which occurs when A and B both occur. Thus,

$P(A \cap B) =$ Probability of simultaneous occurrence of A and B.

(iii) The event "Not A" denoted by $\bar{A}$, occurs when and only when A does not occur. Thus

$P(\bar{A})$ = Probability of non-occurrence of the event A.

(iv) $\bar{A} \cap \bar{B}$ denotes the "non-occurrence of both A and B".

(v) $P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})$ and $P(\overline{A \cap B}) = P(\bar{A} \cap \bar{B})$

ILLUSTRATION 15.1

Consider a single throw of die and following two events

A = the number is even = {2, 4, 6}

B = the number is a multiple of 3 = {3, 6}

Then $P(A \cup B) = \frac{4}{6} = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$, $P(\bar{A}) = \frac{1}{2}$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{2}{3} = \frac{1}{3}$

ADDITION THEOREM OF PROBABILITY :

(i) If A and B are two events associated with a random experiment, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive events then $P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$

(ii) If A, B, C are three events associated with a random experiment, then

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

MULTIPLICATION OF PROBABILITY THEOREM :

If A and B are two events, then

$$P(A\cap B)=P(A)\,.\,P(B/A),\text{ if }P(A)>0$$
$$=P(B)\,.\,P(A/B),\text{ if }P(B)>0$$

Here $P(B/A)$ means probability of occurence of event B, when even A is already occurred. Also $P(A/B)$ means probability of occurrence of event A, when event B is already occurred.

From this theorem we get

$$P(B/A)=\frac{P(A\cap B)}{P(A)}\text{ and }P(A/B)=\frac{P(A\cap B)}{P(B)}$$

If A and B are two independent events, then

$$P(A/B)=P(A)\text{ and }P(B/A)=P(B)$$

Therefore $P(A\cap B)=P(A)\cdot P(B)$

ILLUSTRATION – 15.2

Consider an experiment of throwing a pair of dice. Let A denotes the event " the sum of the point on two dice is 8" and B denotes the event " there is an even number on first die"

Then $A=\{(2,6),(6,2),(3,5),(5,3),(4,4)\}$,

$B=\{(2,1),(2,2),.......,(2,6),(4,1),(4,2),.....,(4,6),(6,1),(6,2),....,(6,6)\}$

$$P(A)=\frac{5}{36},\ P(B)=\frac{18}{36}=\frac{1}{2},\ P(A\cap B)=\frac{3}{36}=\frac{1}{12}$$

Now, $P(A/B)$ = Prob. of occurrence of A when B has already occurred

= Prob. of getting 8 as the sum, when there is an even number on the first die $=\frac{3}{18}=\frac{1}{6}$ and similarly $P(B/A)=\frac{3}{5}$

ILLUSTRATION – 15.3

If A, B are two independent events, then show that

(i) $A,\ \bar{B}$ are independent (ii) $\bar{A},B$ are independent

(iii) $\bar{A},\bar{B}$ are independent.

SOLUTION :

(i) We have

$$P(A\cap\bar{B})=P(A)-P(A\cap B)=P(A)-P(A)P(B)$$
$$=P(A)[1-P(B)]\ =P(A)P(\bar{B})\text{ (Proved)}$$

(ii) Solution is similar to (i)

(iii) $P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)$

$=1-P(A)-P(B)+P(A)P(B)$

$=[1-P(A)][1-P(B)]=P(\bar{A})\,P(\bar{B})$ (Proved)

MISCELLANEOUS SOLVED EXAMPLES

1. Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ? (ii) What is the probability of getting a number less than or equal to 4 ?

Sol. (i) Here, let E be the event 'getting a number greater than 4'. The number of possible outcomes is six : 1, 2, 3, 4, 5 and 6, and the outcomes favourable to E are 5 and 6. Therefore, the number of outcomes favourable to E is 2. So,

$$P(E) = P(\text{number greater than 4}) = \frac{2}{6} = \frac{1}{3}$$

(ii) Let F be the event 'getting a number less than or equal to 4'.
Number of possible outcomes = 6
Outcomes favourable to the event F are 1, 2, 3, 4.
So, the number of outcomes favourable to F is 4.

Therefore, $P(F) = \frac{4}{6} = \frac{2}{3}$

2. A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be (i) white? (ii) blue? (iii) red?

Sol. Saying that a marble is drawn at random is a short way of saying that all the marbles are equally likely to be drawn. Therefore, the number of possible outcomes = 3 + 2 + 4 = 9
Let W denote the event 'the marble is white', B denote the event 'the marble is blue' and R denote the event 'marble is red'.
(i) The number of outcomes favourable to the event $W = 2$

So, $P(W) = \frac{2}{9}$

Similarly, (ii) $P(B) = \frac{3}{9} = \frac{1}{3}$ and (iii) $P(R) = \frac{4}{9}$

3. Savita and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year).

Sol. Out of the two friends, one girl, say, Savita's birthday can be any day of the year. Now, Hamida's birthday can also be any day of 365 days in the year. We assume that these 365 outcomes are equally likely.
(i) If Hamida's birthday is different from Savita's, the number of favourable outcomes for her birthday is 365 – 1 = 364

So, P (Hamida's birthday is different from Savita's birthday) = $\frac{364}{365}$

(ii) P (Savita and Hamida have the same birthday) = 1 – P (both have different birthdays)

$= 1 - \frac{364}{365} = \frac{1}{365}$ [Using $P(\bar{E}) = 1 - P(E)$]

4. One card is drawn at random from a shuffled standard deck of 52 cards. Find the probability that the card selected is not a king.

Sol. There are 52 different possible outcomes. There are 4 kings in a deck, so the other 48 cards are not kings, and these are the successful outcomes. Hence, the probability that the card selected is not a king is $\frac{48}{52}$.

$$P(\text{not } K) = \frac{48}{52} = \frac{12}{13}$$

ALTERNATE SOLUTION:

The probability of selecting a king is $\frac{4}{52}$ or $\frac{1}{13}$. Therefore, the probability that the card selected is not a king is P (not K)

$=1-\frac{1}{13}=\frac{12}{13}$.

5. One hundred cards are numbered from 1 to 100. Find the probability that a card chosen at random has the digit 5.

Sol. Total number of cases $= n = 100$

All the numbers from 50 to 59 have the digit 5. They are 10 in number. Besides these numbers the numbers 5, 15, 25, 35, 45, 65, 75, 85, 95 have the digit 5. These are 9 in number. Number of favourable cases $= m =$ Number of numbers which have the digit $5 = 10 + 9 = 19$

$\therefore$ Probability $= p = \frac{m}{n} = \frac{19}{100}$

6. There are 100 transistors in a box. 20 of them are defective. At random two transistors are taken one by one consecutively without replacement. What is the probability that (i) both of them are good (ii) both of them are defective (iii) one of them is good and the other defective.

Sol. Probability of a transistor to be good $= \frac{80}{100}$

Probability of a transistor to be defective $= \frac{20}{100}$

(i) Probability for the two transistor to be good $= \frac{80}{100}\times\frac{79}{99}=\frac{316}{495}$

(ii) Probability for the two transistor to be defective $= \frac{20}{100}\times\frac{19}{99}=\frac{19}{495}$

(iii) Probability for the first one to be good and the other to be defective $= \frac{80}{100}\times\frac{20}{99}=\frac{80}{495}=\frac{16}{99}$

Probability for the first one to be defective and the second one to be good $= \frac{20}{100}\times\frac{80}{99}=\frac{80}{495}=\frac{16}{99}$

$\therefore$ Probability for one to be good and the other to be defective $\frac{80}{495}+\frac{80}{495}=\frac{160}{495}=\frac{32}{99}$.

7. A bag contains 3 red and 3 white balls. Two balls are drawn one by one. Find the probability that they are of different colours.

Sol. Let $A \equiv$ event that drawn ball is red

$B \equiv$ event that drawn ball is white

Then AB and BA are two disjoint cases of the given event.

$$\therefore P(AB+BA)=P(AB)+P(BA)=P(A)P\left(\frac{B}{A}\right)+P(B)P\left(\frac{A}{B}\right)=\frac{3}{6}\cdot\frac{3}{5}+\frac{3}{6}\cdot\frac{3}{5}=\frac{3}{5}$$

8. A club holds an election for the post of chairperson. The probabilities that the candidates Anjani and Laxmi will be elected are 0.36 and 0.47 respectively. Find the probability that
(a) either Anjani or Laxmi will be elected,
(b) neither Anjani nor Laxmi will be elected.

Sol. (a) Let A be the event that Anjani will be elected,
and B be the event that Laxmi will be elected.

Since there is only 1 chairperson, the events, A and B are mutually exclusive.

P (either Anjani or Laxmi will be elected) = $P(A \text{ or } B) = P(A) + P(B) = 0.36 + 0.47 = 0.83$

(b) P (neither Anjani nor Laxmi will be elected) = $1 - P$ (either Monica or Roland will be elected) = $1 - 0.83 = 0.17$

9. There are n letters and n addressed envelopes. Find the probability that all the letters are not kept in the right envelope.

Sol. Since, there are n letters and n addressed envelopes therefore,

total no. of ways in which letters are kept in the right envelope = $n!$.

There is only one way in which letters takes corresponding envelope.

$\therefore$ P(letters are not in right envelope) = $1 - P$ (All letters in right envelope) $= 1 - \frac{1}{n!}$.

10. A speaks truth in 60% and B in 50% of the cases. Find the probability that they contradict each other discussing the same incident.

Sol. 'A' speaks truth in 60% cases. $\therefore$ P('A' speaks truth) $= \frac{60}{100} = \frac{3}{5}$

Similarly, 'B' speaks truth in 50% cases. $\therefore$ P('B' speaks truth) $= \frac{50}{100} = \frac{1}{2}$

$\therefore$ P(contradict each other) $= P(A\bar{B} + B\bar{A}) = P(A)P(\bar{B}) + P(B)P(\bar{A}) = \left(\frac{3}{5}\right)\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(1-\frac{3}{5}\right) = \frac{3}{5}.\frac{1}{2} + \frac{2}{5}.\frac{1}{2} = \frac{1}{2}$.

11. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these student is selected at random. Find the probability that
(i) The student opted for NCC and NSS.
(ii) The student has opted for neither NCC nor NSS.
(iii) The student has opted NSS but not NCC.

Sol. In a class of 60 students 30 students opted for NCC.

$\therefore$ Probability of opting NCC $= \frac{30}{60}$

Let A be the event that a student opts for NCC.

$\therefore$ $P(A) = 0.5$

If B be the event that a student opts for NSS. $\Rightarrow n(B) = 32$ $\therefore$ $P(B) = \frac{32}{60}$

24 students opt for NCC and NSS both, $P(A \cap B) = \frac{24}{60}$

(i) Probability that a student opts for NSS or NCC = $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{30+32-24}{60} = \frac{38}{60} = \frac{19}{30}$$

(ii) Probability that the student has opted neither NCC non NSS

$= P(A \cap B)' = 1 - P(A \cup B) = 1 - 0.63 = 0.37$

(iii) Probability that the student has opted NSS but not NCC = $P(A' \cap B) = P(B) - P(A \cap B)$

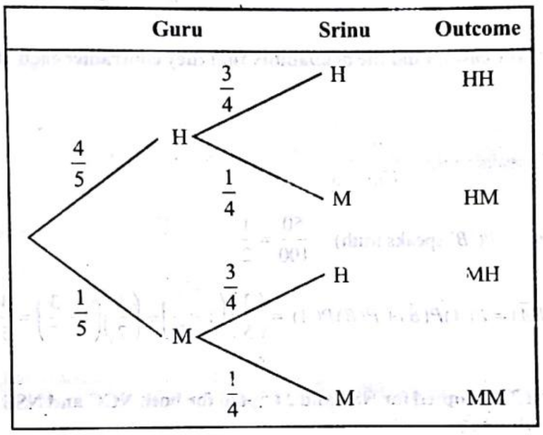
$= \frac{32}{60} - \frac{24}{60} = \frac{8}{60} = \frac{2}{15}$

12. The probability that Guru will hit a target is $\frac{4}{5}$. The probability that Srinu will hit the same target is $\frac{3}{4}$. If each of them fires once, find the probability that the target will be hit by
(a) both of them, (b) only one of them

Sol. (a) P (Guru will miss) $= 1 - P$ (Guru will hit) $= 1 - \frac{4}{5} = \frac{1}{5}$

Similarly, P(Srinu will miss) $= 1 - \frac{3}{4} = \frac{1}{4}$

Hence, we have a tree diagram as shown below.



H – Hit
M – Miss

P (the target will be hit by both) $= P(HH) = \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

(b) P(the target will be hit by only one of them) $= P$(Guru will hit and Srinu will miss) $+ P$(Guru will miss and Srinu will hit)

$= P(HM) + P(MH) = \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{4}{20} + \frac{3}{20} = \frac{7}{20}$

13. A die is thrown. Find the probability of
(a) prime number (b) multiple of 2 or 3 (c) a number greater than 3

Sol. In a single throw of die any one of six numbers 1, 2, 3, 4, 5, 6 can be obtained. Therefore, the total number of elementary events associated with the random experiment of throwing a die is 6.

(a) Let A denote the event "Getting a prime no." Clearly, event A occurs if any one of 2, 3, 5 comes as out come.
$\therefore$ Favourable number of elementary events = 3

Hence, P (Getting a prime no.) $= \frac{3}{6} = \frac{1}{2}$

(b) An multiple of 2 or 3 is obtained if we obtain one of the numbers 2, 3, 4, 6 as out comes.
$\therefore$ Favourable number of elementary events = 4

Hence, P (Getting multiple of 2 or 3) $= \frac{4}{6} = \frac{2}{3}$

(c) The event "Getting a number greater than 3" will occur, if we obtain one of number 4, 5, 6 as an outcome.
$\therefore$ Favourable number of outcomes = 3

Hence, required probability $= \frac{3}{6} = \frac{1}{2}$

14. A box contains 20 balls bearing numbers 1, 2, 3, 4, ...20. A ball is drawn at random from the box. What is the probability that the number on the ball is –
(a) an odd number (b) divisible by 2 or 3 (c) prime number

Sol. Here, total number are 20.

$\therefore$ Total number odd elementary events = 20

(a) The number selected will be odd number, if it is elected from 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

$\therefore$ Favourable number of elementary events = 10

Hence, P (an odd number) $= \frac{10}{20} = \frac{1}{2}$

(b) Number divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

$\therefore$ Favourable number of elementary events = 13

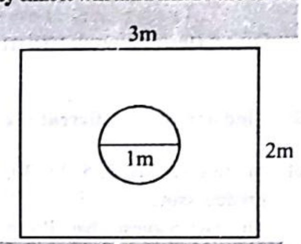
P (Number divisible by 2 or 3) $= \frac{13}{20}$

(c) There are 8 prime number form 1 to 20, i.e., 2, 3, 5, 7. 11. 13, 17, 19

$\therefore$ Favourable number of elementary events = 8

P (prime number) $= \frac{8}{20} = \frac{2}{5}$

15. A die is drop at random on the rectangular region as shown in figure. What is the probability that it will land inside the circle with diameter 1m?



Sol. Area of rectangular region $= 3m \times 2m = 6m^2$

Area of circle $= \pi r^2 = \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} m^2$

$\therefore$ Probability that die will land inside the circle $= \frac{\pi/4}{6} = \frac{\pi}{24}$

16. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing atleast one of them is 0.95. What is the probability of passing both?

Sol. Let A and B be the events of passing I and II examinations respectively.

$\therefore$ $P(A) = 0.8, P(B) = 0.7$

Probability of passing atleast one examination $= 1 - P(A' \cap B') = 0.95$ .... (1)

Now $A' \cap B' = (A \cup B)'$ (De Morgan's Law)

$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

Putting this value in (i), $1 - [1 - P(A \cup B)] = 0.95$ or $P(A \cup B) = 0.95$

Further $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.8 + 0.7 - 0.95 = 1.5 - 0.95 = 0.55$

Thus, probability that the student will pass in both the examinations = 0.55

17. A fair coin is tossed four times, and a person wine ₹ 1 for each head and lose ₹ 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tossed and the probability of having each of these amounts.

Sol. (i) No head and 4 tail appear.

Money lost = ₹ 4 × 1.50 = ₹ 6.00

There is only 1 way when TTTT occurs.

No. of exhaustive cases $= 2^4 = 16$

$\therefore$ Probability of getting no head or 4 tails $= \frac{1}{16}$

(ii) When 1 head and 3 tails appear.
Money lost = ₹ $(-1 \times 1 + 3 \times 1.50)$ = ₹ 3.50
These are 4 ways when 1 head and 3 tails occur i.e.
HTTT, THTT, TTHH, TTTH

Probability of getting 1 head and 3 tails $= \frac{4}{16} = \frac{1}{4}$

(iii) When 2 head and 2 tails appear.
Money lost = ₹ $(2 \times 1.5 - 1 \times 2)$ = ₹ $(3 - 2)$ = ₹ 1
2 heads and 2 tails may occur as
HHTT, HTHT, HTTH, THHT, THTH, TTHH

Thus, 2 heads and 2 tails may appear in 6 $= \frac{6}{10} = \frac{3}{8}$

(iv) When 3 head 1 tail appear, Money gained ₹ $(3 \times 1 - 1 \times 1.5)$ = ₹ 1.50
3 heads and 1 tails may occurs as HHHT, HHTH, HTHH, THHH
∴ 3 heads and 1 tail appear in 4 ways ,

∴ Probability of getting 3 heads and 1 tail $= \frac{4}{16} = \frac{1}{4}$

(v) When all the heads appear , Money gained = ₹ 4×1 = ₹ 4

4 Head occur as HHHH i.e., in one way ; ∴ Probability of getting 4 heads $= \frac{1}{16}$

18. Find how many different +ve integers can be obtained by finding the sum of two or more from the list 2, 5, 15, 30, 55.

Sol. In the given list 2, 5, 15, 30, 55, we see that any member of this list cannot be expressed as the sum of two or more of its predecessors.
This fact suggests that all sums of two or more will give us different + ve integers.
There are 5 elements in the given list.
Sum of two elements : Out of 5 elements, we can choose 2 elements for addition in 10 ways.
Sum of three elements : Out of 5 elements, we can choose 3 elements for addition in 10 ways.
Sum of four elements : Out of 5 elements, we can choose 4 elements for addition in 5 ways.
Sum of five elements : 5 Out of 5 elements can be choosen for addition in only one way.
By the rule of addition, the number of different sums are $10 + 10 + 5 + 1 = 26$

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Probability of an event E + Probability of the event 'not E' = ...
2. The probability of an event that cannot happen is .... Such an event is called ....
3. The probability of an event that is certain to happen is .... Such an event is called ....
4. The sum of the probabilities of all the elementary events of an experiment is ....
5. The probability of an event is greater than or equal to .... and less than or equal to ....
6. The probability of a sure event (or certain event) is ....
7. The probability of an impossible event is ....
8. When an unbiased coin is tossed thrice the probability of getting heads all the time is ....
9. If P(E) = 0.05, the probability of 'not E' is ....
10. A die is thrown once, the probability of getting a prime number is ....
11. If A is an event of a random experiment, then A^C or $\overline{A}$ or A' is called the _______ of the event.
12. If the probability of an event of a random experiment is $P(E) = 0$, then the event is called ....
13. A set of events which have no pair in common are called ....

T/F True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
2. For any event E, P (E) + P ($\overline{E}$) = 1, where $\overline{E}$ stands for 'not E'. E and $\overline{E}$ are called complementary events.
3. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. The probability that the 2 students have the same birthday is 0.008.
4. The probability of an event can be greater than 1.
5. If the probability of an event is 1, then it is an impossible event.
6. The range of probability of any event of a random experiment is [0, 1].
7. If A is any event in a sample space, then $P(\overline{A}) = 1 + P(A)$
8. The sum of probabilities of two students getting distinction in their final examinations is 1.2.
9. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, the number of blue balls in the bag is 10
10. The probabilities that a typist will make 0, 1, 2, 3, 4, 5 or more mistakes in typing a report are, respectively, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.
11. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a two-digit number is 0.9

MTF Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D,............) in column I have to be matched with statements (p, q, r, s,...............) in column II.*

1. Match the proposed probability under Column I with the appropriate written description under column II :

Column I Probability	Column II Written Description
(A) 0.95	(p) An incorrect assignment
(B) 0.02	(q) No chance of happening
(C) –0.3	(r) As much chance of happening as not
(D) 0.5	(s) Very likely to happen
(E) 0	(t) Very little chance of happening

2.

Column-I	Column-II
(A) If E_1 and E_2 are the two mutually exclusive events, then	(p) $E_1 \cap E_2 = E_1$
(B) If E_1 and E_2 are the mutually exclusive and exhaustive events, then	(q) $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$
(C) If E_1 and E_2 have common outcomes, then	(r) $E_1 \cap E_2 = \phi$, $E_1 \cup E_2 = S$
(D) If E_1 and E_2 are two events such that $E_1 \subset E_2$, then	(s) $E_1 \cap E_2 = \phi$

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. A bag contains 5 blue, 6 green and 4 red balls, a ball is drawn at random. Find the probability that it is
(i) blue (ii) green (iii) red
2. It is known that a box of 600 electric bulbs contain 12 defective bulbs. One bulb is taken out at random from this box. What is the probability that it is a non-defective bulb?
3. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?
4. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel, (ii) a consonant.
5. Two coins are tossed simultaneously. Find the probability of getting : atmost one head.
6. Two players, Sania and Sonali, play a tennis match. It is known that the probability of Sania winning the match is 0.62. What is the probability of Sonali winning?
7. Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.
8. If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find P(E or F)
9. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine P(A or B)
10. If 5 coins are tossed, what is the chance that all will show heads?
11. Two unbiased dice are thrown. Find the probability that the total of the numbers on the dice is greater than 10.
12. A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers, 1, 2, 3,...,12 as shown in figure. Find the probability that it will point to a number which is multiple of 3?

13. When a dice is rolled, what is the probability of getting a number 2 or 3?
14. There is a bunch of 10 keys out of which any one of the 4 keys can unlock a door. If a key is selected at random from the bunch and tried on the door, find the probability that the door will be unlocked.
15. Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Harpreet tosses two different coins simultaneously (say, one is of ₹1 and other of ₹2). What is the probability that she gets at least one head ?
2. The probability of occurrence of an event A in one trial is 0.4. Find the probability that the event A happens at least once in three independent trials is.
3. A bag contains 3 blue and 7 red balls. Find the probability that a ball selected at random from the bag will be a blue ball.
4. The probabilities that A, B, C can solve a problem independently are $\frac{1}{3}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that only two of them are able to solve the problem.
5. A box contain 5 red balls, 4 green balls and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drawn is
(a) white (b) neither red nor white
6. A letter is chosen from the word 'TRIANGLE'. What is the probability that it is a vowel?
7. Three dice are thrown together. Find the probability of getting a total of at least 6.
8. The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.
(i) What is the probability that on a given day it was correct?
(ii) What is the probability that it was not correct on a given day?
9. A bag contains 6 red balls and some black balls. If the probability of drawing a black ball is double that of a red ball, find the number of black balls in the bag.
10. Two numbers x and y are selected randomly from the numbers 1, 2, 3 and 1, 4, 9 respectively. Find the probability of getting the product of the numbers x and y less than 9.

11. A black die and a white die are rolled. Find the probability that the number shown by the black die will be more than twice that shown by the white die.
12. A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number is divisible by 4.

LAQ Long Answer Questions

DIRECTIONS : *Give answer in four to five sentences*

1. All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting
   (a) black face card (b) a queen
   (c) a black card
2. From a bag containing 8 green and 5 red balls, there are drawn one after the other. Find the probability of all three balls green if (a) the balls drawn are replaced before the next ball is picked. (b) the balls drawn are not replaced.
3. The king, queen and jack of clubs are removed from a deck of 52 playing well shuffled cards. One card is selected from the remaining cards. Find the probability of getting –
   (i) a heart (ii) a club
   (iii) the '10' of hearts.
4. A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble and a green marble at random from the jar is $\frac{1}{3}$, and $\frac{4}{9}$ respectively. How many white marbles does the jar contain?
5. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?
6. What is the probability that a leap year has 53 Mondays and 53 Sundays?

2 EXERCISE

MCQ Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column –
   (a) 56 (b) 896
   (c) 60 (d) 768
2. If a certain missile will hit the target one out of four times and four such missiles are fired at the same angle, then what is the probability that the target will be hit atleast once–
   (a) 1 (b) 51/256
   (c) 175/256 (d) None of these
3. What is the probability that Gagan will hit the board within the space enclosed by the inner cycle –
   (a) $\frac{3}{4}$ (b) $\frac{33}{350}$
   (c) $\frac{66}{350}$ (d) $\frac{99}{350}$
4. A card is taken out of a full pack of 52 cards. It is replaced. Again a card is taken out. The probability of getting 2 kings is
   (a) $\frac{1}{169}$ (b) $\frac{1}{52}$
   (c) $\frac{1}{26}$ (d) $\frac{2}{51}$
5. A fair die is thrown once. The probability of getting a composite number less than 5 is
   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$
   (c) $\frac{2}{3}$ (d) 0
6. If a letter is chosen at random from the letter of English alphabet, then the probability that it is a letter of the word 'DELHI' is
   (a) $\frac{1}{5}$ (b) $\frac{1}{26}$
   (c) $\frac{5}{26}$ (d) $\frac{21}{26}$
7. The probability of raining on day 1 is 0.2 and on day 2 is 0.3. The probability of raining on both the days is
   (a) 0.2 (b) 0.1
   (c) 0.06 (d) 0.25

8. Suppose six coins are flipped. Then the probability of getting at least one tail is

(a) $\frac{71}{72}$ (b) $\frac{53}{54}$

(c) $\frac{63}{64}$ (d) $\frac{1}{12}$

9. Which of the following cannot be the probability of an event?

(a) 2/3 (b) – 1/5

(c) 15% (d) 0.7

10. If $P(A \cup B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{6}$ and $P(A) = \frac{1}{3}$ then -

(a) A and B are independent events

(b) A and b are disjoint events

(c) A and b are dependent events

(d) none of these

11. A dice is thrown twice. The probability of getting 4, 5 or 6 in the first throw and 1, 2, 3 or 4 in the second throw is–

(a) 1/3 (b) 2/3

(c) 1/2 (d) 1/4

12. The probability that a two digit number selected at random will be a multiple of '3' and not a multiple of '5' is

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$

(c) $\frac{1}{15}$ (d) $\frac{4}{90}$

13. The probability of getting a number greater than 2 in throwing a die is –

(a) 2/3 (b) 1/3

(c) 4/3 (d) 1/4

14. Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{4}{9}$ (d) $\frac{2}{5}$

15. A five digit number is chosen at random. The probability that all the digits are distinct and digits at odd places are odd and digits at even places are even is

(a) $\frac{3}{65}$ (b) $\frac{1}{75}$

(c) $\frac{2}{65}$ (d) $\frac{8}{75}$

16. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is –

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) 0

17. The probability of getting at least one tail in 4 throws of a coin is –

(a) 1/16 (b) 1/4

(c) 15/16 (d) 1/8

18. A three digit number is to be formed using the digits 3, 4, 7, 8 and 2 without repetition. The probability that it is an odd number is

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{4}{5}$ (d) $\frac{3}{5}$

19. An urn contains 6 blue and 'a' green balls. If the probability of drawing a green ball is double that of drawing a blue ball, then 'a' is equal to

(a) 6 (b) 18

(c) 24 (d) 12

20. If events A and B are independent and $P(A) = 0.15$, $P(A \cup B) = 0.45$, Then $P(B) =$ ....

(a) $\frac{6}{13}$ (b) $\frac{6}{17}$

(c) $\frac{6}{19}$ (d) $\frac{6}{23}$

MTOC More than One Correct

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. If $P(A) = \frac{1}{8}$ and $P(B) = \frac{5}{8}$. Which of the following statements is/are not correct?

(a) $P(A \cup B) \le \frac{3}{4}$ (b) $P(A \cap B) \le \frac{1}{8}$

(c) $P(\bar{A} \cap B) \le \frac{5}{8}$ (d) None of these

2. If A and B are two independent events, the probability that both A and B occur is $\frac{1}{8}$ and the probability that neither of them occurs is $\frac{3}{8}$. The probability of the occurrence of A is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

3. A coin is tossed. Then the probability of getting either head or tail is

(a) 1 (b) 2°

(c) $\frac{1}{2}$ (d) $\frac{1}{2^{\circ}}$

4. Two dice are rolled simultaneously. Find the probability that they show different faces.

(a) $\frac{10}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{5}{6}$

5. Which of the following is/are true ?

(a) $O \leq P(E) \leq 1$

(b) $P(\bar{E}) = 1 + P(E)$

(c) For independent events, $P(A \cap B) = P(A) \cdot P(B)$

(d) $P(A \cup B) > P(A) + P(B)$

6. The probability of occurrence of an event A in one trial is 0.4. The probability that the event A happens at least once in three independent trials is –

(a) $1-0.784$ (b) 0.784

(c) 1 0.216 (d) 0.216

7. A bag contains four tickets marked with 112, 121, 211, 222, one ticket is drawn at random from the bag. Let E_i $(i = 1, 2, 3)$ denote the event that i^{th} digit on the ticket is 2. then :

(a) E_1 and E_2 are independent

(b) E_2 and E_3 are independent

(c) E_3 and E_1 are independent

(d) E_1, E_2, E_3 are independent

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. Die is rolled once.

1. The probability of obtaining the number 2 is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) None of these

2. The probability of getting the number 1 or 3 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{2}$ (d) None of these

3. The probability of not getting the number 3 is

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{2}$ (d) None of these

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion is correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason is correct**.

1. Let A and B be two independent events.

Assertion : If $P(A) = 0.3$ and $P(A \cup \bar{B}) = 0.8$, then $P(B)$ is $\frac{2}{7}$.

Reason : $P(\bar{E}) = 1 - P(E)$, where E is any event.

2. **Assertion:** If $P(A) = 0.25, P(B) = 0.50$ and $P(A \cap B) = 0.14$, then the probability that neither A nor B occurs is 0.39.

Reason : $\overline{A \cup B} = \bar{A} \cup \bar{B}$

3. **Assertion :** If A and B are two independent events and it is given that $P(A) = \frac{2}{5}, P(B) = \frac{3}{5}$, then $P(A \cap B) = \frac{6}{25}$.

Reason : $P(A \cap B) = P(A) \cdot P(B)$, where A and B are two independent events.

4. **Assertion :** If a box contains 5 white, 2 red and 4 black marbles, then the probability of not drawing a white marble from the box is $\frac{5}{11}$.

Reason : $P(\bar{E}) = 1 - P(E)$, where E is any event.

5. **Assertion :** In rolling a dice, the probability of getting number 8 is zero.

Reason : Its an impossible event.

MMQ Muliple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r, s, t) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

	Column-I	Column-II
1.	(A) Probability of getting number 5 in throwing a dice	(p) 0

(B) Probability of obtaining three heads in a single throw of a coin	(q)	$\frac{6}{36}$
(C) Probability of getting the sum of the numbers as 7, when two dice are thrown	(r)	1
(D) Probability of occurrence of two sure independent events	(s)	$\left(\frac{1}{2}\right)^0$
	(t)	$\frac{1}{6}$

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. A jar contains only green, white and yellow marbles. The probability of selecting a green marble and white marble randomly from a jar is 1/4 and 1/3 respectively. If this jar contains 10 yellow marbles. What is the total number of marbles in the jar ?

2. Bag A contains 4 chips numbered 1, 3, 5 and 7 respectively. Bag B contains 3 chips numbered 2, 4 and 6 respectively. A chip is drawn at random from each bag.
   (a) Draw a tree diagram to show all the possible outcomes.
   (b) Find the probability that the sum of the two chips drawn is
   (i) 7 (ii) odd (iii) even

3. An unbiased die is rolled twice. Find the probability of getting (i) the sum of two numbers as a prime (ii) the sum of two numbers equal to 9.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. 1
2. 0, impossible event
3. 1, sure or certain event
4. 1
5. 0, 1
6. 1
7. 0
8. $\frac{1}{8}\left[=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right]$
9. .95
10. 1/2
11. complement
12. impossible event
13. mutually exclusive

TRUE / FALSE

1. True 2. True 3. True 4. False
5. False 6. True 7. False 8. True
9. True 10. False 11. True

MATCH THE FOLLOWING :

1. (A) → (s); (B) → (t); (C) → (p); (D) → (r); (E) → (q)
2. (A) → (s); (B) → (r); (C) → (q); (D) → (p)

VERY SHORT ANSWER QUESTIONS :

1. Total number of balls = 5 + 6 + 4 = 15

(i) $\frac{1}{3}$ (ii) $\frac{2}{5}$ (iii) $\frac{4}{15}$

2. Total bulbs = 600, Defective bulbs = 12

$\Rightarrow$ Non-defective bulbs = 588

$\Rightarrow$ P (Non-defective bulb) = $\frac{588}{600}=\frac{49}{50}$

3. There are 10 members of the council. Any one may be selected for a committee.

$\therefore$ No. of exhaustive cases = 10

One woman out of 6 may be selected in 6 ways.

$\therefore$ No. of favourable cases = 6

$\therefore$ Probability of selection of a woman = $\frac{6}{10}=0.6$

4. The word 'ASSASSINATION' has 13 letters in which there are 6 vowels viz. AAAIIO and 7 consonants SSSSNNT.

$\therefore$ n (S) = 13

No. of vowels = 6

$\therefore$ Probability of choosing a vowel = $\frac{6}{13}$

No. of consonants = 7

$\therefore$ Probability of choosing a consonant = $\frac{7}{13}$

5. $\frac{3}{4}$ (The outcomes favourable to the event 'atmost one head' are HT, TH and TT).
6. 0.38
7. Events E and F are not mutually exclusive.
8. $P(E \text{ or } F) = P(E\cup F) = P(E)+P(F)-P(E\cap F)$

$=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$

9. 0.74
10. Each coin turns up in 2 ways.

Hence, the 5 coins can turn up in

$2\times 2\times 2\times 2\times 2=2^5$ ways = 32.

Hence, the probability of a toss of 5 heads = $\frac{1}{32}$

($\because$ there is one possibility of 5 heads falling).

11. $\frac{1}{12}$
12. Given numbers are {1, 2, 3, 4, 5, ..., 12}

$\therefore$ Total number of outcomes = 12

Numbers which are multiple of 3 = {3, 6, 9, 12}

No. of favourable outcomes = 4

$\therefore$ P(a number which is multiple of 3) = $\frac{4}{12}=\frac{1}{3}$.

13. $\frac{1}{3}$
14. $\therefore$ The required probability $=\frac{4}{10}=\frac{2}{5}$
15. Hence, required probability = 8/20 = 2/5

SHORT ANSWER QUESTIONS :

1. $\frac{3}{4}$
2. 0.784
3. A blue ball can be selected from the bag in 3 ways.

Hence, the required probability $=\frac{3}{10}$.

4. The possibilities are (i) A and B solve the problem and C does not solve the problem (ii) B and C solve the problem and A does not solve the problem and (iii) C and A solve the problem and B does not solve the problem.

$\therefore$ The required probability

$=\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{2}{3}=\frac{3}{36}+\frac{2}{36}+\frac{2}{36}=\frac{7}{36}$

5. $\frac{1}{4}$

6. $\frac{3}{8}$

Hint : Total number of letters in the word 'TRIANGLE' is 8 and the number of vowels in it is 3.

$\therefore$ The probability of choosing a vowel $= \frac{3}{8}$.

7. Since one die can be thrown in six ways to obtain any one of the six numbers marked on its six faces.

Therefore, if three dice are thrown, the total number of elementary events

$= 6 \times 6 \times 6 = 216$

Let A be the event of getting a total of at least 6. Then, $\bar{A}$ denotes the event of getting a total of less than 6 i.e. 3, 4, 5.

$\therefore \bar{A} = \{(1,1,1,),(1,1,2),(1,2,1),(2,1,1),(1,1,3),(1,3,1),(3,1,1)(1,2,2,),(2,1,2),(2,2,1)\}$

So, favourable number of elementary events = 10

$\therefore \quad P(\bar{A}) = \frac{10}{216} \Rightarrow P(A) = \frac{103}{108}$

8. $0.7 + 0.3 = 1$

9. Let there be x black balls in the bag.

$\therefore$ Total number of balls in the bag = $(6 + x)$

Now, P_1 = Probability of drawing a black ball = $\frac{x}{6+x}$

P_2 = Probability of drawing a red ball = $\frac{6}{6+x}$

It is given that,

$P_1 = 2P_2 \Rightarrow x = 12.$

Hence, there are 12 black balls in the bag.

10. Number x can be selected in three ways and number y can a so be selected in three ways.

Therefore, two numbers can be selected in $3 \times 3 = 9$ ways as given below :

$(1,1),(1,4),(1,9),(2,1),(2,4),(2,9),(3,1),(3,4),(3,9)$

So, total number of elementary events = 9

We have to find that cases in which the product xy will be less than 9.

This is possible when x and y can be chosen in the given way:

$(1,1),(1,4),(2,1),(2,4),(3,1).$

$\therefore$ Favourable number of elementary events = 5

Hence, required probability $= \frac{5}{9}$.

11. The number of favourable cases are shown below:

Number on white die	Number on black die
1	3
1	4
1	5
1	6
2	5
2	6

There are 6 favourable cases in which the number on black die is more than twice the number on the white die.

$\therefore m = 6$

n = Total number of cases = 6×6

($\because$ with each die there are six possibilities)

$\therefore$ Probability, $p = \frac{m}{n} = \frac{6}{6 \times 6} = \frac{1}{6}$.

12. Total number of five digit numbers formed by the digits 1, 2, 3, 4, 5, is 5!.

$\therefore$ Total number of elementary events = 5! = 120.

We know that a number is divisible by 4 if the number formed by last two digits is divisible by 4. Therefore last two digits can be 12, 24, 32, 52 that is, last two digits can be filled in 4 ways. but corresponding to each of these ways there are 3! = 6 ways of filling the remaining three places. Therefore the total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 and divisible by 4 is $4 \times 6 = 24$

$\therefore$ Favourable number of elementary events = 24.

So, required probability $= \frac{24}{120} = \frac{1}{5}$

LONG ANSWER QUESTIONS :

1. After removing three face cards of spades (king, queen, jack) from a deck of 52 playing cards, there are 49 cards left in the pack. Out of these 49 cards one card can be chosen in 49 ways.

$\therefore$ Total number of elementary events = 49

(i) Hence, P (Getting a black card) $= \frac{3}{49}$

(ii) Hence, P (Getting a queen) $= \frac{3}{49}$

(iii) Hence, P (Getting a black card) $= \frac{23}{49}$

2. (a) Required probability $= \frac{8}{13} \times \frac{8}{13} \times \frac{8}{13} = \frac{8^3}{13^3}$

(b) Required probability $= \frac{8}{13} \times \frac{7}{12} \times \frac{6}{11}$

3. After removing king, queen and jack of clubs from a deck of 52 playing cards.

There are 49 cards left in the deck. Out of these 49 cards one card can be chosen in 49 ways.

$\therefore$ Total number of elementary events = 49

(i) Hence, P (Getting a heart) $= \frac{13}{49}$.

(ii) Hence, P (Getting a club) $= \frac{10}{49}$.

(iii) There is only one '10' of hearts.

$\therefore$ Favourable number of elementary events = 1

Hence, P (Getting the '10' of hearts) $= \frac{1}{49}$.

4. Let there be x blue, y green and z white marbles in the jar.
Then, $x+y+z=54$ ...(i)

$\therefore$ P(selecting a blue marble) $= \frac{x}{54}$

But, probability (blue marble) $= \frac{1}{3}$. [given]

$\therefore \Rightarrow x=18$

Similarly, P(selecting a green marble)$=\frac{y}{54}$. $\Rightarrow$ $y=24$
Substituting the values of x and y in (i),
$z=12$
Hence, the jar contains 12 white marbles.

5. The probability that the student will pass in Hindi is 0.65.

6. A leap year has 52 weeks and 2 more days.
The two days can be :
Mon-Tue, Tue-Wed, Wed-Thur, Thur-Fri, Fri-Sat, Sat-Sun, Sun-Mon.
Total number of outcomes = 7
Let 'A' be the event which contains 53 Sundays.

$\therefore$ Favourable outcomes $= 2 \Rightarrow P(A) = \frac{2}{7}$

Similarly, $P(B)=\frac{2}{7}$

Now, '$A \cup B$' be the event contains either Sunday or Monday or both.

$\Rightarrow P(A \cup B)=\frac{3}{7}$

Now, P(53 Sundays and 53 Mondays) $= P(A \cap B) = ?$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{3}{7}=\frac{2}{7}+\frac{2}{7}-P(A\cap B)$$

$$\Rightarrow \quad P(A\cap B)=\frac{4}{7}-\frac{3}{7}=\frac{1}{7}.$$

Hence, P(53 Sundays and 53 Mondays) $= \frac{1}{7}$.

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (d) A white square can be selected in 32 ways. There will be 32 black squares on the board. Of these 4 black squares will be in the row in which the selected white square.
And 4 black squares will be in the column in which the selected white square is.
So if we do not take these 8, we have 24 squares to choose from $\Rightarrow$ Number of outcomes $= 32 \times 24 = 768$.

2. (c) Required probability = 1 – prob (that target won't be hit) $= 1-(3/4)^4 = 175/256$

3. (b) Area of the board $= 5 \times 5 = 25\ m^2$
Area of inner circle $= \pi \times 1^2 = \pi\ m^2$
Area of outer circle $= \pi \times 2^2 = 4\pi\ m^2$
Area between the two circle $= 4\pi - \pi = 3\pi$
Probability (Hitting the inner circle)
= Probability (Hitting the dart board) × Probability of hitting the area enclosed by the inner circle

$$= 0.75\times\frac{\text{Area of inner circle}}{\text{Area of dart board}} = 0.75\times\frac{\pi}{25}$$

$$= \frac{3\pi}{100}=\frac{3\times 22}{100\times 7}=\frac{33}{350}$$

4. (a)
5. (b) [**Hint.** The outcomes are 1, 2, 3, 4, 5, 6. Out of these, 4 is the only composite number which is less than 5]
6. (c) [**Hint.** The English alphabet has 26 letters in all. The word 'DELHI' has 5 letter, so the number of favourable outcomes = 5.]
7. (d) 8. (c) 9. (b)
10. (a) $\therefore$ A and B are independent events.
11. (a) Let $P(A)$ and $P(B)$ be the probability of the events then $P(A \text{ and } B)=P(A).P(B)=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$
12. (b) 24 out of the 90 two digit numbers are divisible by '3' and not by '5'.
The required probability is therefore, $\frac{24}{90}=\frac{4}{15}$.
13. (a) Required probability $=\frac{4}{6}=\frac{2}{3}$
14. (b) [**Hint.** One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.]
15. (b)

Odd digits = 1, 3, 5, 7, 9
Even digits = 0, 2, 4, 6, 8
Since odd digits at odd place and even digit at even place
Places of odd digit = 3 and places of even digits = 2
$\therefore$ Favourable number of ways = 1200
Total five digit numbers $= 9 \times 10 \times 10 \times 10 \times 10$

$\therefore$ Required probability $= \frac{1200}{9\times 10\times 10\times 10\times 10} = \frac{1}{75}$.

16. (c) required probability $= \frac{1+2+1}{11}=\frac{4}{11}$
17. (c) Required probability $= 1-\left(\frac{1}{2}\right)^4 = \frac{15}{16}$
18. (a) 19. (d) 20. (b)

MORE THAN ONE CORRECT :

1. (a, b, c) 2. (a, c)
3. (a, b, d) 4. (a, d)
5. (a, c)
6. (b, c) Here $P(A) = 0.4$ and $P(\bar{A}) = 0.6$
Probability that A does not happen at all $= (0.6)^3$
Thus required probability $= 1-(0.6)^3 = 0.784$
7. (a, b, c)

PASSAGE BASED QUESTIONS :

1. (a) As three faces are marked with number '2', so number of favourable cases = 3.

   $\therefore$ Required probability, $P(2)=\frac{3}{6}=\frac{1}{2}$

2. (c) No. of favourable cases = No. of events of getting the number 1 + no. of events of getting the number $3=2+1=3$

   $\therefore$ Required probability, $P(1 \text{ or } 3)=\frac{3}{6}=\frac{1}{2}$

3. (b) Only 1 face is marked with 3, so there are 5 faces which are not marked with 3.

   $\therefore$ Required probability, $P(\text{not } 3)=\frac{5}{6}$

ASSERTION & REASON :

1. (a) $\frac{2}{7}$
2. (c) Here, Assertion is correct, but reason is not true.
3. (a) Both assertion and reason are correct. Also, reason is the correct explanation of the assertion.

$$P(A\cap B)=\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)=\frac{6}{25}$$

4. (d) Assertion is not correct, but reason is correct.

$$P(\text{white marble})=\frac{5}{5+2+4}=\frac{5}{11}$$

$$P(\text{not white marble})=1-\frac{5}{11}=\frac{11-5}{11}=\frac{6}{11}$$

5. (a) Assertion and reason both are correct. Also reason is the correct explanation of the assertion.

MULTIPLE MATCHING QUESTIONS :

1. (A) $\to$ q, t; (B) $\to$ p; (C) $\to$ q, t; (D) $\to$ r, s

HOTS SUBJECTIVE QUESTIONS :

1. Let the no. of green marbles $=x$
   Let the no. of white marbles $=y$
   $\therefore$ Total no. of marbles $=x+y+10$

Now, P (selecting a green marble) $=\frac{x}{x+y+10}$

But it is given that P (green marble) $=\frac{1}{4}$

$$\therefore \frac{x}{x+y+10}=\frac{1}{4}\Rightarrow 4x=x+y+10\Rightarrow 3x-y=10 \text{ ...... (1)}$$

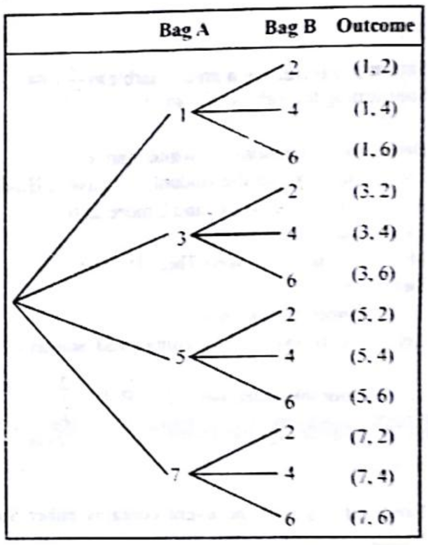
Similarly, P(selecting a white marble) $=\frac{y}{x+y+10}$

$$\therefore \frac{y}{x+y+10}=\frac{1}{3}\Rightarrow 2y-x=10 \text{ ..... (2)}$$

On solving equation (1) and (2) we get, $y=8$ and $x=6$.

$\therefore$ Total no. of marbles $=x+y+10=24$

2. (a)



The above diagram is the required tree diagram. There are 12 equally likely outcomes in the sample space.

(b) (i) The favourable outcomes for a sum of 7 are : (1, 6), (3, 4) and (5, 2)

$\therefore P(\text{the sum is } 7)=\frac{3}{12}=\frac{1}{4}$

(ii) Since all the sums of the outcomes are odd,

$P(\text{the sum is odd})=\frac{12}{12}=1$

(iii) There are no even sums.

$\therefore P(\text{the sum is even})=\frac{0}{12}=0$

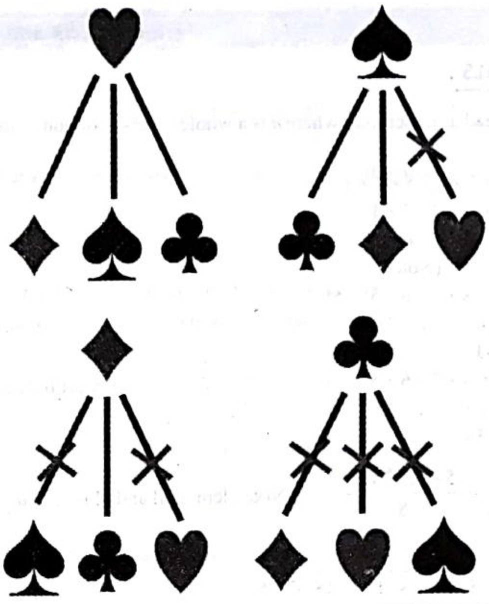
3. (i) The sum of the two numbers lies between 2 and 12. So the primes are 2, 3, 5, 7, 11.
   No. of ways for getting 2 = (1, 1) = 1
   No. of ways of getting 3 = (1, 2), (2, 1) = 2
   No. of ways of getting 5 = (1, 4), (4, 1), (2, 3), (3, 2) = 4
   No. of ways of getting 7
   = (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3) = 6
   No. of ways of getting 11 = (5, 6), (6, 5) = 2
   No. of favourable ways = 1 + 2 + 4 + 6 + 2 = 15
   No. of exhaustive ways = $6\times 6=36$

   Probability of the sum as a prime $=\frac{15}{36}=\frac{5}{12}$

   (ii) No. of ways of getting a sum of 9
   = (3, 6), (6, 3), (4, 5), (5, 4) = 4

   Probability of getting a sum of 9 $=\frac{4}{36}=\frac{1}{9}$

chapter 16



PERMUTATIONS AND COMBINATIONS

Introduction

In this chapter, you will study how to arrange several things (which are distinct or non distinct) in some specific order and number of ways in which the specific things arranged in some given order. You will also study in this chapter that how many groups are formed from the given various things, when specific thing can be selected in the groups.

This chapter is very important in the process of reasoning to arrive at particular conclusion from the general.

FACTORIALS :

$n!$ or $\lfloor n$ is read as n factorial, where n is a whole number (or non-negative integer) and $n! = n.(n-1).(n-2) \ldots 3.2.1$

Examples :

(i) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 1$

(ii) $5! = 5 \times 4 \times 3 \times 2 \times 1$

(iii) $11! = 11 \times 10 \times 9 \times 8!$

(iv) $1! = 1 = 0!$ (Note)

(v) $8! + 5! = 8 \times 7 \times 6 \times 5! + 5!$
$= 5! \times (8 \times 7 \times 6 + 1)$
$= 5! \times 337$

(vi) $8! - 5! = 8 \times 7 \times 6 \times 5! - 5!$
$= 5! \times (8 \times 7 \times 6 - 1)$
$= 5! \times 335$

(vii) $\frac{5}{8!} + \frac{3}{6!} = \frac{5 + (8 \times 7) \times 5}{8!}$ [Note : lcm of $a!$ and $b!$ is $a!$, if $a > b$]

(viii) $\frac{5!}{9!} = \frac{5!}{9 \times 8 \times 7 \times 6 \times 5!} = \frac{1}{9 \times 8 \times 7 \times 6}$

(ix) $5! \times 3! \neq 15!$

FUNDAMENTAL PRINCIPLES OF COUNTING :

(a) Principle of Addition :

If an event can occur in 'm' ways and another event can occur in 'n' ways independent of the first event, then either of the two events can occur in $(m + n)$ ways.

(b) Principle of Multiplication :

If an operation can be performed in 'm' ways and after it has been performed in any one of these ways, a second operation can be performed in 'n' ways, then the two operations in succession can be performed is $(m \times n)$ ways.

PERMUTATIONS AND COMBINATIONS :

(a) Permutations :

Each of the arrangements, which can be made by taking some or all of a number of things is called permutation.

(b) Combinations :

Each of the selections that can be made with a given number of objects taken some or all of them is called a combination.

DIFFERENCE BETWEEN PERMUTATION AND COMBINATION :

To understand the difference between permutation and combinations clearly consider the following example.

If we have 4 objects A, B, C and D the possible selection (or combination) and arrangements (or permutations) of 3 objects out of 4 are given below.

Selection (Combination)	Arrangement (Permutation)
ABC	*ABC, ACB, BAC, BCA, CAB, CBA*
ABD	*ABD, ADB, BAD, BDA, DAB, DBA*
ACD	*ACD, ADC, CAD, CDA, DAC, DCA*
BCD	*BCD, BDC, CBD, CDB, DBC, DCB*
No. of combinations = 4	No. of Permutations = 24

NUMBER OF PERMUTATIONS OF N ELEMENTS

(a) The number of permutations of n different things taken all at a time when repetition is not allowed is given by

$n.(n-1)...4.3.2.1 = n!$

(b) The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ^nP_r [or $P(n, r)$]

and is given by $^nP_r = \frac{n!}{(n-r)!}$, where $0 \le r \le n$.

(c) The number of permutations of n different things, taken r at a time, where repetition is allowed, is $(n)^r$.

(d) The number of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{p_1!p_2!......p_k!}$.

(e) The number of circular permutations of n different things taken all at a time is $\frac{^nP_n}{n} = (n-1)!$, if clockwise and anticlockwise orders are taken as different.

(f) Arrangements of beads or flowers (all different) around a circular necklace or garland:

The number of circular permutations of 'n' different things taken all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken to be same.

NUMBER OF COMBINATION OF *N* ELEMENTS :

(a) The number of combinations of n different things taken r at a time, denoted by ^nC_r [or $C(n, r)$], is given by

$$^nC_r = \frac{n!}{r!(n-r)!}, 0 \le r \le n$$

(b) The total number of selections of r things from n different things when each thing may be repeated any number of times is $^{n+r-1}C_r$.

NOTE: (i) $^nC_r = {^nC_{n-r}}$; if $1 < r < n$

(ii) $^nC_r = {^nC_{r-1}} = {^{n+1}C_r}$

(iii) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

MISCELLANEOUS SOLVED EXAMPLES

1. In a class there are 10 boys and 8 girls. The class teacher wants to select a student for monitor of the class. In how many ways the class teacher can make this selection ?

Sol. The teacher can select a student for monitor in two exclusive ways

(i) Select a boy among 10 boys, which can be done in 10 ways OR

(ii) Select a girl among 8 girls, which can be done in 8 ways.

Hence, by the fundamental principle of addition, either a boy or a girl can be selected in $10 + 8 = 18$ ways.

2. In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection?

Sol. The teacher has to perform two jobs :

(i) To select a boy among 10 boys, which can be done in 10 ways.

(ii) To select a girl, among 8 girls, which can be done in 8 ways.

Hence, the required number of ways $= 10 \times 8 = 80$.

3. How many words with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Sol. The word DAUGHTER consists of 3 vowels and 5 consonants

2 vowels and 3 consonants may be selected in ${}^3C_2 \times {}^5C_3$ ways

$$= {}^3C_1 \times {}^5C_2 \text{ ways } = \frac{3}{1} \times \frac{5 \times 4}{1 \times 2} = 30 \text{ ways}$$

5 letters can be arranged in 5! ways or in $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways

Number of words formed by 2 vowels and 3 consonants which are selected from the word DAUGHTER $= 30 \times 120 = 3600$

4. How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together ?

Sol. The word EQUATION consists of 5 vowels and 3 consonants

5 words can be arranged in $5! = 120$ ways

3 consonants can be arranged in $3! = 6$ ways

The two block of vowels and consonants can be arranged in $2! = 2$ ways

$\therefore$ the number of words which can be formed with letters of the word EQUATION so that vowels and consonants occur together $= 120 \times 6 \times 2 = 1440$.

5. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which will divisible by 10 and no digit is repeated?

Sol. A number is divisible by 10 if zero occurs at the units place. Now, we have to fill up 5 place with the digits 1, 3, 5, 7 and 9.

This can be done in $5! = 120$ ways

$\therefore$ Required number $= 120$

6. In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II containing 5 and 7 questions, respectively. A student is required to atempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Sol. Student may select 8 questions according to following scheme

	I (5 questions)	II (7 questions)
(a)	3	5
(b)	4	4
(c)	5	3

Required number of ways

$P = {}^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3 = {}^5C_2 \times {}^7C_2 + {}^5C_1 \times {}^7C_3 + {}^5C_0 \times {}^7C_3$

$= \frac{5.4}{2.1} \times \frac{7.6}{2.1} + 5 \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} + 1 \times \frac{7 \times 6 \times 5}{3.\,2.\,1} = 10 \times 21 + 5 \times 35 + 35 = 210 + 175 + 35 = 420$ ways

7. Determine the number of 5- cards combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Sol. 1 King can selected in ${}^4C_1$ ways

4 other cards can be selected in ${}^{48}C_4$ ways

$\therefore$ No. of card combination out of a deck of 52 cards each combination having exactly one king $= {}^4C_1 \times {}^{48}C_4$

8. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Sol. The word ASSASSI NATION has $4S, 3A, 2I, 2N, T, O$ are together. 4s considered as one block or 1 letter

Now, we have $3A, 2I, 2N, S, T, O$.

$\therefore$ Number of words = Number of permutations of $3A, 2I, 2N, S, T, O$.

Required no. of ways $= \frac{10!}{3!2!2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1} = 10 \times 9 \times 8 \times 7 \times 5 \times 3 \times 2 = 151200$

9. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x.

Sol. $\frac{1}{6!} + \frac{1}{7!} = \frac{1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} + \frac{1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{7+1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{8}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{8 \times 8}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{64}{8!}$

$\Rightarrow \frac{64}{8!} = \frac{x}{8!} \Rightarrow x = 64$

10. Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$

Sol. $\frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{9}$ or $\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{9}$

$\frac{1}{n} = \frac{1}{9}$ or $n = 9$

11. In how many ways can a student choose a programme of 5 courses if 9 courses are available and two courses are compulsory for every students?

Sol. Out of available nine courses, two are compulsory

Hence, the student is free to select 3 courses out of 7 remaining courses. If P is the number of ways of selecting 3 courses out of 7 courses, then

$P = C(7, 3) = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 7 \times 5 = 35$ ways

12. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5; assume that
(i) repetition of the digits is allowed
(ii) repetition of the digits is not allowed.

Sol. (i) There are five digits viz 1, 2, 3, 4 and 5. Every digit can be selected any number of times . Hence, we can select first digit 5 ways. The second digit of 5 ways and the third digit 5 ways. Hence, number of three digits number formed $= 5 \times 5 \times 5$ ways $= 125$ ways.

(ii) Under the restriction, first digit can be selected 5 ways. After the selection of first digit four digits are left. Second digit can be selected in 4 ways and third digit can be selected in 3 ways. Hence, number of three digits number formed $= 5 \times 4 \times 3 = 60$

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Permutation of n different things taken r at a time is denoted by ....
2. The circular permutations of n different things taken all at a time is ....
3. ${}^nP_r = {}^{n-1}P_{r-1}$ + ...
4. The number of combination of n dissimilar things taken r at a time is given by ....
5. ${}^nC_r + {}^nC_{r-1}$ = ...
6. The number of permutations of n objects, where p_1 object are of one kind, p_2 are of second kind .... p_k are of kth kind and the rest, if any, are of different kind, is ....
7. The number of ways in which $(p+q)$ dissimilar objects can be partitioned into two groups consisting of p and q objects respectively is ...., when $p \neq q$.
8. If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in .... ways.
9. Total no. of combinations of four letters taken two at a time is ....
10. For $0 \le r \le n$, ${}^nC_r$ = ...
11. If ${}^nC_x = {}^nC_y$, then $x + y$ = ...

T/F True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. $n! = n(n-1)(n-2)$......... 3. 2. 1
2. $\frac{20!}{18!} = 380$
3. If there are three objects, then the number of permutations of these objects, taken two at a time is 8.
4. Value of 0! is always 1.
5. If $2P(5,3) = P(n,4)$, then $n = 6$.
6. In a combination, the ordering of the selected objects is immaterial whereas in a permutation, the ordering is essential.
7. Each combination corresponds to many permutations.
8. ${}^{20}C_{14} = {}^{20}C_7$
9. If ${}^{10}C_x = {}^{10}C_{x+4}$, then $x = 3$.
10. Factorial of negative numbers is always greater than 1.

MTF Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1.

Column I	Column II
(A) 6!	(p) $\frac{P(4,2)}{2!}$
(B) P(5, 5)	(q) 720
(C) ${}^4C_2$	(r) 1
(D) C(5, 5)	(s) 120
	(t) P(4, 2)

2.

Column I	Column II
(A) Given ${}^nC_7 = {}^nC_4$ $n = ?$	(p) 3
(B) ${}^nP_r = 720$ and ${}^nC_r = 120$ $r = ?$	(q) 4
(C) $P(22, r+1) : P(20, r+2) = 11 : 52$ r = ?	(r) 11
(D) Given, ${}^{10}P_r = 5040$ r = ?	(s) 7

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Compute: $\frac{10!}{6!4!}$
2. Find the LCM of 4!, 5! and 6!
3. Find n, if $(n+1)! = 12 \times (n-1)!$
4. In a class there are 10 boys and 8 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways can the teacher make this selection?

5. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than cnce.

6. If $\frac{1}{6!}+\frac{1}{7!}=\frac{x}{8!}$, find x.

7. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 5, 6, 7, if no. digit is repeated.

8. Find n, if ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$

9. In how many ways 5 rings of different types can be worm in 4 fingers?

10. Find the total number of ways in which n distinct objects can be put into two different boxes.

11. If $P(n, 4) = 20 \times P(n, 2)$, find n.

12. In how many ways can 6 persons stand in a queue?

13. How many different words can be formed with the letters of the word 'MISSISSIPPI'?

14. Determine the number of 5 cards combinations out of a deck of 52 cards if there is exactly one ace in each combination.

SAQ Short Answer Questions

DIRECTIONS : *Give answer in 2-3 sentences.*

1. How many 4-letter code words are possible using the first 10 letters of the English alphabet if no letter can be repeated?

2. A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

3. Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other ?

4. How many 3-digit number can be formed by using the digits 1 to 9, if no digit is repeated?

5. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

6. How many chords can be drawn through 21 points on a circle?

7. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

8. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

9. In how many ways can one select a cricket team of eleven from 17 players in which 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

10. A bag contain 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

11. In a class of 10 students, there are 3 girls. In how many different ways can all the students be arranged in a row such that no two of the three girls are consecutive?

12. The word 'PATALIPUTRA' without changing the relative order of the vowels and consonants then how many words can be formed is ?

13. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ and ${}^{n}C_{r} = {}^{n}C_{r-1}$, then the values of n and r are?

LAQ Long Answer Questions:

DIRECTIONS : *Give answer in four to five sentences.*

1. Suppose m men and n women are to be seated in a row so that no two women sit together. If $m > n$, show that the number of ways in which they can be seated is

$$\frac{m!(m+1)!}{(m-n+1)!}$$

2. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

3. In a small village, there are 87 families, of which 52 families have atmost 2 children. In a rural development programme 20 families are to be chosen for assistance, of which atleast 18 families must have at most 2 children. In how many ways can the choice be made?

4. In an examination hall there are four rows of chairs. Each row has 8 chairs one behind the other. There are two classes sitting for the examination with 16 students in each class. It is desired that in each row, all students belong to the same class and that no two adjacent rows are allotted to the same class. In how many ways can these 32 students be seated?

5. A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways this can be done, when

(i) at least two ladies are included.

(ii) at most two ladies are included.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. Number of different permutations, each containing all letters of the word "STATESMAN" is
(a) 90720 (b) 45360
(c) 22680 (d) None of these

2. The total number of 9 digit numbers, which have all different digits is
(a) 9! (b) 8!
(c) $9 \times 9!$ (d) none of these

3. Number of 6-digit telephone numbers, which can be constructed with digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, if each number starts with 35 and no digit appears more than once is
(a) 1680 (b) 8!
(c) 6! (d) 6.6!

4. Numbers which can be formed with digits 1, 2, 3, 4, 3, 2, 1 using all digits so that odd digits occupy always the odd place are
(a) 6 (b) 12
(c) 15 (d) 18

5. No. of such numbers between 5000 and 10000 that can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, each digit appearing not more than once in each number is
(a) $5 \times {}^8P_3$ (b) $5 \times {}^8C_8$
(c) $5! \times {}^8P_3$ (d) $5! \times {}^8C_3$

6. A father with 8 children takes 3 at a time to the Zoological Gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is
(a) 336 (b) 112
(c) 56 (d) None of these

7. Out of 7 consonants and 4 vowels, no. of words that can be made each containing 3 consonants and 2 vowels is
(a) 120 (b) 25200
(c) 4200 (d) None of these

8. There are n points in a plane, no three being collinear except m of them which are collinear. The number of triangles that can be drawn with their vertices at three of the given points is
(a) ${}^{n-m}C_3$ (b) ${}^nC_3 - {}^mC_3$
(c) ${}^nC_{m-3}$ (d) None of these

9. In a plane there are 37 straight lines of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B, and no two are parallel. Then the number of intersecting points the lines have is equal to
(a) 535 (b) 601
(c) 728 (d) None of these

10. From six gentlemen and four ladies, a committee of five is to be formed. Number of ways in which this can be done if the committee is to include at least one lady is
(a) 252 (b) 246
(c) 248 (d) 250

11. Given five different green dyes, four different blue dyes and three different red dyes. The number of combinations of dyes which can be chosen, taking at least one green and one blue dye, is
(a) 3240 (b) 3570
(c) 3720 (d) 3880

12. Six persons A, B, C, D, E and F are to be seated at a circular table. The number of ways, this can be done if A must have either B or C on his right and B must have either C or D on his right is
(a) 36 (b) 12
(c) 24 (d) 18

13. Given five line segments of length 2, 3, 4, 5, 6 units. Then the number of triangles that can be formed by joining these lines is
(a) ${}^5C_3 - 3$ (b) ${}^5C_3 - 1$
(c) ${}^5C_3$ (d) ${}^5C_3 - 2$

14. The number of values of r satisfying the equation ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is
(a) 1 (b) 2
(c) 3 (d) 4

15. If 12 persons are seated in a row, the number of ways of selecting 3 persons from them, so that no two of them are seated next to each other is
(a) 85 (b) 100
(c) 120 (d) 240

16. A teaparty is arranged for 16 people along two sides of a large table with 8 chais on each side. Four men want to sit on one particular side and two on the other side. The number of ways in which they can be seated is

(a) $\frac{6!8!10!}{4!6!}$ (b) $\frac{8!8!10!}{4!6!}$

(c) $\frac{8!8!6!}{6!4!}$ (d) None of these

17. Ten different letters of an alphabet are given, words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

(a) 69760 (b) 30240

(c) 99784 (d) None of these

18. A box contains two white balls, three black balls and four red balls. No. of ways can three balls be drawn from the box if atleast one black ball is to be included in the draw is

(a) 129 (b) 84

(c) 64 (d) None

19. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card

(a) $\frac{52!}{(17!)^3}$ (b) $\frac{52!}{(17!)^3 3!}$

(c) $\frac{51!}{(17!)^3}$ (d) $\frac{51!}{(17!)^3 3!}$

20. The number of ways in which four letters of the word MATHEMATICS can be arranged is given by

(a) 136 (b) 192

(c) 1680 (d) 2454

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which among the following is/ are correct ?

(a) If an operation can be performed in 'm' different ways and a second operation can be performed in 'n' different ways, then both of these operations can be performed in '$m \times n$' ways together.

(b) The number of arrangements of n different objects taken all at a time is $n!$.

(c) The number of permutations of n different things taken r at a time, when each thing may be repeated any number of times is n^r.

(d) The number of circular permutations of 'n' different things taken all at a time is $\frac{1}{2}(n-1)!$, if clockwise and anticlockwise orders are taken as different.

2. $C(8,3) = ?$

(a) $\frac{P(8,3)}{3!}$ (b) $C(8,5)$

(c) 56 (d) $P(8,5)$

3. To fill up 12 vacancies, there are 25 candidates of which 5 are from SC. If 3 of these vacancies are reserved for SC candidates while the remaining are open to all then the number of ways in which the selection can be made is

(a) ${}^5C_2 \times {}^{22}C_{13}$ (b) ${}^5C_3 \times {}^{22}C_9$

(c) ${}^5C_3 \times {}^{23}C_8$ (d) ${}^8C_5 \times {}^{21}C_7$

4. ${}^{10}P_0$ corresponds to which value among the following given options.

(a) 0 (b) ${}^{10}C_0$

(c) 1 (d) 10

5. Which among the following is/are not correct?

(a) ${}^6C_2 + {}^6C_1 = {}^7C_2$ (b) ${}^6C_1 + {}^6C_2 = {}^6C_2$

(c) ${}^6C_2 + {}^6C_1 = {}^7C_1$ (d) ${}^6C_2 - {}^6C_1 = {}^7C_2$

6. The number of ways in which a team of eleven players can be selected from 22 players, always including 2 of them and excluding 4 of them is

(a) ${}^{16}C_{11}$ (b) ${}^{16}C_7$

(c) ${}^{16}C_9$ (d) ${}^{20}C_9$

Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Passage I

There are 4 candidates for a Natural science scholarship, 2 for a classical and 6 for a Mathematical scholarship.

1. No. of ways these scholarships can be awarded is

(a) 48 (b) 12

(c) 24 (d) 8

2. No. of ways one of these scholarship can be awarded is

(a) 6 (b) 10

(c) 48 (d) 12

Passage II

In how many ways can a cricket eleven be chosen out of a batch of 15 players if

1. there is no restriction on the selection;

(a) ${}^{15}C_0$ (b) ${}^{15}C_1$

(c) ${}^{15}C_{11}$ (d) none of these

2. a particular player is always chosen;

(a) ${}^{14}C_4$ (b) ${}^{14}C_1$

(c) ${}^{14}C_{13}$ (d) none of these

3. a particular player is never chosen?

(a) ${}^{14}C_3$ (b) ${}^{14}C_0$

(c) ${}^{14}C_{14}$ (d) none of these

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect.**

(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** The maximum number of points of intersection of 8 circles of unequal radii is 56.
   Reason : The maximum number of points into which 4 circles of unequal radii and 4 non coincident straight lines intersect, is 50.

2. Consider the word 'SMALL'
   Assertion : Total number of 3 letter words from the letters of the given word is 13.
   Reason : Number of words having all the letters distinct = 4 and number of words having two are alike and third different = 9

3. **Assertion :** The number of ways of selecting 5 students from 12 students (of which six are boys and six are girls), such that in the selection there are at least three girls is ${}^6C_3\ {}^9C_2$.
   Reason : If a work has two independent parts, of which first part can be done in m way and for each choice of first part, the second part can be done in n ways, then the work can be completed in $m \times n$ ways.

4. **Assertion :** A polygon has 44 diagonals and number of sides are 11.
   Reason : From n distinct objects r objects can be selected in ${}^nC_r$ ways.

5. **Assertion :** $\left(\sum_{r=0}^{100} {}^{500-r}C_3\right) + {}^{400}C_4 = {}^{501}C_4$
   Reason : ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column I	Column II
(A) ${}^{20}C_{19}$	(p) 20!
(B) ${}^{20}P_{19}$	(q) ${}^{20}C_1$
(C) ${}^6C_3 + {}^6C_2$	(r) 5
(D) P(5, 1)	(s) $\dfrac{P(20,19)}{19!}$

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. In how many ways can 3 prizes be distributed among 4 boys, when
   (i) no boy gets more than one prize?
   (ii) a boy may get any number of prizes?
   (iii) no boy gets all the prizes?

2. A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if at least five women have to be included in a committee? In how many of these committees
   (i) the women are in majority
   (ii) the men are in majority?

3. The sum of all the numbers of four different digits that can be made by using the digit 0, 1, 2 and 3.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. ${}^nP_r$ 2. $(n-1)!$ 3. $rx^{n-1}P_{r-1}$
4. ${}^nC_r$ 5. ${}^{n+1}C_r$ 6. $\frac{n!}{p_1!p_2!p_3!......p_k!}$
7. $\frac{(p+q)!}{p!.q!}$ 8. $m+n$ 9. 6.
10. ${}^nC_{n-r}$ 11. n

TRUE / FALSE

1. True 2. True 3. False 4. True
5. False 6. True 7. True 8. False
9. True 10. False.

MATCH THE FOLLOWING :

1. (A) → q (B) → s (C) → p (D) → r

(A) $6! = 6\times5\times4\times3\times2\times1 = 720$

(B) $P(5,5) = \frac{5!}{(5-5)!} = 5\times4\times3\times2\times1 = 120$

(C) ${}^4C_2 = \frac{4!}{2!\times2!} = \frac{P(4,2)}{2!}$

(D) $C(5,5) = \frac{5!}{5!\times0!} = 1$

2. (A) → r (B) → p (C) → s (D) → q

(A) $n = 7+4 = 11$

(B) ${}^nC_r = 120 = \frac{{}^nP_r}{r!} = \frac{720}{r!}$

$\frac{720}{r!} = 120$

$r! = \frac{720}{120} = 6 = 3!$

$r = 3$

(C) $\frac{22!}{(21-r)!}\times\frac{(18-r)!}{20!} = \frac{11}{52}$

$$\frac{22\times21\times20!}{(21-r)(20-r)(19-r)(18-r)!}\times\frac{(18-r)!}{20!} = \frac{11}{52}$$

$$\frac{22\times21}{(21-r)(20-r)(19-r)} = \frac{11}{52}$$

$(21-r)(20-r)(19-r) = 2\times21\times52$
$= 2\times3\times7\times4\times13$
$= 12\times13\times14$

$(21-r)(20-r)(19-r) = (21-7)(20-7)(19-7)$
$r = 7$

(D) $\frac{10!}{(10-r)!} = 5040 = 10\times504 = 10\times9\times8\times7$

$\frac{10!}{(10-r)!} = \frac{10\times9\times8\times7\times6!}{6!} = \frac{10!}{6!}$

$(10-r)! = 6! = (10-4)!$
$r = 4$

VERY SHORT ANSWER QUESTIONS :

1. $\frac{10!}{6!4!} = \frac{10\times9\times8\times7\times6!}{6!\times(4\times3\times2\times1)} = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

2. We have, $5! = 5\times4!$ and $6! = 6\times5\times4!$

∴ L.C.M. of $4!, 5!, 6!$ = L.C.M. $[4!, 5\times4!, 6\times5\times4!]$
$= (4!)\times5\times6 = 6! = 720$

3. $n = 3$

4. Here the teacher is to perform either of the following two jobs

(i) selecting a boy among 10 boys.

or

(ii) selecting a girl among 10 girls.

The first of these can be performed in 10 ways and the second in 8 ways. Therefore, by fundamental principle of addition either of the two jobs can be performed in $(10+8) = 18$ ways. Hence, the teacher can make the selection of either a boy or a girl in 18 ways.

5. There are 10 digits form 0 to 9

6	7	I	II	III

I place after 67 can be filled in 8 ways
II place after 67 can be filled in 7 ways
III place after 67 can be filled in 6 ways
∴ No. of telephone numbers that can be constructed
$= 8\times7\times6 = 336$

6. $x = 64$

7. One of the digit 2, 4 and 6 will come at unit place
Now, we have 6 digits and 2 places are to be filled up
This can be alone in ${}^6P_2$ ways.
$\therefore$ No. of 3-digit even numbers are $3 \times {}^6P_2 = 3 \times 6 \times 5 = 50$

8. $\frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{9}$ or $\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{9}$

$\frac{1}{n} = \frac{1}{9}$ or $n = 9$

9. The first ring can be worn in any of the 4 fingers. So, there are 4 ways of wearing it. Similarly, each one of the other rings can be worn in 4 ways.
Hence, the requisite number of ways $= 4\times4\times4\times4\times4 = 4^5$.

10. Let the two boxes be B_1 and B_2. We observe that there are two choices for each of the n objects. Therefore, by fundamental priniciple of counting
Total number of ways $= 2\times2\times.....n\text{-times} \times 2 = 2^n$

11. $n = 7$

12. The number of ways in which 6 persons can stand in a queue is same as the number of arrangements of 6 different things taken all at a time.
Hence, the required number of ways $= {}^6P_6 = 6! = 720$

13. There are 11 letters in the given word, of which 4 are S' s, 4 are I's and 2 are P's. So, total number of words is the number of arrangements of 11 things, of which 4 are similar of one kind, 4 are similar of second kind and 2 are similar of third kind i.e., $\frac{11!}{4!4!2!}$.

Hence, the total number of words $= \frac{11!}{4!4!2!} = 34650$.

14. One ace will be selected from four aces and four cards will be selected from $52 - 4 = 48$ cards
If P is the required number of ways, then
$P = C(4, 1) \times C(48, 4)$

$= \frac{4!}{1!(4-1)!} \times \frac{48!}{4!(48-4)!} = \frac{4\times3!}{1!\times3!} \times \frac{48\times47\times46\times45\times44!}{4\times3\times2\times1\times44!}$

$= 4\times2\times47\times46\times45 = 778320$ ways

SHORT ANSWER QUESTIONS :

1. Hence number of four letters code words
$= 10 \times 9 \times 8 \times 7$
$= 5040$

2. Total number of outcomes in three toss $= 2 \times 2 \times 2 = 8$

3. The total number of signals which can be generated
$= 5 \times 4 = 20$

4. 3 digit number are to be formed with digits 1 to 9.
This can be done in $9 \times 8 \times 7 = 504$ ways

5. Out of 5 digits, 4-digit numbers are to be formed. No. of such numbers are ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$
When 2 is at units place then remaining three places are filled in ${}^4P_3$ ways $= 4 \times 3 \times 2 = 24$
When 4 is at unit's place, then 4-digit numbers are again $= 24$
$\therefore$ Even 4- digit numbers $= 2 \times 24 = 48$

6. Number of chords from 21 points

$= C(21, 2) = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!}$

$= \frac{21\times20(19!)}{2\times(19!)} = 21\times10 = 210$ chords

7. 3 boys out of 5 boys can be selected in ${}^5C_3$ ways
3 girls out of 4 girls can be selected in ${}^4C_3$ ways
Number of ways in which 3 boys and 3 girls are selected

$= {}^5C_3 \times {}^4C_3 = {}^5C_2 \times {}^4C_1 = \frac{5\times4}{1\times2} \times \frac{4}{1} = 40$

8. The number of ways of selecting 3 balls of each colour $= {}^6C_3 \times {}^5C_3 \times {}^5C_3 = {}^6C_3 \times {}^5C_2 \times {}^5C_2$

$= \frac{6\times5\times4}{1\times2\times3} \times \frac{5\times4}{1\times2} \times \frac{5\times4}{1\times2} = 20 \times 10 \times 10 = 2000$

9. Number of ways of selecting the cricket team of eleven
$= C(5, 4) \times C(12, 7)$

$= \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!}$

$= \frac{5\times4!}{1\times4!} \times \frac{12\times11\times10\times9\times8\times7!}{5!\times7!} = 3960$

10. The number of ways in which 2 black and 3 red balls can be selected $= {}^5C_2 \times {}^6C_3$

$= \frac{5\times4}{1\times2} \times \frac{6\times5\times4}{1\times2\times3} = 10\times20 = 200$

11. Number of girls = 3, number of boys = 7. Since there is no restriction on boys, therefore first of all arrange the 7 boys in ${}^7P_7 = 7!$ ways.
$\times B \times B \times B \times B \times B \times B \times B \times$
If the girls are arranged at the places (including the two ends) indicated by crosses, no two of three girls will be consecutive.
Now there are 8 places for 3 girls
$\therefore$ 3 girls can be arranged in ${}^8P_3$ ways
$\therefore$ Required number $= {}^8P_3 \times 7!$

$= \frac{8!}{5!} \times 7! = 4281.$

12. There are eleven letters in the word 'PATALIPUTRA' and there are two P's, two T's, three A's and four other different letters.

Number of consonants = 6,

number of vowels = 5

Since relative order of the vowels and consonants remains unchanged, therefore, vowels will occupy only vowels's place and consonants will occupy only consonant's place. Now 6 consonants can be arranged among themselves in

$\therefore \frac{6!}{2!2!}$ ways [since there are two P's and two T's]

and five vowels can be arranged among themselves in

$\frac{5!}{3!}$ ways, since A occurs thrice

$\therefore$ Required number $= \frac{6!}{2!2!} \cdot \frac{5!}{3!} = 3600$

13. We have, ${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!} \Rightarrow \frac{1}{(n-r)} = 1$$

or $n - r = 1$ ...(1)

Also, ${}^nC_r = {}^nC_{r-1} \Rightarrow r + r - 1 = n \Rightarrow 2r - n = 1$ ...(2)

Solving (1) and (2), we get $r = 2$ and $n = 3$

LONG ANSWER QUESTIONS :

1. Let the men take their seats first. They can be seated in ${}^mP_m$ ways as shown in the following figure.

□ M □ M □ .... □ M □

1^{st} 2^{nd} m^{th}

From the above figure, we observe, that there are $(m + 1)$ places for n women. It is given that $m > n$ and no two women can sit together. Therefore, n women can take their seats in ${}^{(m+1)}P_n$ ways and hence the total number of ways so that no two women sit together is

$$({}^mP_m) \times ({}^{m+1}P_n) = \frac{m!(m+1)!}{(m-n+1)!}$$

2. Therefore, the total number of ways where ladies sit together is $3! \times 3! \times 2! = 72$.

3. Hence, the total number of possible choices is

$${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$$

4. Let the two classes be C_1 and C_2 and the four rows be R_1, R_2, R_3, R_4. There are 16 students in each class. So, there are 32 students. According to the given conditions there are two different ways in which 32 students can be seated:

	R_1	R_2	R_3	R_4
I	C_1	C_2	C_1	C_2
II	C_2	C_1	C_2	C_1

Since, the seating arrangement can be completed by using any one of these two ways. So, by the fundamental principle of addition, we have

Total number of seating arangements

= No. of arrangements in I case + No. of arrangements in II case.

Now, 16 students of class C_1 can be seated in 16 chairs in ${}^{16}P_{16} = 16!$ ways.

And, 16 students of class C_2 can be seated in 16 chairs in ${}^{16}P_{16} = 16!$ ways.

Hence, Total number of seating arrangements

$= (16! \times 16!) + (16! \times 16!) = 2(16! \times 16!)$

5. (i) A committee of 5 persons, consisting of at least two ladies, can be formed in the following ways:

I. Selecting 2 ladies out of 4 and 3 gents out of 6. This can be done in ${}^4C_2 \times {}^6C_3$ ways.

II. Selecting 3 ladies out of 4 and 2 gents out of 6. This can be done in ${}^4C_3 \times {}^6C_2$ ways.

III. Selecting 4 ladies out of 4 and 1 gent out of 6. This can be done in ${}^4C_4 \times {}^6C_1$ ways.

Since, the committee is formed in each case, therefore, by the fundamental principle of addition, the total number of ways of forming the committee

$= {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$

$= 120 + 60 + 6 = 186$

(ii) A committee of 5 persons, consisting of at most two ladies, can be constituted in the following ways:

I. Selecting 5 gents only out of 6. This can be done in ${}^6C_5$ ways.

II. Selecting 4 gents only out of 6 and one lady out of 4. This can be done in ${}^6C_4 \times {}^4C_1$ ways.

III. Selecting 3 gents only out of 6 and two ladies out of 4. This can be done in ${}^6C_3 \times {}^4C_2$ ways.

Since the committee is formed in each case, so, the total number of ways of forming the committee

$= {}^6C_5 + {}^6C_4 \times {}^4C_1 + {}^6C_3 \times {}^4C_2 = 6 + 60 + 120 = 186.$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (b) "STATESMAN"
Total number of letters = 9,
Number of S's = 2, Number of T's = 2,
Number of A's = 2
$\therefore$ Required number of permutations
$$= \frac{9!}{(2)!\,(2)!\,(2)!} = 45360$$

2. (c) Total number of digits is 10, i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In order to find the total number of 9 digit numbers we have to find the number of ways of filling up 9 places out of these 10 digits, but 0 cannot be in the first place. Out of the remaining 9 digits any one can be put in the first place.
$\therefore$ The first place can be filled in 9 ways.
The remaining 8 places can be filled by any eight of the remaining 9 digits which can be done in ${}^9P_8$ ways.
Hence, the total number of 9 digit numbers are
$9 \times {}^9P_8 = 9 \times 9!$

3. (a) Since, each number consisting of 6 digits starts with 35, 3 and 5 are fixed in the first and second places.
The other four places can be filled up with remaining 8 digits in ${}^8P_4$ ways $= 8 \times 7 \times 6 \times 5 = 1680$ ways.
Hence, the required number of telephone numbers is 1680.

4. (d)

5. (a)

6. (c) The number of times he will go to the garden is same as the number of ways of selecting 3 children from 8.
So, the required number $= {}^8C_3 = 56$.

7. (b) Hence, the required no. of words $= 35 \times 6 \times 120 = 25200$.

8. (b) Hence, required number of triangles $= {}^nC_3 - {}^mC_3$.

9. (a) Hence, the number of intersection points of the lines is
${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535$.

10. (b) We can select 5 members for the committee to include at least one lady in the following four ways :
(1) 1 lady and 4 gentleman
(2) 2 ladies and 3 gentlemen
(3) 3 ladies and 2 gentlemen
(4) 4 ladies and 1 gentleman.
Hence, required number of committees
$$= {}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$
$$= 60 + 120 + 60 + 6 = 246$$

11. (c)

12. (d) When A has B or C to his right we have the order :
AB or AC ...(1)
When B has C or D to his right, we have the order :
BC or BD ...(2)
Taking these two possibilities together, we must have ABC or ABD or AC and BD.
For ABC, D, E, F to arrange along a circle, number of way = 3 ! = 6, where three persons A,B,C together are treated as single.
For ABD, C, E, F, the number of ways = 6.
For AC, BD, E,F the number of ways = 6.
Hence, total number of ways = 18.

13. (a) We know that in any triangle the sum of two sides is always greater than the third side.
$\therefore$ The triangle will not be formed if we select segments of length (2, 3, 5), (2, 3, 6) or (2, 4, 6).
Hence, number of triangles formed $= {}^5C_3 - 3$.

14. (b) ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$
$\Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$
$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$
$\Rightarrow r^2 = 3r \text{ or } r^2 = 40 - 3r \Rightarrow r = 0, 3 \text{ or } -8, 5$
3 and 5 are the values as the given equation is not defined by $r = 0$ and $r = -8$. Hence, the number of values of r is 2.

15. (c)

16. (b) There are 8 chairs on each side of the table. Let the sides be represented by A and B. Let four persons sit on side A, then number of ways of arranging 4 persons on 8 chairs on side $A = {}^8P_4$ and then two persons sit on side B. The number of ways of arranging 2 persons on 8 chairs on side $B = {}^8P_2$ and the remaining 10 persons can be arranged in remaining 10 chairs in 10! ways.
Hence, the total number of ways in which the persons can be arranged
$$= {}^8P_4 \times {}^8P_2 \times 10! = \frac{8!\,8!\,10!}{4!\,6!}$$

17. (a)

18. (c) At least one black ball can be drawn in the following ways:
(i) one black and two other colour balls
(ii) two black and one other colour balls, and
(iii) all the, three black balls
Therefore the required number of ways is
$${}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3 = 64.$$

19. (b)

20. (d) Two pairs of identical letters can be arranged in $^3C_2 \frac{4!}{2!2!}$ ways. Two identical letters and two different letters can be arranged in $^3C_1 \times {^7C_2} \times \frac{4!}{2!}$ ways. All different letters can be arranged in $^8P_4$ ways.

$\therefore$ Total no. of arrangements

$$= {^3C_2} \frac{4!}{2!2!} + {^3C_1} \times {^7C_2} \times \frac{4!}{2!} + \frac{8!}{4!} = 2454.$$

MULTIPLE CHOICE QUESTIONS MORE THAN ONE CORRECT ANSWER :

1. (a, b, c).
2. (a, b, c)
3. (a, b)

3 vacancies for SC candidates can be filled up from 5 candidates in $^5C_3$ ways.

After this for remaining $12 - 3 = 9$ vacancies, there will be $25 - 3 = 32$ candidates. These vacancies can be filled up in $^{22}C_9$ ways.

Hence, required number of ways $= {^5C_3} \times {^{22}C_9}$

or $^5C_3 \times {^{22}C_9} = {^5C_{5-3}} \times {^{22}C_{22-9}} = {^5C_2} \times {^{22}C_{13}}$

4. (b, c)
5. (b, c, d).
6. (b, c)

PASSAGE BASED QUESTIONS :

Passage I

1. (a) Natural science scholarship can be awarded to any one of the four candidates. So, there are 4 ways of awarding the natural science scholarships. Similarly, mathematical and classical scholarships can be awarded in 2 and 6 ways respectively. Hence, by fundamental principle of multiplication, number of ways of awarding these scholarships $= 4\times2\times6 = 48$

2. (d) By fundamental principle of addition, number of ways of awarding one of the three scholarships $= 4+2+6 = 12$

Passage II

1. (c) The total number of ways of selecting 11 players out of 15 is

$$^{15}C_{11} = {^{15}C_{15-11}} = {^{15}C_4} = \frac{15\times14\times13\times12}{4\times3\times2\times1} = 1365$$

2. (a) If a particular player is always chosen. This means that 10 players are selected out of the remaining 14 players.

$\therefore$ Required number of ways

$= {^{14}C_{10}} = {^{14}C_{14-10}} = {^{14}C_4} = 1001$

3. (a) If a particular player is never chosen. This means that 11 players are selected out of the remaining 14 players.

$\therefore$ Required number of ways $= {^{14}C_{11}} = {^{14}C_{14-11}}$

$= {^{14}C_3} = 364$

ASSERTION & REASON :

1. (b) Two circles intersect in 2 points.

$\therefore$ Maximum number of points of intersection

$= 2 \times$ number of selections of two circles from 8 circles

$= 2 \times {^8C_2} = 2 \times 28 = 56$

Statement 2 : 4 lines intersect each other in $^4C_2 = 6$ points

4 circles intersect each other in $2 \times {^4C_2} = 12$ points.

Further, one lines and one circle intersect in two points.

So 4 lines will intersect four circles in 32 points.

Maximum number of points $= 6 + 12 + 32 = 50$-

2. (a) Number of words having all the letters distinct $= {^4P_1} = 4$

Number of words having two are alike and third different

$= {^1C_1} \cdot {^3C_1} \cdot \frac{3!}{2!} = 9$

3. (d) Statement – II is true, known as the rule of product. Statement – I is not true, as the two parts of the work are not independent. Three girls can be chosen out of six girls in $^6C_3$ ways, but after this choosing 3 students out of remaining nine students depends on the first part.

4. (a) Let no of sides are n.

$^nC_2 - n = 44$

$\Rightarrow n = -8$ or $11 \Rightarrow n = 11$.

5. (a) $({^{400}C_4} + {^{400}C_3}) + {^{401}C_3} + ... + {^{500}C_3}$

$= ({^{401}C_4} + {^{401}C_3}) + {^{402}C_3} + ... + {^{500}C_3}$ ...

$= ({^{500}C_4} + {^{500}C_3}) = {^{501}C_4}$

MULTIPLE MATCHING QUESTIONS :

1. (A) $\to$ q, s, (B) $\to$ p (C) $\to$ t (D) $\to$ r

HOTS SUBJECTIVE QUESTIONS :

1. (i) The number of ways in which all the prizes can be given away $= 4 \times 3 \times 2 = 24$.

(ii) The number of ways in which all the prizes can be given away $= 4 \times 4 \times 4 = 4^3 = 64$.

(iii) The number of ways in which a boy does not get all the prizes $= 64 - 4 = 60$.

2. There are 9 women and 8 men. A committee of 12, consisting of at least 5 women, can be formed by choosing:
(I) 5 women and 7 men
(II) 6 women and 6 men
(III) 7 women and 5 men
(IV) 8 women and 4 men
(V) 9 women and 3 men
∴ Total number of ways of forming the committee

$= {}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$

$= 126 \times 8 + 84 \times 28 + 36 \times 56 + 9 \times 70 + 1 \times 56 = 6062$

Clearly, women are in majority in (iii), (iv) and (v) cases as discussed above.

So, total number of committee in which women are in

$= {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$

$= 36 \times 56 + 9 \times 70 + 1 \times 56 = 2702$

Clearly, men are in majority in only (I) case as discussed above.

So, total number of committees in which men are in majority

$= {}^9C_5 \times {}^8C_7 = 126 \times 8 = 1008.$

3. The number of numbers with 0 in the units place $= 3! = 6$
The number of numbers with 1 or 2 or 3 in the units place $= 3! - 2! = 4$
∴ The sum of the digits in the unit place
$= 6 \times 0 + 4 \times 1 + 4 \times 2 + 4 \times 3 = 24$
Similarly, for the tens and the hundreds places.
The number of numbers with 1 or 2 or 3 in the thousands place $= 3!$
∴ The sum of the digits in the thousands place
$= 6 \times 1 + 6 \times 2 + 6 \times 3 = 36$
∴ The required sum
$= 36 \times 1000 + 24 \times 100 + 24 \times 10 + 24 = 38664$

chapter 17

$$(1+x)^n$$
$$= 1 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_r x^r + \ldots + x^n$$
$$= 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{r}x^r + \ldots + x^n$$
$$= 1 + nx + \left(\frac{n(n-1)}{1\times 2}\right)x^2 + \ldots$$
$$+ \left(\frac{n(n-1)(n-2)\ldots(n-4+1)}{1\times 2 \times \ldots \times r}\right)x^r + \ldots + x^n$$
$$= 1 + \frac{n!}{1!(n-1)!}x + \frac{n!}{2!(n-2)!}x^2 + \ldots$$
$$+ \frac{n!}{r!(n-r)!}x^r + \ldots + x^n.$$

MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

Introduction

Mathematical Induction is a way by which we arrive at a general from particular conclusion. The principle of mathematical induction is one such tool, which can be used to prove a wide variety of mathematical statements. Each such statement is assumed as $P(n)$ associated. With positive integer n, for which the correctness for the case $n = 1$ is examined. Then assuming the truth of $P(K + 1)$ is established. In this way the statement is proved for any natural number n. For any given two numbers a and b, if we required to find $(a + b)^2$, we can just add a and b then multiply the sum by itself. Another way of doing it is to find $a^2 + 2ab + b^2$. We also know that $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Now we may need to use the expression for $(a + b)^5$, $(a + b)^{30}$, etc. But we can not remember all the expressions. Binomial Theorem helps to find these expressions. Binomial Theorem is the theorem which gives the expansion of $(x + a)^n$, where 'n' is a natural number. This theorem was first given by Sir Issac Newton.

MATHEMATICAL INDUCTION :

Induction' means the generalisation from particular cases of facts.
The principle of mathematical induction is one such tool which can be used to prove a wide variety of mathematical statements. Each such statement is assumed as $P(n)$ associated with positive integer n
If $P(1)$ is true and $P(k+1)$ is true whenever $P(k)$ is true, then $P(n)$ is true for all natural numbers n.

BINOMIAL THEOREM :

(i) The expansion of a binomial for any positive integral power n is given by Binomial theorem, which is
$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \text{..........} + {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n;$

where ${}^nC_r = \dfrac{n!}{(n-r)!r!}$, $n! = n(n-1)(n-2)(n-3)\text{.......}2.1$ and $0! = 1! = 1$

(ii) The general term of an expansion $(a+b)^n$ is
$T_{r+1} = {}^nC_ra^{n-r}.b^r.$

(iii) In the expansion $(a+b)^n$, if n is even, then the middle term is the $\left(\dfrac{n}{2}+1\right)^{th}$ term. If n is odd, then the middle terms are

$\left(\dfrac{n+1}{2}\right)^{th}$ and $\left(\dfrac{n+1}{2}+1\right)^{th}$ terms.

Some Important Expansions Based on Binomial Theorem :

(a) $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \ldots + {}^nC_nx^n = 1 + nx + \dfrac{n(n-1)}{2\times1}x^2 + \dfrac{n(n-1)(n-2)}{3\times2\times1}x^3 + \ldots + x^n$

(b) $(1-x)^n = {}^nC_0 - {}^nC_1x - {}^nC_2x^2 - {}^nC_3x^3 + \ldots + (-1)n\, {}^nC_nx^n = 1 - nx + \dfrac{n(n-1)}{2\times1}x^2 - \dfrac{n(n-1)(n-2)}{3\times2\times1}x^3 + \ldots + (-1)^nx^n$

(c) ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n$

(d) ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^n\, {}^nC_n = 0$

NOTE:

(i) ${}^nC_r = {}^nC_{r-r}$ [where, $r<n$]

(ii) ${}^nC_0 = {}^nC_n = 1$

(iii) ${}^nC_1 = {}^nC_{n-1} = n$

(iv) ${}^nC_2 = {}^nC_{n-2} = \dfrac{n(n-1)}{2\times1}$

(v) ${}^nC_3 = {}^nC_{n-3} = \dfrac{n(n-1)(n-2)}{3\times2\times1}$

(vi) ${}^nC_4 = {}^nC_{n-4} = \dfrac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1}$

MISCELLANEOUS SOLVED EXAMPLES

1. Use mathematical induction to prove the following formula.

$$S_n : 1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sol. (1) When n = 1, the formula is valid, because $S_1 = 1^2 = \frac{1(2)(3)}{6}$

(2) Assuming that $S_k : 1^2+2^2+3^2+4^2+\ldots+k^2 = \frac{k(k+1)(2k+1)}{6}$

you must show that $S_{k+1} : \frac{(k+1)(k+2)(2k+3)}{6}$

To do this, write the following.

$S_{k+1} : S_k + a_{k+1} = (1^2+2^2+3^2+4^2+\ldots+k^2)+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6}+(k+1)^2 = \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$= \frac{(k+1)[k(2k+1)+6(k+1)]}{6} = \frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$

Combining the results of parts (1) and (2), you can conclude by mathematical induction that the formula is valid for all n $\geq$ 1.

2. Prove using mathematical Induction that $n(n+1)(n+2)$ is divisible by 6.

Sol. Let $P(n)$ be the statement "$n(n+1)(n+2)$ is divisible by 6", i.e. $P(n) = n(n+1)(n+2)$ is divisible by 6.

For $n = 1$, we have

$P(1) = 1(1+1)(1+2) = 1 \times 2 \times 3 = 6$, which is divisible by 6. Thus, $P(1)$ is true for $n = 1$.

For $n = k$, Let $P(k)$ be true,

i.e., $P(k) = k(k+1)(k+2)$ is divisible by 6.

Let, $k(k+1)(k+2) = 6\lambda$, for some $\lambda \in N$. .... (1)

For $n = k+1$, we have to show that $P(k+1)$ is true,

i.e. $P(k+1)$ is divisible by 6.

$P(k+1) = (k+1)(k+1+1)(k+1+2) = (k+1)(k+2)(k+3)$

$= k(k+1)(k+2)+3(k+1)(k+2)$

$= 6\lambda + 3(k+1)(k+2)$ [From (1)]

$= 6\lambda + 6t = 6(\lambda + t)$

Thus, $P(k+1)$ is divisible by 6.

Therefore, $P(k+1)$ is true.

$\therefore$ $P(k)$ is true $\Rightarrow$ $P(k+1)$ is true.

Hence, by principle of mathematical induction, $P(n)$ is true for all natural number n.

3. If $P(n)$ is a statement ($n \in N$) such that if $P(k)$ is true, $P(k+1)$ is true of $k \in N$, then $P(n)$ is true,

(a) for all n **(b)** for all $n > 1$ **(c)** for all $n > 2$ **(d)** nothing can be said

Sol. (d) Unless we prove $P(1)$ is true, nothing can be said.

4. Let $P(n) : n^2+n+1$ is an even integer. If $P(k)$ is assumed true $\Rightarrow P(k+1)$ is true. Therefore $P(n)$ is true.

(a) for $n>1$ (b) for all $n \in N$ (c) for $n>2$ (d) none of these

Sol. (d) $P(1)$ is not true (Principle of induction is not applicable). Also $n(n+1)+1$ is always an odd integer.

5. If $P(n): 2+4+6+...+(2n), n \in N$, then $P(k)=k(k+1)+2$ implies $P(k+1)=(k+1)(k+2)+2$ is true for all $k \in N$. So statement $P(n)=n(n+1)+2$ is true for:

(a) $n \geq 1$ (b) $n \geq 2$ (c) $n \geq 3$ (d) none of these

Sol. (d) $P(1)=2$ and $k(k+1)+2=4$, So $P(1)$ is not true. Mathematical Induction is not applicable.

6. Write the expression of $(2x-3)^6$.

Sol. $(2x-3)^6 = {}^6C_0(2x)^6 + {}^6C_1(2x)^5(-3) + {}^6C_2(2x)^4(-3)^2 + {}^6C_3(2x)^3(-3)^3 + {}^6C_4(2x)^2(-3)^4 + {}^6C_5(2x)(-3)^5 + {}^6C_6(2x)^0(-3)^6$

$$= 64x^6 + \frac{6}{1}(32x^5)(-3) + \frac{6\ .\ 5}{1.2}(16x^4)9 + \frac{6.5.4}{1.2.3}(8x^3)(-27) + \frac{6\ .5}{1.2}(4x^2)81 + \frac{6}{1}(2x)(-243) + 729$$

$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

7. Write the expression of $\left(x+\frac{1}{x}\right)^6$.

Sol. $\left(x+\frac{1}{x}\right)^6 = {}^6C_0x^6\left(\frac{1}{x}\right)^0 + {}^6C_1x^5\left(\frac{1}{x}\right) + {}^6C_2x^4\left(\frac{1}{x}\right)^2 + {}^6C_3x^3\left(\frac{1}{x}\right)^3 + {}^6C_4x^2\left(\frac{1}{x}\right)^4 + {}^6C_5x\left(\frac{1}{x}\right)^5 + {}^6C_6x^0\left(\frac{1}{x}\right)^6$

$$= x^6 + \frac{6}{1}x^4 + \frac{6.5}{1.2}x^3 + \frac{6.5.4}{1.2.3} + \frac{6.5}{1.2}\left(\frac{1}{x^2}\right) + \frac{6}{1}\left(\frac{1}{x^4}\right) + \left(\frac{1}{x^6}\right)$$

$$= x^6 + 6x^4 + 15x^3 + 20 + 15\frac{1}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

8. Using binomial theorem, find the value of $(102)^5$.

Sol. $(102)^5 = (100+2)^5 = 100^5 + {}^5C_1(100)^4 2 + {}^5C_2(100)^3 2^2 + {}^5C_3(100)^2 2^3 + {}^5C_4(100)2^4 + {}^5C_5(100)^0 2^5$

$$= 10000000000 + 5\times(100000000)\times 2 + \frac{5\ .\ 4}{1\ .\ 2}(1000000)\times 4 + \frac{5\ .\ 4}{1\ .\ 2}(10000)\times 8 + \frac{5}{1}(100)\times 16 + 32$$

$$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 = 11040808032$$

9. Find the middle terms in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$.

Sol. Number of terms in the expansion is $10+1=11$

Middle term of the expansion is $\frac{11+1}{2} = T_6$

$$T_{r+1} = C(10, r)\left(\frac{x}{3}\right)^{10-r}(9y)^r \qquad \ldots\text{(i)}$$

But $T_{r+1} = T_6$ or $r+1=6$, $r=5$

Putting $r = 5$ in (i), we have

$$T_6 = T_{5+1} = C(10, 5)\left(\frac{x}{3}\right)^{10-5}(9y)^5 = C(10, 5)\frac{x^5}{3^5}9^5 y^5 = C(10, 5)3^5 x^5 y^5 = \frac{10!}{5!(10-5)!}3^5 x^5 y^5 = \frac{10!}{5!\,5!}3^5 x^5 y^5$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}3^5 x^5 y^5 = 61236x^5y^5$$

10. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

Sol. Coefficient of x^2 in the expansion of $(1+x)^m$ is $C(m, 2)$

According to the question, $C(m,2)=6$

or $\frac{m(m-1)}{2!} = 6 \Rightarrow m^2 - m = 12$

or $m^2 - m - 12 = 0 \Rightarrow m^2 - 4m + 3m - 12 = 0$ or $(m-4)(m+3) = 0$

$\therefore$ $m = 4$ [since $m \neq -3$]

1 EXERCISE

FIB Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. In the binomial expansion of $(x-a)^n$, the general term T_{r+1} is given by ....
2. In the binomial expansion of $(x+a)^n$, the rth term from the end is .... th term from the beginning.
3. Total no. of terms in the binomial expansion of $(x + a)^n$ is ....
4. Middle term in the expansion of $\left(\frac{2}{3}x^2 - \frac{3}{2x}\right)^{20}$ is ....
5. In the expansion of $\left(x - \frac{1}{x}\right)^{12}$, the term independent of x is ....
6. The coefficients of terms equidistant from the beginning and the end are ....
7. In the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$, the value of constant term is ....
8. Middle term in the expansion of $(a^3 + ba)^{28}$ is ....
9. The ratio of the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ is ....

   [Hint: $^{p+q}C_p = {}^{p+q}C_q$]
10. If $P(n) : 2n < n!$, $n \in N$, then $P(n)$ is true for all $n \geq$ ....

T/F True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. In the binomial expansion of $(a+b)^n$, if n is odd then there will be two middle terms.
2. An algebric expression containing more than two terms are known as binomial expression.
3. Binomial expansion of $(2x - 3y)^9$ has 9 terms.
4. In the binomial expansion of $(a+b)^n$, total no. of terms are n.
5. In the expansion of $(x-y)^7$, 4th term from the end is 6th term from the beginning.
6. Coefficient of $(r+1)$th term in the expansion of $(1-x)^n$ is $(-1)^r\ {}^nC_r$
7. The number of terms in the expansion of $[2x + y^3)^4]^7$ is 8
8. Number of terms in the expansion of $(a+b)^n$ where $n \in N$ is one less than the power n.
9. Let $P(n)$ be a statement and let $P(k) \Rightarrow P(k+1)$, for some natural number k, then $P(n)$ is true for all $n \in N$.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s ....) in column II.*

1.

Column-I	Column-II
(A) $\left(1+5\sqrt{2}x\right)^9$ number of terms is	(p) ${}^{11}C_4\times2^7\times3^4$
(B) $\left(\frac{3}{x^2}-\frac{x^3}{6}\right)^7$ 4th term from the end is	(q) 10
(C) $\left(2x^2-\frac{3}{x}\right)^{11}$ coefficient of x^{10} is	(r) $\frac{35}{48}x^6$
(D) $\left(x-\frac{1}{x}\right)^{12}$ term independent of x is	(s) 5
	(t) ${}^{12}C_6$
	(u) ${}^{11}C_3\times2^7\times7^4$

Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Using Binomial theorem evaluate $(99)^5$
2. Using Binomial theorem indicate which number is larger $(1.10)^{10000}$ or 1000.?
3. Using binomial theorem, evaluate the value of $(102)^5$.
4. Prove that $\sum_{r=0}^{n}3^r\,{}^nC_r=4^n$
5. Find the 4^{th} term in the expansion of $(x-2y)^{12}$.
6. Find the 13^{th} term in the expansion of $\left(9x-\frac{1}{3\sqrt{3}}\right)^{18}$, $x\neq0$
7. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.
8. Find the number of terms in the expansions of the $\left(\sqrt{x}+\sqrt{y}\right)^{10}+\left(\sqrt{x}-\sqrt{y}\right)^{10}$
9. Using binomial theorem, expand $\{(x+y)^5+(x-y)^5\}$ and hence find the value of $\left\{\left(\sqrt{2}+1\right)^5+\left(\sqrt{2}-1\right)^5\right\}$.
10. Find the number of non-zero terms in the expansion of $(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$
11. Given positive integers $r>1$, $n>2$ and the coefficients of $(3r)^{th}$ and $(r+2)^{th}$ terms in the Binomial expansion of $(1+x)^{2n}$ are equal. Find the relation between n and r.
12. Write the general term in the expansion of $(x^2-y)^6$.
13. Find the coefficient of x^n in the expansion of $(1+x)(1-x)^n$.

Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find a if the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal.
2. Prove $1+3+5+....+(2n-1)=n^2$
3. Find the coefficient of x^{-12} in the expansion of $\left(x+\frac{y}{x^3}\right)^{20}$.
4. Find the middle term in the expansion of $\left(\frac{2}{3}x-\frac{3}{2}y\right)^{20}$.
5. Find the 7th term from the end in the expansion of $\left(x-\frac{2}{x^2}\right)^{10}$.
6. Find the coefficient of x^{-8} in the expansion of $\left(x-\frac{1}{2x^2}\right)^{10}$
7. If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then find n.

8. Find the term which has the greatest binomial coefficient in the expansion of $(x^2 + 2/x)^6$

9. Find the term independent of x in the expansion of $\left(\frac{4}{3}x^2 - \frac{3}{2x}\right)^9$.

10. Write the general term in the expansion of $(x^2 - yx)^{12}$, $x \neq 0$.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences*

1. Prove by principal of mathematical induction :

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

2. Prove by using the principle of mathematical induction for all $n \in N$

$$1 + 3 + 3^2 + ... + 3^{n-1} = \frac{(3^n - 1)}{2}$$

3. Find the middle term in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$.

4. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$.

5. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The term independent of x in the expansion of $\left(2x + \frac{1}{3x^2}\right)^9$ is

(a) 2^{nd} (b) 3^{rd}
(c) 4^{th} (d) 5^{th}

2. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$ then, $a_1 + a_3 + a_5 + ... + a_{39} = 2$

(a) $2^{20} - 1$ (b) $2^{20}(2^{20} - 1)$
(c) $2^{19}(2^{20} - 1)$ (d) 2^{19}

3. The term independent of x in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, is a times the corresponding binomial coefficient. Then a is

(a) 3 (b) 1/3
(c) –1/3 (d) None of these

4. The middle term in the expansion of $\left(1 + \frac{1}{x^2}\right)(1 + x^2)^n$ is

(a) ${}^{2n}C_n\, x^{2n}$ (b) ${}^{2n}C_n\, x^{-2n}$
(c) ${}^{2n}C_n$ (d) ${}^{2n}C_{n-1}$

5. In the expansion of $\left(\sqrt[3]{\frac{x}{3}} - \sqrt{\frac{3}{x}}\right)^{10}$, $x > 0$, the constant term is

(a) –70 (b) 70
(c) 210 (d) –210

6. If in the expansion of $\left(2^x + \frac{1}{4^x}\right)^n$, $T_3 = 7T_2$ and sum of the binomial coefficients of second and third terms is 36, then the value of x is –

(a) –1/3 (b) – 1/2
(c) 1/3 (d) 1/2

7. The greatest coefficient in the expansion of $(1+x)^{2n}$ is

(a) $\frac{1.3.5........(2n-1)}{n!}.2^n$
(b) ${}^{2n}C_{n-1}$
(c) ${}^{2n}C_{n+1}$
(d) None of these

8. If the r^{th} term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to

(a) 2 (b) 3
(c) 4 (d) 5

9. The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is

(a) 5/4 (b) 7/4
(c) 9/4 (d) none of these

10. The coefficient of x^5 in the expansion of $(x^2-x-2)^5$ is

(a) -83 (b) -82
(c) -81 (d) 0

11. If the sum of the coefficients in the expansion of $(\alpha^2x^2-2\alpha x+1)^{51}$ vanishes, then the value of α is

(a) 2 (b) -1
(c) 1 (d) -2

12. The coefficient of x^4 in the expansion $(1+5x+9x^2+..........\infty)(1+x^2)^{11}$ is–

(a) ${}^{11}C_2+4\,{}^{11}C_1+3$ (b) ${}^{11}C_2+3\,{}^{11}C_1+4$
(c) $3\,{}^{11}C_2+4\,{}^{11}C_1+3$ (d) 171

13. Let $S(k)=1+3+5...+(2k-1)=3+k^2$. Then which of the following is true

(a) Principle of mathematical induction can be used to prove the formula
(b) $S(k)\Rightarrow S(k+1)$
(c) $S(k)\not\Rightarrow S(k+1)$
(d) $S(1)$ is correct

14. If P (n) is statement such that $P(3)$ is true. Assuming $P(k)$ is true $\Rightarrow P(k+1)$ is true for all $k\geq 3$, then P(n) is true.

(a) for all n (b) for $n\geq 3$
(c) for $n\geq 4$ (d) none of these

MTOC More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is/are correct ?

(a) The number of terms in the expansion of $(x+a)^n$ is one more than the index n.
(b) The binomial coefficient of terms equidistant from the beginning and end are same.
(c) Binomial expansion of $(x+a)^n$ is a homogeneous expansion.
(d) The rth, i.e. t_r in the binomial expansion is called the general term.

2. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

(a) $\frac{405}{256}$ (b) $\frac{900}{526}$
(c) $\frac{450}{263}$ (d) $\frac{810}{512}$

3. The coefficient of a^5b^7 in the expansion of $(a-2b)^{12}$ is

(a) ${}^{12}C_7\cdot(-2)^7$ (b) ${}^{12}C_5\cdot(-2)^5$
(c) ${}^{12}C_5\cdot(-2)^7$ (d) ${}^{12}C_7\cdot(-2)^5$

4. Let $f(x)=\left(3-\frac{x^3}{6}\right)^7$. Then

(a) $f(x)$ has exactly two terms.
(b) Expansion of $f(x)$ contains two middle terms.
(c) Coefficient of x^{12} is $\frac{35}{48}$
(d) $f(x)$ is not a polynomial in x

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Passage-I

Let $f(x)=\left(x^4-\frac{1}{x^3}\right)^{15}$ be a polynomial function in x.

1. Number of terms in the expansion of $f(x)$ is

(a) 15 (b) 16
(c) 8 (d) 9

2. Coefficient of x^{32} in the expansion of $f(x)$ is

(a) ${}^{15}C_4$ (b) $-{}^{15}C_4$
(c) ${}^{15}C_5$ (d) ${}^{15}C_6$

3. Among which of the following given options is the middle term for the expansion of $f(x)$?

(a) t_8 only (b) t_9 only
(c) Both t_8 and t_9 (d) None of these

Passage-II

Let $f(x)=\left(3x-\frac{x^3}{6}\right)^7$ be a given polynomial function.

1. Number of middle terms in the expansion of $f(x)$ is.

(a) 2 (b) 1
(c) 3 (d) None of these

2. 4th term in the expansion of $f(x)$ is

(a) $\frac{-105}{4}x^{13}$ (b) $\frac{-105}{8}x^{13}$

(c) $\frac{105}{8}x^{13}$ (d) $\frac{105}{4}x^{13}$

3. No. of terms in the expansion of $f(x)$ is

(a) 7 (b) 6

(c) 8 (d) None of these

Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** Number of terms in the expansion of $(3x+y)^8-(3x-y)^8$ is 4.
Reason : In the expansion of $(ax+b)^n$, if n is even, then number of terms is $\frac{n}{2}$.

2. **Assertion :** 10th term in the binomial expansion of $\left(2x^2+\frac{1}{x}\right)^{12}$ is $\frac{1760}{x^3}$.
Reason : $T_{r+1}={}^nC_r\cdot x^{n-r}\cdot a^r$

3. **Assertion :** In the expansion of $\left(2x-\frac{1}{x^2}\right)^{25}$, the 11th term from the end is 15th term from the beginning.
Reason : In the expansion of $(ax+b)^n$, rth term from the end is $(n-r+2)$th term from the beginning.

4. **Assertion :** The term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ is 84.
Reason : In the expansion of $(ax+b)^n$, if the number of terms is even, then middle term is given by $\left(\frac{n}{2}+1\right)^{th}$ term.

5. **Assertion :** The coefficient of x^{40} in the expansion of $(1+2x+x^2)^{27}$ is ${}^{54}C_{40}$.
Reason : Coefficient of $(r+1)$th term in the binomial expansion of $(1+x)^n$ is ${}^nC_r$.

Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r, s ....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column-I	Column-II
(A) $\left(2x-\frac{1}{x^2}\right)^{25}$, 11th term from the end is	(p) ${}^5C_1$
(B) $\left(\frac{x}{a}-\frac{3a}{x^2}\right)^{12}$, 9th term is	(q) $-{}^{25}C_{15}\cdot\frac{2^{10}}{x^{20}}$
(C) $(2x-3y)^4$, No. of terms n is	(r) Solution of $2n-10=0$
(D) $\left(2x-\frac{1}{x}\right)^{10}$, term independent of x is	(s) $\left({}^{12}C_4x^{-12}a^4\right)3^8$
	(t) $-{}^{10}C_5\times2^5$
	(u) -8064

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Find numerically the greatest term in the expansion of $(2+3x)^9$, where $x=\frac{3}{2}$

2. Prove by principle of mathematical induction :

$$1.2+2.3+3.4+\ldots\ldots+n.(n+1)=\left[\frac{n(n+1)(n+2)}{3}\right]$$

3. Prove by principle of mathematical induction :

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$$

4. Find a, b and n in the expansion of $(a+b)^n$. If the first three terms of the expansion are 729, 7290 and 30375, respectively.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. $T_{r+1}=(-1)^r\ {}^nC_r x^{n-r}\cdot a^r$
2. $(n-r+2)$
3. $n+1$
4. ${}^{20}C_{10}\cdot x^{10}$
5. ${}^{12}C_6$
6. Equal
7. ${}^{16}C_8$
8. ${}^{28}C_{14}\ a^{56}\ b^{14}$
9. 1
10. 4

TRUE / FALSE

1. True 2. False 3. False 4. False
5. False 6. True 7. False 8. False
9. False

MATCH THE FOLLOWING :

1. (A) → q; (B) → r; (C) → p; (D) → t

(A) number of terms $=9+1=10$

(B) 4th term from the end $=(7-4+2)$th term from the beginning $=5$th term from the beginning.

$$t_5=t_{4+1}={}^7C_4\left(\frac{3}{x^2}\right)^{7-4}\left(\frac{-x^3}{6}\right)^4$$

$$={}^7C_4\left(\frac{3}{x^2}\right)^3\left(\frac{x^3}{6}\right)^4=\frac{35}{48}x^6$$

(C) $T_{r+1}={}^{11}C_r\left(2x^2\right)^{11-r}\left(\frac{-3}{x}\right)^r$

$$=(-1)^r\ {}^{11}C_r\cdot x^{(22-2r-r)}\cdot 2^{(11-r)}\cdot 3^r$$

$$=(-1)^r\cdot{}^{11}C_r\cdot x^{(22-3r)}\cdot 2^{(11-r)}\cdot 3^r$$

Now, $22-3r=10$

$r=4$

So, $(4+1)=5$th term contains x^{10}.

$t_5=t_{4+1}={}^{11}C_4 x^{10}\cdot 2^7\cdot 3^4$

$\therefore$ coefficient of $x^{10}={}^{11}C_4\cdot 2^7\cdot 3^4$

(D) $T_{r+1}={}^{12}C_r(x)^{12-r}\left(\frac{-1}{x}\right)^r=(-1)^r\cdot{}^{12}C_r\,(x)^{12-r-r}$

$$=(-1)^r\cdot{}^{12}C_r\,(x)^{12-2r}$$

For this term to be independent of x,

$12-2r=0\Rightarrow r=6$

So, $(6+1)$th, i.e. 7th term is independent of x.

Hence, $t_7=t_{6+1}={}^{12}C_6$

VERY SHORT ANSWER QUESTIONS :

1. $(99)^5=(100-1)^5$

$={}^5C_0(100)^5+{}^5C_1(100)^4(-1)+{}^5C_2(100)^3(-1)^2+{}^5C_3(100)^2(-1)^3+{}^5C_4(100)(-1)^4+(-1)^5$

$=10000000000-500000000+10000000-100000+500-1=9509900499$

2. $(1.1)^{10000}=[1+(0.1)]^{10000}$

Expanding by binomial theorem

$=C(10000,0)(1)^{10000}+C(10000,1)(1)^{10000-1}(0.1)$

$=1+10000\times 0.1+$ other terms

$=1001+$ other term

Hence, $(1.1)^{10000}>1000$.

3. $(102)^5=(100+2)^5=100^5+{}^5C_1(100)^4 2+{}^5C_2(100)^3 2^2+{}^5C_3(100)^2 2^3+{}^5C_4(100)2^4+{}^5C_5(100)^0 2^5$

$= 10000000000 + 5\times(100000000)\times 2 + \frac{5.4}{1.2}(1000000)\times$

$4 + \frac{5.4}{1.2}(10000)\times 8 + \frac{5}{1}(100)\times 16 + 32$

$= 10000000000 + 1000000000 + 40000000 +$

$800000 + 8000 + 32 = 11040808032$

4. $\sum_{r=0}^{n} 3^r\, {}^nC_r = {}^nC_0 + {}^nC_1.3^1 + {}^nC_2 3^2 + ...$

$+ {}^nC_r.3^r + ... + {}^nC_n.3^n$

$= (1+3)^n = 4^n$

5. 4^{th} term $= T_{3+1}$ in the expansion of $(x + (-2y))^{12}$

$= {}^{12}C_3 x^{12-3}[-2y]^3 = \frac{12.11.10}{1.2.3}x^9(-1)^3.2^3.y^3$

$= -220\times 8.x^9.y^3 = -1760\,x^9y^3.$

6. 13^{th} term, $T_{13} = T_{12+1} = {}^{18}C_{12}(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$= {}^{18}C_6 9^6.x^6(-1)^{12}.\frac{1}{3^{12}}\times\frac{1}{x^6} = 18564\times(3^2)^6.\frac{1}{3^{12}}\times\frac{x^6}{x^6}$

$= 18564\times\frac{3^{12}}{3^{12}} = 18564$

7. $(0.99)^5 = (1-0.01)^5$

$= 1 - {}^5C_1\times(0.01) + {}^5C_2\times(0.01)^2$ ....

$= 1 - 0.05 + 10\times 0.0001$............... $= 1.001 - 0.05 = 0.951$

8. If n is even, then the expansion of $\left\{(x+a)^n + (x-a)^n\right\}$ has $\left(\frac{n}{2}+1\right)$ terms.

So, $\left(\sqrt{x}+\sqrt{y}\right)^{10} + \left(\sqrt{x}-\sqrt{y}\right)^{10}$ has 6 terms.

9. We have,

$(x+y)^5 + (x-y)^5 = 2\left[{}^5C_0x^5 + {}^5C_2x^3y^2 + {}^5C_4x^1y^4\right]$

$= 2(x^5 + 10x^3y^2 + 5xy^4)$

Putting $x = \sqrt{2}$ and $y = 1$, we get

$\left(\sqrt{2}+1\right)^5 + \left(\sqrt{2}-1\right)^5 = 2\left[\left(\sqrt{2}\right)^5 + 10\left(\sqrt{2}\right)^3 + 5\sqrt{2}\right]$

$= 2\left[4\sqrt{2} + 20\sqrt{2} + 5\sqrt{2}\right] = 58\sqrt{2}$

10. Given expression

$= 2[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 +$

${}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8]x$

$\therefore$ the number of non-zero terms is 5

11. $n = 2r$

Coefficient of $(3r)^{th}$ term $=$ Coefficient of $(r+2)^{th}$ term

$\Rightarrow {}^{2n}C_{3r-1} = {}^{2n}C_{r+1} \Rightarrow (3r-1) + (r+1) = 2n \Rightarrow n = 2r$

12. We have, $\left(x^2-y\right)^6 = \left\{x^2 + (-y)\right\}^6$

The general term in the expansion of the above binomial is given by

$T_{r+1} = {}^6C_r\left(x^2\right)^{6-r}(-y)^r \qquad \left[\because T_{r+1} = {}^nC_r x^{n-r}a^r\right]$

$\Rightarrow T_{r+1} = (-1)^r\,{}^6C_r x^{12-2r}y^r$

13. We have,

Coefficient of x^n in $(1+x)(1-x)^n$

$=$ Coefficient of x^n in $(1-x)^n +$ coefficient of x^{n-1} in $(1-x)^n$

$= (-1)^n\,{}^nC_n + (-1)^{n-1}\,{}^nC_n - 1 = (-1)^n(1-n)$

SHORT ANSWER QUESTIONS :

1. $a = \frac{9}{7}$

2. Let the given statement $P(n)$ be defined as

$P(n): 1 + 3 + 5 + ... + (2n-1) = n^2$, for $n \in N$. Note that $P(1)$ is true, since

$P(1): 1 = 1^2$

Assume that $P(k)$ is true for some $k \in N$, i.e.,

$P(k): 1 + 3 + 5 + ... + (2k-1) = k^2$

Now, to prove that $P(k+1)$ is true, we have

$1 + 3 + 5 + ... + (2k-1) + (2k+1) = k^2 + (2k+1)$

$= k^2 + 2k + 1 = (2k+1)^2$

Thus, $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all $n \in N$.

3. Suppose x^{-12} occurs in $(r+1)$th term. We have

$T_{r+1} = {}^{20}C_r\,x^{20-r}\left(\frac{y}{x^3}\right)^r = {}^{20}C_r\,x^{20-4r}\,y^r$

This term contains x^{-12} if $20 - 4r = -12$ or $r = 8$.

$\therefore$ The coefficient of x^{-12} is ${}^{20}C_8\,y^8$.

4. The binomial expansion of $\left(\frac{2}{3}x - \frac{3}{2}y\right)^{20}$ consists of 21 terms. Therefore $\left(\frac{20}{2}+1\right)$th term, i.e., 11th term is the middle term.

Hence the middle term

$= T_{11} = {}^{20}C_{10}\left(\frac{2}{3}x\right)^{20-10}\left(-\frac{3}{2}y\right)^{10} = {}^{20}C_{10}\,x^{10}y^{10}.$

5. The 7th term from the end = 5th term from beginning

$$T_5 = {}^{10}C_4 x^6 \left(-\frac{2}{x^2}\right)^4 = {}^{10}C_4 \cdot 2^4\left(\frac{1}{x^2}\right)$$

6. The 7th term has x^{-8} and its coefficient is

$${}^{10}C_6(-1)^6\, 2^{-6} = \frac{105}{32}.$$

7. $n = 12$

8. We know that Binomial Coefficient of middle term is the greatest Binomial coefficient.
Since $n = 6$ is even, So the middle term is $T_{n/2+1}$
$\therefore$ middle term $= n/2 + 1 = 3 + 1 \Rightarrow 4^{th}$ term

9. 7^{th} term is independent of x.

10. Binomial expansion is $(x^2 - yx)^{12}$
General term $T_{r+1} = {}^{12}C_r(x^2)^{12-r}.(-yx)^r$

$$= \frac{12!}{r!(12-r)!}.x^{24-2r}.(-1)^r.y^r x^r = \frac{(-1)^r 12!}{r!(12-r)!}.x^{24-r}.y^r$$

LONG ANSWER QUESTIONS :

1. Let $P(n) = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + n^3 = \dfrac{n^2(n+1)^2}{4}$ ...(i)

For $n = 1$, L.H.S. $= 1^3 = 1$

And RHS $= \dfrac{1^2(1+1)^2}{4} = \dfrac{1\times 4}{4} = 1$

$\therefore$ LHS = RHS i.e $P(1)$ is true

Let us suppose that $P(k)$ is true

$\therefore$ putting $n = k$ in (i), we have

$$P(k): 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + k^3 = \frac{k^2(k+1)^2}{4} \quad \ldots\text{(ii)}$$

Adding $(k+1)^3$ on both sides,

$$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{(k+1)^2(k^2+2)^2}{4} = \frac{(k+1)^2(\overline{k+1}+1)^2}{4} \quad \ldots\text{(iii)}$$

$\therefore P(n)$ is true for $n = k + 1$ i.e., $p(k+1)$ is true

$\therefore$ By principle of Mathematical Induction. $P(n)$ is true for all natural numbers n.

2. Let $P(n)$ be the given statement

i.e. $P(n): 1 + 3 + 3^2 + \ldots\ldots\ldots + 3^{n-1} = \dfrac{(3^n - 1)}{2}$

Putting n = 1, $P(1) = \dfrac{3-1}{2} = 1$

$P(n)$ is true for $n = 1$

Assume that $P(k)$ is true

$$P(k): 1 + 3 + 3^2 + \ldots\ldots\ldots + 3^{k-1} = \frac{(3^k - 1)}{2}$$

we shall prove that P (k + 1) is true whenever P(k) is true.

Adding 3^k to both sides

$$1 + 3 + 3^2 + \ldots\ldots\ldots + 3^{k-1} + 3^k = \frac{(3^k - 1)}{2} + 3^k$$

$$= \frac{3^k - 1 + 2.3^k}{2} = \frac{(1+2)3^k - 1}{2} = \frac{3.3^k - 1}{2} = \frac{3^{k+1} - 1}{2}$$

$\therefore P(k+1)$ is also true whenever $P(k)$ is true.

Hence, $P(n)$ is true for all $n \in N$

3. Number of terms in the expansion is $7 + 1 = 8$
There are two middle terms which are T_4 and T_5.
Hence, we are to find T_4 and T_5 in the given expansion

$$\left(3 - \frac{x^3}{6}\right)^7 = \left[3 + \left(-\frac{x^3}{6}\right)\right]^7$$

$$T_{r+1} = C(7, r)3^{7-r}\left(-\frac{x^3}{6}\right)^r \quad \ldots\text{(i)}$$

Now $T_{r+1} = T_4$ or $r + 1 = 4$ $\therefore r = 3$

Putting r = 3, we have

$$T_{3+1} = C(7, 3)3^{7-3}\left(-\frac{x^3}{6}\right)^3 = C(7, 3)3^4(-1)^3\frac{x^9}{6^3}$$

$$= \frac{-7!}{3!\,4!}\frac{3}{2^3}x^9$$

$$= \frac{-7\times6\times5\times4!}{3\times2\times1\times4!}.\frac{3}{2^3}.x^9 = \frac{-105}{8}x^9$$

Again $T_{r+1} = T_5$ or $r + 1 = 5$ or $r = 4$

Putting $r = 4$ in (i), we have

$$T_{4+1} = T_5 = C(7, 4)\,3^{7-4}(-1)^4\frac{x^{12}}{6^4}$$

$$= \frac{7!}{4!\,3!}\frac{3^3 x^{12}}{3^4\, 2^4} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!}\times\frac{x^{12}}{3\times2^4} = \frac{35}{48}x^{12}$$

4. General term in the expansion of $(1+x)^{2n}$

$T_{r+1} = C(2n, r)x^r$

Putting $r = n$, we have

$T_{n+1} = C(2n, n)\,x^n$

Coefficient of $x^n = C(2n, n)$

Again general term in the expansion of

$(1+x)^{2n-1}$ is $T_{r+1} = C(2n-1, r)\,x^r$

Putting $r = n$, we have

$T_{n+1} = C(2n-1, n)\,x^n$

Coefficient of x^n in the expansion of

$(1+x)^{2n-1}$ is $C(2n-1, n)$

According to the problem, we have to proce that

$C(2n, n) = 2\times C(2n-1, n)$

$$\frac{2n!}{n!n!} = 2.\frac{(2n-1)!}{n!(n-1)!}$$

Multiplying N^r and D^r by n on RHS, we have

$$= \frac{2n(2n-1)!}{n!n(n-1)!}$$

i.e $\frac{2n!}{n!n!} = \frac{2n!}{n!n!}$, Which is true. Hence proved.

5. T_5 in $\left(2^{1/4}+\frac{1}{3^{1/4}}\right)^n = {}^nC_4.(2^{1/4})^{n-4}.\left(\frac{1}{3^{1/4}}\right)^4$

$$= {}^nC_4.2^{(n-4)/4}.\frac{1}{3} \quad \ldots(i)$$

Total number of terms $= n+1$

Fifth term from the end

$= [(n+1)-5+1]^{th}$ term from the beginning

$= (n-3)^{th}$ term $= {}^nC_{n-4}.(2^{1/4})^{n-(n-4)}.\left(\frac{1}{3^{1/4}}\right)^{n-4}$

$$= {}^nC_{n-4}.2.\left(\frac{1}{3}\right)^{n-4/4} = {}^nC_n.2.\left(\frac{1}{3}\right)^{n-4/4} \ldots(ii)$$

Dividing (i) by (ii)

$$\frac{{}^nC_4 2^{(n-4)/4}.\frac{1}{3}}{{}^nC_4.2\left(\frac{1}{3}\right)^{(n-4)/4}} = \frac{\sqrt{6}}{1} \text{ or } \frac{2^{\frac{n}{4}-2}}{\left(\frac{1}{3}\right)^{\frac{n}{4}-2}} = \frac{\sqrt{6}}{1}$$

or $2^{\frac{n}{4}-2}.3^{\frac{n}{4}-2} = 6^{\frac{1}{2}}$ or $6^{\frac{n}{4}-2} = 6^{\frac{1}{2}}$

$\Rightarrow \frac{n}{4}-2 = \frac{1}{2}$ or $\frac{n}{4} = \frac{5}{2}$ or $n = \frac{5}{2}\times 4 = 10$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (c) Suppose $(r+1)$th term is independent of x. We have

$$T_{r+1} = {}^9C_r\,(2x)^{9-r}\left(\frac{1}{3x^2}\right)^r = {}^9C_r 2^{9-r}\,\frac{1}{3^r}\,.\,x^{9-3r}$$

This term is independent of x if $9-3r=0$ i.e., $r=3$.
Thus, 4th term is independent of x.

2. (c)

3. (d) $T_{r+1} = {}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

$$= (-r)^r\,{}^{18}C_r\,9^{18-\frac{3r}{2}}\,x^{18-\frac{3r}{2}}$$

is independent of x provided $r=12$ and then $\alpha = 1$.

4. (c)

5. (c) The constant term

$$= {}^{10}C_6\left(\sqrt[3]{\frac{x}{3}}\right)^6\left(-\sqrt{\frac{3}{x}}\right)^4 = {}^{10}C_4\,\frac{1}{3^2}\,.\,3^2 = 210$$

6. (a)

7. (a) The greatest coefficient = the coefficient of the middle term

$$= {}^{2n}C_n = \frac{1.3.5.........(2n-1)}{n!}.2^n$$

8. (a)

9. (a) The $(r+1)$th term in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}$$

$$\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r\,\frac{x^{5-(r/2)}}{3^{5-(r/2)}}.\frac{3^r}{2^r x^{2r}}$$

$$= {}^{10}C_r\,\frac{3^{(3r/2)-5}}{2^r}x^{5-(5r/2)}.$$

For T_{r+1} to be independent of x, we must have $5-(5r/2)=0$ or $r=2$.

Thus, the 3rd term is independent of x and is equal to

$${}^{10}C_2\,\frac{3^{3-5}}{2^2} = \frac{10\times 9}{2}\times\frac{3^{-2}}{4} = \frac{5}{4}$$

10. (c) 11. (c)

12. (d) Coefficient of $x^4 = {}^{11}C_2 + 9\times 11 + 17 = 171$

13. (b) $S(k) = 1+3+5+...+(2k-1) = 3+k^2$

$S(1): 1 = 3+1$, which is not true

$\because S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(k)$ is true, i.e.

$1+3+5....+(2k-1) = 3+k^2$

$\Rightarrow 1+3+5....+(2k-1)+2k+1$

$= 3+k^2+2k+1 = 3+(k+1)^2$

$\therefore S(k) \Rightarrow S(k+1)$

14. (b) $P(3)$ is true

Assume $P(k)$ is true $\Rightarrow P(k+1)$ is true means if P (3) is true $\Rightarrow P(4)$ is true $\Rightarrow P(5)$ is true and so on. So statement is true for all $n \geq 3$.

MORE THAN ONE CORRECT :

1. (a, b, c)
2. (a, d)

$$\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10} = {}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(\frac{-3}{x^2}\right)^r$$

$$= {}^{10}C_r x^{(10-r-2r)}\frac{(-1)^r\cdot 3^r}{2^{10-r}} = {}^{10}C_r x^{10-3r}\frac{(-1)^r\cdot 3^r}{2^{10-r}}$$

For coefficient of x^4, $10-3r=4$
$r=2$

$$\therefore \text{ Coefficient of } x^4 = {}^{10}C_2\frac{(-1)^2\cdot 3^2}{2^8} = \frac{405}{256}$$

$$= \frac{405\times 2}{256\times 2} = \frac{810}{512}$$

3. (a, c)

$$[a+(-2b)]^{12} = {}^{12}C_r a^{12-r}(-2b)^r$$

Putting $12-r=5$
$r=7$

$$T_8 = T_{7+1} = {}^{12}C_7\cdot a^5(-2b)^7 = {}^{12}C_7\cdot(-2)^7 a^5b^7$$

Hence, coefficient of $a^5b^7 = {}^{12}C_7\cdot(-2)^7$

$$= {}^{12}C_{12-7}\cdot(-2)^7 = {}^{12}C_5\cdot(-2)^7$$

4. (a, b, c)

PASSAGE BASED QUESTIONS :

Passage-1

1. (b)
2. (a) $T_{r+1} = (-1)^r\cdot{}^{15}C_r\left(x^4\right)^{15-r}\cdot\left(\frac{1}{x^3}\right)^r$

$$= (-1)^r\cdot{}^{15}C_r x^{60-7r}$$

Putting, $60-7r=32$
$r=4$

So, $(4+1)$th, i.e., 5th term contains x^{32}.

$$t_5 = t_{4+1} = {}^{15}C_4\cdot(-1)^4\cdot x^{60-28} = {}^{15}C_4\cdot x^{32}$$

Hence, coefficient of x^{32} is ${}^{15}C_4$.

3. (c) Total no. of terms = 16 i.e. odd

Middle term $= \left(\frac{15+1}{2}\right)^{th}$ term and $\left(\frac{15+3}{2}\right)^{th}$ term

i.e, 8th and 9th term

Passage-2

1. (a)
2. (b) $T_4 = T_{3+1} = {}^7C_3(3x)^{7-3}\left(-\frac{x^3}{6}\right)^3$

$$= (-1)^3\,{}^7C_3(3x)^4\left(\frac{x^3}{6}\right)^3$$

$$T_4 = -35\times 81x^4\times\frac{x^9}{216} = -\frac{105x^{13}}{8}$$

3. (c)

ASSERTION & REASON :

1. (a)
2. (a)
3. (d) 11th term from the end is $(25-11+2)=16$th term from the beginning.
4. (b) Assertion and reason both are correct, but reason is not the correct explanation of the given assertion.
5. (a)

MULTIPLE MATCHING QUESTIONS :

1. (A)→q; (B)→s; (C)→p,r; (D)→t, u;

(A) Clearly, the given expansion contains 26 terms.
So, 11th term from the end = (26 – 11 + 1)th term from the beginning i.e. 16th term from the beginning

$$\therefore \text{ Required term} = T_{16} = T_{15+1} = {}^{25}C_{15}(2x)^{25-15}\left(-\frac{1}{x^2}\right)^{15}$$

$$= {}^{25}C_{15}\cdot 2^{10}\cdot x^{10}\frac{(-1)^{15}}{x^{30}}$$

$$= -\,{}^{25}C_{15}\cdot\frac{2^{10}}{x^{20}}$$

(B) We know that the $(r+1)$th term in the expansion of $(x+a)^n$ is given by $T_{r+1} = {}^nC_r x^{n-r}a^r$

In the expansion of $\left(\frac{x}{a}-\frac{3a}{x^2}\right)^{12}$, we have

$$\Rightarrow T_9 = T_{8+1} = {}^{12}C_8\left(\frac{x}{a}\right)^{12-8}\left(\frac{-3a}{x^2}\right)^8 = {}^{12}C_8\left(\frac{x}{a}\right)^4\left(\frac{-3a}{x^2}\right)^8$$

$$\Rightarrow T_9 = {}^{12}C_4\cdot 3^8\frac{a^4}{x^{12}} = \left({}^{12}C_4 x^{-12}a^4\right)3^8$$

(C) No. of terms = 4 + 1 = 5

$${}^5C_1 = \frac{5!}{4!} = 5 \Rightarrow 2n-10=0 \Rightarrow n=5$$

(D) Let $(r+1)$th term be independent of x in the given expression.

Now, $T_{r+1} = {}^{10}C_r\,(2x)^{10-r}\left(-\dfrac{1}{x}\right)^r$

$= (-1)^r\,{}^{10}C_r\,2^{10-r}x^{10-2r}$ ...(i)

For this term to be independent of x, we must have

$10-2r=0 \Rightarrow r=5$

So, $(5+1)$th i.e. 6th term is independent of x.

Putting $r=5$ in (i), we get

$$T_6 = (-1)^5\,{}^{10}C_5\cdot 2^{10-5} = -{}^{10}C_5\times 2^5$$

$$= -\frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}\times 32 = -8064$$

Hence, required term $= -8064$

HOTS SUBJECTIVE QUESTIONS :

1. We have $(2+3x)^9 = 2^9\left(1+\dfrac{3x}{2}\right)^9$

Now, $$\frac{T_{r+1}}{T_r} = \frac{2^9\left[{}^9C_r\left(\frac{3x}{2}\right)^r\right]}{2^9\left[{}^9C_{r-1}\left(\frac{3x}{2}\right)^{r-1}\right]}$$

$$= \frac{10-r}{r}\left|\frac{3x}{2}\right| = \frac{10-r}{r}\left(\frac{9}{4}\right) \text{ Since } x=\frac{3}{2}$$

Therefore, $\dfrac{T_{r+1}}{T_r}\ge 1 \Rightarrow \dfrac{90-9r}{4r}\ge 1$

$\Rightarrow 90-9r\ge 4r \Rightarrow r\le\dfrac{90}{13} \Rightarrow r\le 6\dfrac{12}{13}$

Thus the maximum value of r is 6. Therefore, the greatest term is $T_{r+1}=T_7$.

Hence, $T_7 = 2^9\left[{}^9C_6\left(\dfrac{3x}{2}\right)^6\right]$, where $x=\dfrac{3}{2}$

$$= 2^9\cdot{}^9C_6\left(\frac{9}{4}\right)^6 = 2^9\cdot\frac{9\times8\times7}{3\times2\times1}\left(\frac{3^{12}}{2^{12}}\right) = \frac{7\times3^{13}}{2}$$

2. Let $P(n)$: $1.2+2.3+3.4+\ldots\ldots\ldots\ldots+n.(n+1)$

$= \left[\dfrac{n(n+1)(n+2)}{3}\right]$

for $n=1$, LHS $= 1.2 = 2$

RHS $= \left[\dfrac{1(n+1)(n+2)}{3}\right] = \dfrac{1.2.3}{3} = 1.2 = 2$

We assume that $P(n)$ is true for $n=k$

i.e $1.2+2.3+3.4+\ldots\ldots\ldots\ldots+k(k+1) = \left[\dfrac{k(k+1)(k+2)}{3}\right]$

last term is $k(k+1)$

Adding $(k+1)(k+2)$ on both sides,

LHS $= 1.2+2.3+3.4+\ldots\ldots\ldots\ldots+k(k+1)+(k+1)(k+2)$

RHS $= \dfrac{k(k+1)(k+2)}{3}+(k+1)(k+2)$

$= \dfrac{(k+1)(\overline{k+1}+1)(\overline{k+1}+2)}{3}$

$\therefore$ $1.2+2.3+3.4+\ldots\ldots\ldots\ldots+k$

$$(k+1)+(k+1)(k+2) = \frac{(k+1)(\overline{k+1}+1)(\overline{k+1}+2)}{3}$$

Thus, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by Principle of mathematical induction

$P(n)$ is true for all values of $n\in N$

3. Let $P(n)$: $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\ldots\ldots+\dfrac{1}{2^n} = 1-\dfrac{1}{2^n}$

Putting $n=1$, LHS $=\dfrac{1}{2}$, RHS $= 1-\dfrac{1}{2}=\dfrac{1}{2}$ i.e.

LHS $=$ RHS $=\dfrac{1}{2}$

$\therefore$ $P(n)$ is true for $n=1$

Suppose $P(n)$ is true for $n=k$

$\therefore$ $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\ldots\ldots+\dfrac{1}{2^k} = 1-\dfrac{1}{2^k}$

last term $=\dfrac{1}{2^k}$; Replacing k by $k+1$, last term $=\dfrac{1}{2^{k+1}}$

Adding $\dfrac{1}{2^{k+1}}$ to both sides,

LHS $=\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\ldots\ldots+\dfrac{1}{2^k}+\dfrac{1}{2^{k+1}}$

RHS $= 1-\dfrac{1}{2^k}+\dfrac{1}{2^{k+1}} = 1-\dfrac{1}{2^k}\left(1-\dfrac{1}{2}\right)$

$= 1-\dfrac{1}{2^k}\cdot\dfrac{1}{2} = 1-\dfrac{1}{2^{k+1}}$

This shows $P(n)$ is true for $n = k+1$

Thus $P(k+1)$ is true whenever $P(k)$ is true

Hence, $P(n)$ is true for all $n \in N$

4. T_1 of $(a+b)^n = a^n = 729$ ...(i)

T_2 of $(a+b)^n = {}^nC_1 a^{n-1} b = 7290$ ...(ii)

T_3 of $(a+b)^n = {}^nC_2 a^{n-2} b^2 = 30375$ ...(iii)

Dividing (i) by (ii),

$$\frac{a^n}{{}^nC_1 a^{n-1} b} = \frac{729}{7290} = \frac{1}{10} \text{ or } \frac{a}{nb} = \frac{1}{10} \quad \text{...(iv)}$$

Dividing (ii) by (iii)

$$\frac{{}^nC_1 a^{n-1} b}{{}^nC_2 a^{n-2} b^2} = \frac{7290}{30375} \text{ or}$$

$$\frac{na^{n-1}b}{\frac{n(n-1)}{2} a^{n-2} b^2} = \frac{7290}{30375} = \frac{6}{25} \text{ or } \frac{2}{n-1} \times \frac{a}{b} = \frac{6}{25} \quad \text{...(v)}$$

Dividing (iv) by (v)

$$\frac{a}{nb} \times \frac{(n-1)b}{2a} = \frac{1}{10} \times \frac{25}{6} = \frac{5}{12} \text{ or } \frac{n-1}{2n} = \frac{5}{12}$$

or $12n - 12 = 10n$ or $2n = 12$ or $n = 6$

Also, putting $n = 6$ in (i) $a^6 = 729$ $\therefore$ $a = 3$

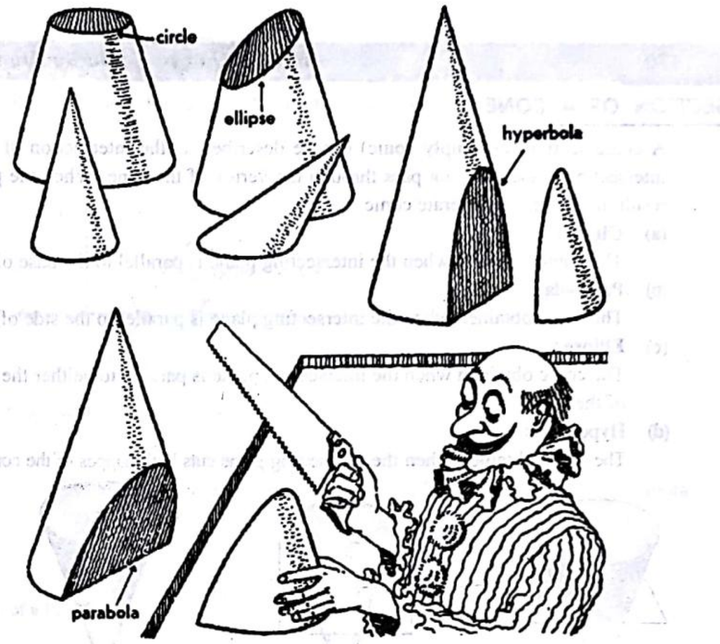
Putting $n = 6, a = 3$ in eqn (iv)

$$\frac{3}{6b} = \frac{1}{10} \therefore b = \frac{3 \times 10}{6} = 5$$

Thus $a = 3, b = 5, n = 6$

chapter

18



Conic sections have different shapes and different names.

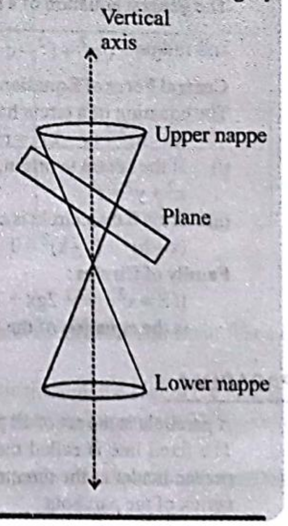
INTRODUCTION TO CONIC SECTIONS

Introduction

Conic sections are among the oldest curves, and is a oldest mathematics subject studied systematically and thoroughly. The conics seems to have been discovered by Menaechmus, tutor to Alexander the Great.

Conic sections are important in astronomy the orbits of two massive objects that interact according to Newton's law of universal gravitation are conic sections.

Conic sections are the curves obtained by intersecting a double napped right circular hollow cone by a plane. We obtain different kinds of conic sections depending on the position of the intersecting plane with respect to the cone and the angle made by it with the vertical axis of the cone.



SECTION OF A CONE :

A conic section (or simply conic) can be described as the intersection of a plane and a double-napped cone. When the intersecting plane does not pass through the vertex of the cone. When the plane does pass through the vertex, we call the resulting figure a degenerate conic.

(a) Circle :
The conic obtained when the intersecting plane is parallel to the base of the cone.

(b) Parabola :
The conic obtained when the intersecting plane is parallel to the side of the cone.

(c) Ellipse :
The conic obtained when the intersecting plane is parallel to neither the base nor the side and intersects only one nappe of the cone.

(d) Hyperbola :
The conic obtained when the intersecting plane cuts both nappes of the cone.

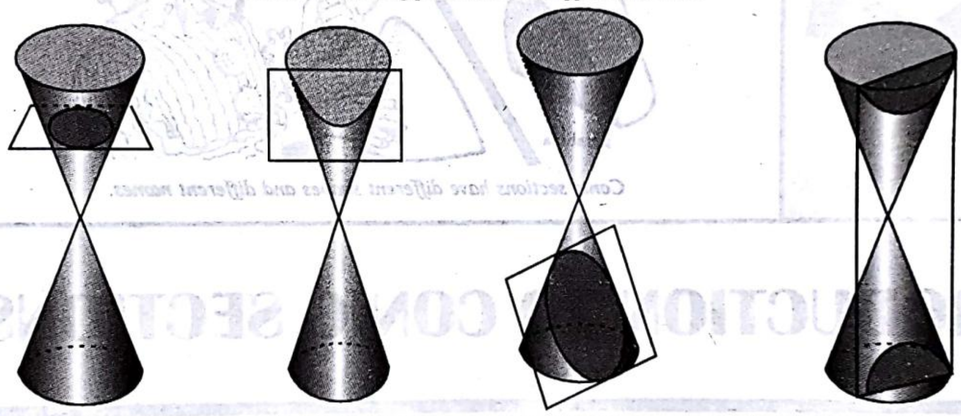


Fig. (a) : Circle **Fig (b) : Parabola** **Fig. (c) : Ellipse** **Fig. (d) : Hyperbola**

CIRCLE :

A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

General Equation of a Circle :
The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$, where g, f, c are constants. Its centre = $(-g, -f)$ and radius = $\sqrt{g^2+f^2-c}$ units

Central Form of Equation of a Circle:
The equation of a circle having centre (h, k) and radius r is
$(x-h)^2+(y-k)^2=r^2$. Its centre = (h, k) and radius = r units

(i) If the centre is origin, then the equation of the circle is
$x^2+y^2=r^2$

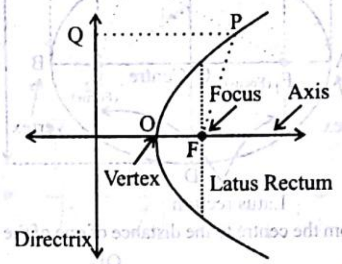
(ii) If r = 0 than circle is called point circle and its equation is
$(x-h)^2+(y-k)^2=0$

Family of Circles :
If $S \equiv x^2+y^2+2gx+2fy+c=0$ and $S' \equiv x^2+y^2+2g'x+2f'y+c'=0$ are two intersecting circles, then $S+\lambda S'=0, \lambda \neq -1$, is the equation of the family of circles passing through the points of intersection of $S=0$ and $S'=0$.

PARABOLA :

A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane. The fixed line is called the directrix of the parabola and the fixed point F is called the focus. A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola with its axis is called the vertex of the parabola.

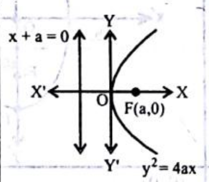
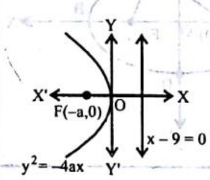
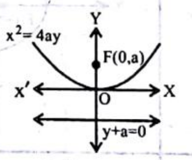
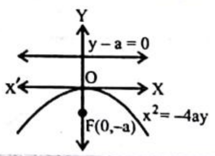
A line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola is called the latus rectum of the parabola.



Ratio of the distances of any point on the parabole from the focus and the directric is called Ecentricity of the parabola.

Ecentricity $(e) = \frac{PF}{PQ} = 1$ $[\because PQ = PF]$

Main Facts About The Standard Forms of a Parabola :

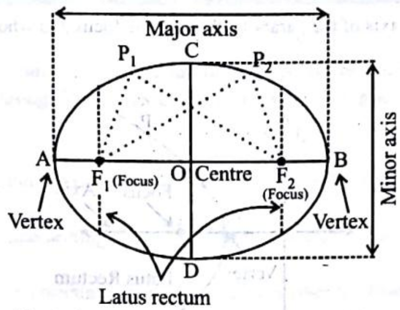
Equation	$y^2 = 4ax$ $(a>0)$	$y^2 = -4ax$ $(a>0)$	$x^2 = 4ay$ $(a>0)$	$x^2 = -4ay$ $(a>0)$
Graph				
Vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Focus(F)	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Equation of Directrix	$x+a=0$	$x-a=0$	$y+a=0$	$y-a=0$
Equation of Axis	$y=0$	$y=0$	$x=0$	$x=0$
Length of latus rectum	4a	4a	4a	4a

ELLIPSE :

An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. The two fixed points are called the foci (plural of focus) of the ellipse.

The mid point of the line segment joining the foci is called the centre of the ellipse. The line through the foci of the ellipse is called the major axis and the line through the centre and perpendicular to the major axis is called the minor axis. The points of intersection of the major axis and the ellipse are called the vertices of the ellipse.

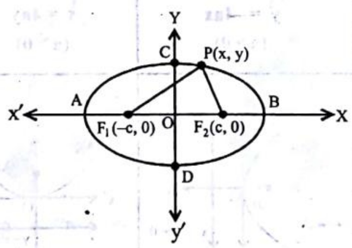
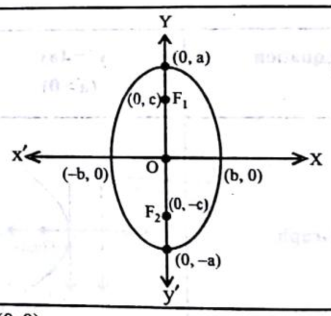
A line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse is called the latus rectum.



The ratio of distance of one of the foci from the centre to the distance of one of the vertices from the centre of the ellipse is called the eccentricity (e) of the ellipse. Ecentricity(e) of the ellipse is $= \frac{OF_2}{OB} < 1$

$P_1F_1 + P_1F_2 = P_2F_1 + P_2F_2$ [By definition of ellipse]

Main Facts About The Standard Forms of an Ellipse :

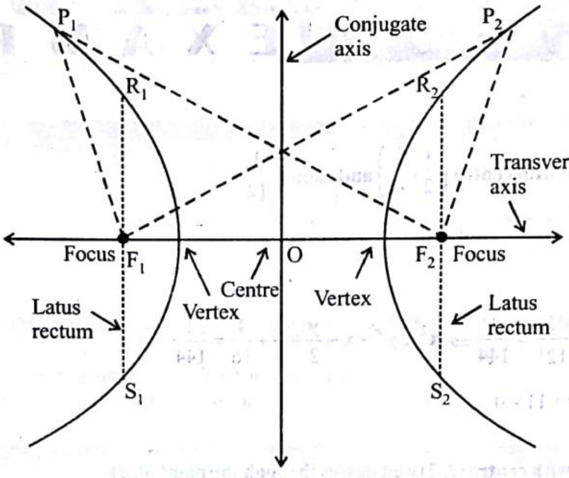
Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a^2>b^2$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a^2>b^2$
Graph		
Centre	$(0,0)$	$(0,0)$
Equation of major axis	$y=0$	$x=0$
Equation of minor axis	$x=0$	$y=0$
Length of minor axis	$2b$	$2b$
Length of major axis	$2a$	$2a$
Foci	$(\pm ae, 0)$ or $(\pm c, 0)$ where, $c^2=a^2-b^2$	$(0, \pm ae)$ or $(0, \pm c)$ where, $c^2=a^2-b^2$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

For each of the standard form of the Ellipse, $b^2 = a^2 (i - e^2)$

HYPERBOLA :

A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant. The two fixed points are called the foci of the hyperbola. The mid-point of the line segment joining the foci is called the centre of the hyperbola. The line through the foci is called the transverse axis and the line through the centre and perpendicular to the transverse axis is called the conjugate axis. The point at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.

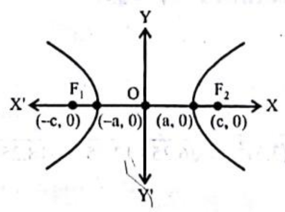
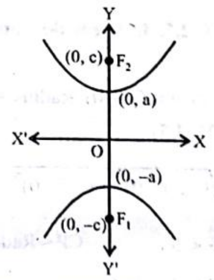
A line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola is called the latus rectum of the hyperbola.



$P_1F_2 - P_1F_1 = P_2F_1 - P_2F_2$ [By definition of Hyperbola]

The ratio of the distance of one of the foci from the centre to the distance of one of the vertices from the centre of the ellipse is called the eccentricity (e) of the hyperbole. Ecentriciry(e) of the Hyperbola is greater than 1.

Main Facts about The Standard Forms of a Hyperbola:

Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$	$\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$
Graph		
Centre	(0,0)	(0,0)
Eqn. of transverse axis	y=0	x=0
Eqn. of conjugate axis	x=0	y=0
Length of transverse axis	2a	2a
Length of conjugate axis	2b	2b
Foci	$(\pm ae, 0)$ or $(\pm c, 0)$ where $c^2=a^2+b^2$	$(\pm O, ae)$ or $(0, \pm c)$ where, $c^2=a^2+b^2$
Vertices	$(\pm a, 0)$	$(0, \pm a)$
Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

For each of the two standard form of Hyperbola, $b^2 = a^2(e^2 - 1)$

MISCELLANEOUS SOLVED EXAMPLES

1. Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Sol. Equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\frac{1}{12^2}=\frac{1}{144} \Rightarrow x^2+y^2-x-\frac{y}{2}+\frac{1}{4}+\frac{1}{16}=\frac{1}{144}$$

$\Rightarrow\ 36x^2+36y^2-36x-18y+11=0$

2. Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Sol. Let the centre of the circle C(2, 2), and P(4, 5) is a point on the circle

$\therefore$ radius, CP $=\sqrt{(4-2)^2+(5-2)^2}=\sqrt{4+9}=\sqrt{13}$

$\therefore$ Equation of the circle is $(x-2)^2+(y-2)^2=13 \Rightarrow x^2+y^2-4x-4y=5$

3. Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2+y^2=25$?

Sol. Centre of the circle is C(0, 0), Radius = 5

The point P(–25, 3.5)

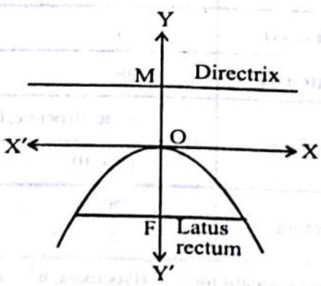
$$(CP)=\sqrt{(-2.5-0)^2+(3.5-0)^2}=\sqrt{(2.5)^2+(3.5)^2}=\sqrt{6.25+12.25}=\sqrt{18.25}$$

CP $=\sqrt{18.25}<5$, $\therefore$ CP < Radius

$\therefore$ P lies inside the circle.

4. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum for given curve $x^2=-9y$.

Sol. Equation of the parabola is $x^2=-9y$, $4a=9$, $a=\frac{9}{4}$



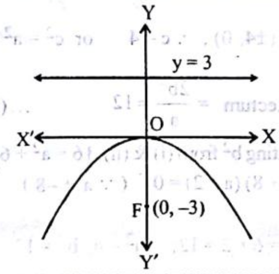
Focus, $\left(0, -\frac{9}{4}\right)$

Axis of parabola is y-axis i.e $x=0$, Directrix is $y=\frac{9}{4}$

and length of latus rectum = 9

5. Find the equation of parabola that satisfies the given focus (0, –3) and directrix y = 3.

Sol. Focus (0, –3) and directrix y = 3
Vertex is the mid point of (0, –3), (0, 3)
i.e. vertex is (0, 0)
and a = 3, $\therefore$ 4a = 12
$\therefore$ Equation of parabola $x^2 = -12y$



6. Find the equation of the parabola whose vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis.

Sol. Vertex (0, 0), parabola passes through (5, 2) symmetric about Y-axis
Let the equation of parabola be $x^2 = 4ay$. (5, 2) lies on it

$\therefore \quad 25 = 4a.\,2 \therefore a = \dfrac{25}{8}$

$\therefore \quad$ Equation of parabola is $x^2 = \dfrac{25}{2}y$ or $2x^2 = 25y$

7. Find the equation of the parabola with focus (2, 0) and directrix x = –2

Sol. Since, the focus (2, 0) lies on the x-axis, the X-axis itself is the axis of the parabola. Hence, the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$. Since, the directrix is x = –2 and the focus is (2, 0), the parabola is to be of the form $y^2 = 4ax$ with a = 2. Hence the required equation is

$$y^2 = 4(2)x = 8x$$

8. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).

Sol. Since the vertex is at (0, 0) and the focus is at (0, 2) which lies on y-axis, hence y-axis is the axis of the parabola. Therefore, equation of the parabola is the form $x^2 = 4ay$. Thus, we have

$$x^2 = 4(2)y, \text{ i.e., } x^2 = 8y.$$

9. Find the equation of the ellipse whose centre is at (0, 0), major axis on the Y-axis and passes through the points (3, 2) and (1, 6).

Sol. Major axis is y-axis.

let the ellipse be $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$.

Since (3, 2) and (1, 6) lies on the ellipse.

$\therefore \quad \dfrac{9}{b^2} + \dfrac{4}{a^2} = 1 \quad \ldots \text{(i)} ; \quad \dfrac{1}{b^2} + \dfrac{36}{a^2} = 1 \quad \ldots \text{(ii)}$

Subtracting eq. (ii) from (i), we get

$\dfrac{9-1}{b^2} + \dfrac{4-36}{a^2} = 0 \Rightarrow \dfrac{8}{b^2} - \dfrac{32}{a^2} = 0 \Rightarrow 8a^2 = 32b^2, \therefore a^2 = 4b^2$

Putting the value, it become $\dfrac{9}{b^2} + \dfrac{4}{4b^2} = 1, \qquad \therefore \dfrac{10}{b^2} = 1$

$\therefore \quad b^2 = 10$
Now, $a^2 = 4b^2 = 4 \times 10 = 4$

$\therefore \quad$ Equation of the ellipse is $\dfrac{x^2}{10} + \dfrac{y^2}{40} = 4$

10. Find the equation of ellipse with foci $(\pm4, 0)$, the latus rectum is of length 12.

Sol. Foci = $(\pm4, 0)$, $\therefore c = 4$ or $c^2 = a^2 + b^2$ $\therefore 16 = a^2 + b^2$ ...(i)

Latus Rectum $= \dfrac{2b^2}{a} = 12$ ...(ii)

Eliminating b^2 from (i) & (ii), $16 = a^2 + 6a$ or $a^2 + 6a - 16 = 0$

or $(a+8)(a-2) = 0$ $(\because a \neq -8)$ $\therefore a = 2$

$b^2 = 6a = 6 \times 2 = 12$; $a^2 = 4$, $b^2 = 12$, $\dfrac{x^2}{4} + \dfrac{y^2}{12} = 1$.

11. Find the equations of the hyperbola satisfying the given condition. Foci $(0, \pm\sqrt{10})$, passing through $(2, 3)$

Sol. Let the equation of hyperbola be $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$ ...(i)

$\therefore$ $ae = \sqrt{10}$ Also, $b^2 = a^2(e^2 - 1) = a^2e^2 - a^2 = 10 - a^2$ ...(ii)

Thus, the equation of the hyperbola is $\dfrac{y^2}{a^2} - \dfrac{x^2}{10 - a^2} = 1$

As, it passes through the point (2, 3)

$\therefore$ $\dfrac{9}{a^2} - \dfrac{4}{10 - a^2} = 1 \Rightarrow a^4 - 23a^2 + 90 = 0 \Rightarrow (a^2 - 5)(a^2 - 18) = 0 \Rightarrow a^2 = 18, 5$

When $a^2 = 18$, then from (ii), $b^2 = -8$ which is not possible and when $a^2 = 5$, then from (ii), $b^2 = 5$

Hence, the required equation of the hyperbola is $\dfrac{y^2}{5} - \dfrac{x^2}{5} = 1 \Rightarrow y^2 - x^2 = 5$.

12. Find the equation of the circle whose centre is (2, –3) and radius is 8.

Sol. The equation of the circle,

$(x-2)^2 + (y-(-3))^2 = 8^2$ [Using: $(x-h)^2 + (y-k)^2 = a^2$]

$\Rightarrow$ $(x-2)^2 + (y+3)^2 = 8$ $\Rightarrow$ $x^2 + y^2 - 4x + 6y + 5 = 0$.

13. Find the equation of a circle whose radius is 6 and the centre is at the origin.

Sol. The equation of the required circle is

$x^2 + y^2 = 6^2$ $\Rightarrow$ $x^2 + y^2 = 36$

14. Find the equation of the circle that passes through the points (1, 0), (–1, 0) and (0, 1).

Sol. Let the, required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since it passes through (1, 0), (–1, 0) and (0, 1).

$\therefore$ $1 + 2g + c = 0$ ...(ii), $1 - 2g + c = 0$ ...(iii), $1 + 2f + c = 0$ ...(iv)

Subtracting equation (iii) from (ii), we get

$4g = 0 \Rightarrow g = 0$.

Putting $g = 0$ in (ii), we get $c = -1$.

Now, putting $c = -1$ in (iv), we get $f = 0$.

Substituting the values of g, f and c in (i), we get the equation of the required circle as $x^2 + y^2 = 1$.

15. Find the equation of the parabola whose focus at (–3, 0) and the directrix, $x + 5 = 0$.

Sol. Let P (x, y) be any point on the parabola having its focus at S (–3, 0) and directrix, $x + 5 = 0$. Then

SP = PM, where PM is the length of the perpendicular from P on the directrix

$\Rightarrow$ $SP^2 = PM^2$

$$\Rightarrow \quad (x+3)^2 + (y-0)^2 = \left|\frac{x+0y+5}{\sqrt{1+0}}\right|^2 \quad \Rightarrow \quad y^2 = 4x + 16$$

This is the equation of the required parabola.

16. For the following parabolas find the coordinates of the foci, the equations of the directrices and the lengths of the latus rectum: (i) $x^2 = 6y$ (ii) $y^2 = -12x$

Sol. (i) The given parabola $x^2 = 6y$ is of the form $x^2 = 4ay$, where $4a = 6$ i.e., $a = 3/2$.

Clearly, the coordinates of the focus are $(0, a) = (0, 3/2)$ and the equation of the directrix is $y = -a$ i.e., $y = -3/2$.

Length of the latus rectum = 4a = 8.

(ii) The given parabola $y^2 = -12x$ is of the form $y^2 = -4ax$, where $4a = 12$ i.e., $a = 3$.

Clearly, the coordinates of the focus are $(-a, 0) = (-3, 0)$ and the equation of the directrix is $x = a$ i.e., $y = 3$.

Length of the latus rectum = 4a = 12.

17. Find the equation of the ellipse whose axes are along the coordinate axes, vertices at (±5, 0) and foci at (±4, 0).

Sol. Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(i)$$

The coordinates of its vertices and foci are (±a, 0) and (±ae, 0), respectively.

$\therefore$ a = 5 and ae = 4 $\Rightarrow$ $e = \frac{4}{5}$

Now, $b^2 = a^2(1 - e^2) = 25\left(1 - \frac{16}{25}\right) = 9.$

Substituting the values of a^2 and b^2 in (i), we get

$\frac{x^2}{25} + \frac{y^2}{9} = 1$, which is the equation of the required ellipse.

18. Find the equation of the ellipse whose axes are along the coordinate axes, vertices are (0, ±10) and eccentricity $e = 4/5$.

Sol. Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(i)$$

Since, the vertices of the ellipse are on y-axis. So, the coordinates of the vertices are $(0, \pm b)$

$\therefore$ $b = 10$.

Now, $a^2 = b^2(1 - e^2) \Rightarrow a^2 = 100\left(1 - \frac{16}{25}\right) = 36$

Substituting the values of a^2 and b^2 in (i), we obtain

$\frac{x^2}{36} + \frac{y^2}{100} = 1$ as the equation of the required ellipse.

19. The foci of an ellipse are (±2, 0) and its eccentricity is 1/2, find its equation.

Sol. Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Then coordinates of foci = (± ae, 0).

$\therefore\ ae=2 \Rightarrow a\times\frac{1}{2}=2 \qquad [\because e=1/2]$

$\Rightarrow\ a=4$.

We have, $b^2=a^2(1-e^2) \Rightarrow b^2=16\left(1-\frac{1}{4}\right)=12$.

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$.

20. Find the equation of the hyperbola, the length of whose latus rectum is 8 and eccentricity is $3/\sqrt{5}$.

Sol. Let the equation of the hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad ...(i)$$

Length of the latus rectum = 8

$\Rightarrow\ \frac{2b^2}{a}=8$

$\Rightarrow\ b^2=4a \Rightarrow a^2(e^2-1)=4a \Rightarrow a(e^2-1)=4 \Rightarrow a\left(\frac{9}{5}-1\right)=4 \Rightarrow a=5$

Putting $a=5$ in $b^2=4a$, we get $b^2=20$.

Hence, the equation of the required hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1$

21. Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

Sol. Let $2a$ and $2b$ be the transverse and conjugate axes and e be the eccentricity. Let the centre be the origin and the transverse and the conjugate axes the coordinate axes.

Then the equation of the hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad ...(i)$$

We have, $2b=5$ and $2ae=13$

Now, $b^2=a^2(e^2-1)$

$\Rightarrow\ b^2=a^2e^2-a^2$

$\Rightarrow\ \frac{25}{4}=\frac{169}{4}-a^2$

$\Rightarrow\ a^2=\frac{144}{4} \Rightarrow a=6$

Substituting the values of a and b in (i), the equation of the hyperbola is

$$\frac{x^2}{36}-\frac{y^2}{25/4}=1 \Rightarrow 25x^2-144y^2=900$$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. A parabola is the set of all points in a plane that are .... from a fixed line and a fixed point in the plane.
2. A line through the focus and perpendicular to the directrix is called the .... of the parabola.
3. Latus rectum of a parabola is a line segment to the axis of the parabola through the focus and whose end points lie on the parabola.
4. A chord of the parabola is a focal chord if it passes through the ....
5. The constant ratio denoted by 'e' is known as .... of the ellipse.
6. Sum of the focal distances of a point on the ellipse is constant and is equal to the length of the .... axis of the ellipse.
7. In the equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if a > b, then the major axes lie along the ....
8. In an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the coordinates of vertices are ....
9. In an ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a < b, eccentricity 'e' is given by ....
10. The conic obtained is a parabola, if 'e' = ...
11. Length of transverse axis of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is ....
12. The point at which the hyperbola intersects the transverse axis are called the .... of the hyperbola.

True / False :

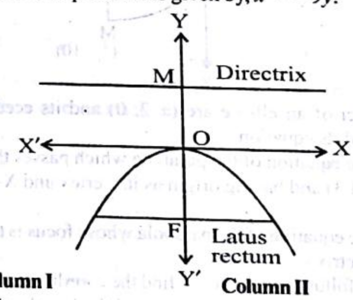
DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The point which bisects every chord of the conic passing through it, is called the centre of the conic.
2. The points of intersection of the conic section and the axis are called vertices of the conic section.
3. The equation of a parabola in its standard form is $x^3 = 4ay$.
4. For the parabola, $y^2 = 4ax$, where $a > 0$, the curve passes through the origin and the tangent at the origin is $x = 0$.
5. For the standard form of the parabola, $y^2 = -4ax$. Equation of the directrix is $x + a = 0$
6. In a parabola, length of the latus rectum is $4a$.
7. Equation of the ellipse in its standard form is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.
8. In an ellipse, the curve does not pass through the origin.
9. In an ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a < b)$, length of the major axis is $2a$.
10. Coordinates of foci of an ellipse having standard form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, (a > b), is given by $(\pm ae, 0)$
11. The equation of circle with its centre at the origin is $x^2+y^2=r^2$.
12. Eccentricity of a hyperbola is always less than 1.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1. Equation of a parabola is given by, $x^2 = -9y$.



Column I	Column II
(A) Focus	(p) $y = 9/4$
(B) Axis	(q) 9
(C) Directrix	(r) $(0, -9/4)$
(D) Length of latus rectum	(s) $x = 0$
	(t) $x = 9/4$
	(u) $y = 0$.

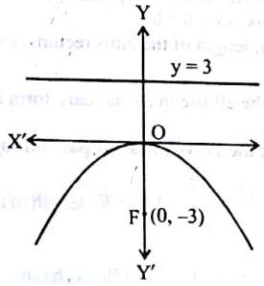
2. Given the ellipse with equation $9x^2 + 25y^2 = 225$.

Column I	Column II
(A) Major axis	(p) 6
(B) Minor axis	(q) $(\pm4, 0)$
(C) Foci	(r) 10
(D) Vertices	(s) $(\pm5, 0)$

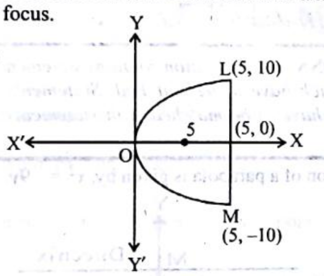
VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Find the equation of parabola that satisfies the given condition : focus (0, –3) and directrix $y = 3$.



2. Find the equation of the parabola whose vertex (0, 0), passes through (5, 2), and symmetric with respect to Y-axis.
3. Find the equation of the parabola whose focus is (3, –4) and directrix is $x - y + 5 = 0$
4. If the line $2x + 3y = 1$ touches the parabola $y^2 = 4ax$, find the length of the latus rectum.
5. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.



6. The foci of an ellipse are (± 2, 0) and its eccentricity is 1/2, find its equation.
7. Find the equation of the parabola which passes through the point (4,3) and having origin as its vertex and X-axis as its axis.
8. Find the equation of the parabola whose focus is (–3, 0) and the directrix $x + 5 = 0$.
9. For the following parabolas, find the coordinates of the foci, the equations of the directrices and the lengths of the latus rectum: $y^2 = 8x$.
10. Find the equation of the ellipse whose axes are along the coordinate axes, vertices are $(0, \pm 10)$ and eccentricity $e = 4/5$.
11. The foci of an ellipse are $(\pm 2, 0)$ and its eccentricity is 1/2, find its equation.
12. Find the equation of an ellipse whose foci are at $(\pm 3, 0)$ and which passes through (4, 1).
13. Find the equation of an ellipse whose eccentricity is 2/3, the latus rectum is 5 and the centre is at the origin.
14. Find the equation of ellipse whose Foci (±4, 0), the latus rectum is of length 12.
15. If LR of an ellipse is half of its minor axis, then find its eccentricity.
16. Find the equation of the circle whose centre is (2, –3) and radius is 8.
17. Find the equation of a circle whose radius is 6 and the centre is at the origin.
18. Find the equation of the hyperbola with vertices at $(0, \pm 6)$ and $e = \frac{5}{3}$. Find its foci.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find the centre and radius of the given circle $2x^2 + 2y^2 - x = 0$.
2. Find the equation of the ellipse whose axes are along the coordinate axes, vertices are $(0, \pm 10)$ and eccentricity $e = 4/5$.
3. Find the equation of the parabola whose focus is at (–3, 0) and the directrix, $x + 5 = 0$.
4. For the following parabolas, find the coordinates of the foci, the equations of the directrices and the lengths of the latus rectum:
   (i) $x^2 = 6y$ (ii) $y^2 = -12x$
5. Find the equation of the ellipse whose axes are along the coordinate axes, vertices at (±5, 0) and foci at (±4, 0).
6. The foci of an ellipse are (±2, 0) and its eccentricity is 1/2, find its equation.
7. Find the equation of the hyperbola, the length of whose latus rectum is 8 and eccentricity is $3/\sqrt{5}$.
8. For the hyperbola $9x^2 - 16y^2 = 144$, find the vertices, foci and eccentricity.
9. The focal distance of a point on the parabola $y^2 = 12x$ is 4. Find the abscissa of this point.
10. Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, for the given case:

    Vertices at $(0, \pm 7), e = \frac{4}{3}$

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Find the equation of the circle that passes through the points (1, 0), (–1, 0) and (0, 1).
2. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ whose one vertex is at the vertex of the parabola. Find the length of the side of the triangle.
3. If major axis lies on the X-axis and passes through the points (4, 3) and (6, 2), then find the equation for the ellipse that satisfies the given condition.
4. Find the equation of the ellipse which passes through the point (–3, 1) and has eccentricity $\frac{\sqrt{2}}{5}$, with X-axis as its major axis and centre at the origin.

2 EXERCISE

MCQ Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The equation of the parabola with vertex at origin, which passes through the point $(-3, 7)$ and axis along the X-axis is
(a) $y^2 = 49x$ (b) $3y^2 = -49x$
(c) $3y^2 = 49x$ (d) $x^2 = -49y$

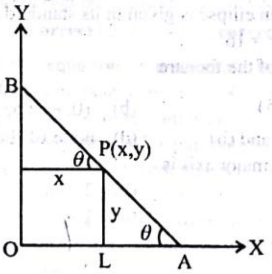
2. The locus of the points which are equidistant from $(-a, 0)$ and $x = a$ is
(a) $y^2 = 4ax$ (b) $y^2 + 4ax = 0$
(c) $x^2 + 4ay = 0$ (d) $x^2 - 4ay = 0$

3. The parabola $y^2 = 4ax$ passes through the point $(2, -6)$ then the length of its latus rectum is
(a) 9 (b) 16
(c) 18 (d) $\frac{9}{2}$

4. The equations of the lines joining the vertex of the parabola $y^2 = 6x$ to the points on it which have abscissa 24 are
(a) $y \pm 2x = 0$ (b) $2y \pm x = 0$
(c) $x \pm 2y = 0$ (d) $2x \pm y = 0$

5. The equation of the ellipse whose centre is at the origin and the X-axis, the major axis, which passes through the points $(-3, 1)$ and $(2, -2)$ is
(a) $5x^2 + 3y^2 = 32$ (b) $3x^2 + 5y^2 = 32$
(c) $5x^2 - 3y^2 = 32$ (d) $3x^2 + 5y^2 + 32 = 0$

6. A bar of given length moves with its extremities on two fixed straight lines at right angles. Any point of the bar describes



(a) parabola (b) ellipse
(c) hyperbola (d) circle

7. If e is the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a < b)$, then
(a) $b^2 = a^2(1 - e^2)$ (b) $a^2 = b^2(1 - e^2)$
(c) $a^2 = b^2(e^2 - 1)$ (d) $b^2 = a^2(e^2 - 1)$

8. If the focus of a parabola is $(0, -3)$ and its directrix is $y = 3$, then its equation is
(a) $x^2 = -12y$ (b) $x^2 = 12y$
(c) $y^2 = -12x$ (d) $y^2 = 12x$

9. If the parabola $y^2 = 4ax$ passes through the point $(3, 2)$, then the length of its latus rectum is
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) 4

10. The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is
(a) 4 (b) 3 (c) 8 (d) $\frac{4}{\sqrt{3}}$

11. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is:
(a) $4x^2 + 3y^2 = 1$ (b) $3x^2 + 4y^2 = 12$
(c) $4x^2 + 3y^2 = 12$ (d) $3x^2 + 4y^2 = 1$

12. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is
(a) $\frac{3}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

13. If $P \equiv (x, y)$, $F_1 \equiv (3, 0)$, $F_2 \equiv (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals
(a) 8 (b) 6 (c) 10 (d) 12

14. Eccentricity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if it passes through point $(9, 5)$ and $(12, 4)$ is
(a) $\sqrt{3/4}$ (b) $\sqrt{4/5}$ (c) $\sqrt{5/6}$ (d) $\sqrt{6/7}$

15. The coordinates of a point on the parabola $y^2 = 8x$, whose focal distance is 4, is :
(a) $(2, \pm 4)$ (b) $(4, 2)$ (c) $(4, -2)$ (d) $(2, 4)$

16. The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is:
(a) $x^2 + y^2 - 2x - 2y + 1 = 0$ (b) $x^2 + y^2 - 2x - 2y - 1 = 0$
(c) $x^2 + y^2 - 2x - 2y = 0$ (d) $x^2 + y^2 - 2x + 2y - 1 = 0$

17. The equation of the circle having centre $(1, -2)$ and passing through the point of intersection of the lines $3x + y = 14$ and $2x + 5y = 18$ is

(a) $x^2 + y^2 - 2x + 4y - 20 = 0$
(b) $x^2 + y^2 - 2x - 4y - 20 = 0$
(c) $x^2 + y^2 + 2x - 4y - 20 = 0$
(d) $x^2 + y^2 + 2x + 4y - 20 = 0$

18. The length of the transverse axis along X-axis with centre at origin of a hyperbola is 7 and it passes through the point $(5, -2)$. The equation of the hyperbola is

(a) $\frac{4}{49}x^2 - \frac{196}{51}y^2 = 1$ (b) $\frac{49}{4}x^2 - \frac{51}{196}y^2 = 1$

(c) $\frac{4}{49}x^2 - \frac{51}{196}y^2 = 1$ (d) none of these

19. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$ (d) none of these

20. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(a) $x^2 - y^2 = 32$ (b) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(c) $2x - 3y^2 = 7$ (d) none of these

MTOC More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. The locus of a point, such that the sum of its distance from the points $(0, 2)$ and $(0, -2)$ is 6, is:

(a) $9x^2 - 5y^2 = 45$ (b) $9(5 - x^2) = 5y^2$
(c) $9x^2 + 5y^2 = 45$ (d) $5x^2 - 9y^2 = 45$

2. The length of major and minor axis of an ellipse are 10 and 8 respectively and its major axis lies along the Y-axis then the equation of the ellipse referred to its centre as origin is

(a) $25x^2 + 16y^2 = 400$ (b) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

(c) $16x^2 + 25y^2 = 1600$ (d) $\frac{x^2}{64} + \frac{y^2}{100} = 1$

3. Which of the following is/are correct?

(a) Parabola is symmetric with respect to the axis of the parabola.
(b) Length of latus rectum of a parabola, $y^2 = 4ax$ is $2a$.
(c) A line through the focus and perpendicular to the directrix is called the axis of the parabola.
(d) The point of intersection of a parabola with the axis is called the vertex of the parabola.

4. Which of the following is/are not false ?

(a) The mid point of the line segment joining the foci is called the centre of the ellipse.
(b) The line segment through the foci of the ellipse is called the major axis.
(c) The end points of the major axis are called the vertices of the ellipse.
(d) Ellipse is symmetric with respect to Y-axis only.

5. Find the equation of a parabola with vertex at the origin, the axis along X-axis and passing through (2,3).

(a) $2x^2 = 9y$ (b) $2y^2 = 9x$
(c) $6y^2 - 27x = 0$ (d) $6x^2 - 27y = 0$

6. Which of the given statements is/are not correct?

(a) The equation of an ellipse with foci on the X-axis to is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(b) Length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2a^2}{b}$.

(c) Equation of the parabola with vertex at the origin, focus at (a, 0) and directrix $x = -a$ is $y^2 = 4ax$.
(d) Length of the latus rectum of the parabola $x^2 = -4ay$ is $2a$.

7. Equation of the hyperbola with vertices at $(\pm 5, 0)$ and foci at $(\pm 7, 0)$ is

(a) $24x^2 - 25y^2 = 600$ (b) $25x^2 - 24y^2 = 600$

(c) $\frac{x^2}{25} - \frac{y^2}{24} = 1$ (d) $\frac{x^2}{24} - \frac{y^2}{25} = 1$

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Passage 1

Equation of an ellipse is given in its standard form as,
$16x^2 + y^2 = 16$.

1. Coordinates of the foci are

(a) $(0, +\sqrt{15})$ (b) $(0, -\sqrt{15})$
(c) both (a) and (b) (d) none of these

2. Length of the major axis is

(a) 8 (b) 2
(c) 4 (d) 1

3. Length of the latus rectum is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 4 (d) 2

Passage II

A parabola is given in its standard form as $y^2 = 8x$.

1. Coordinates of the foci are
(a) $(2,0)$ (b) $(0,2)$
(c) $(-2,0)$ (d) $(0,-2)$

2. Equation of the axis is
(a) $x=0$ (b) $y=2$
(c) $x=2$ (d) $y=0$

3. Equation of the directrix is
(a) $y=-2$ (b) $x=2$
(c) $x=-2$ (d) $y=2$

Passage III

The equation of a hyperbola is given in its standard form as $16x^2-9y^2-144$.

1. Eccentricity of the given hyperbola is
(a) $\frac{5}{4}$ (b) $\frac{4}{5}$
(c) $\frac{1}{4}$ (d) none of these

2. Coordinates of foci is
(a) $(0,\pm1)$ (b) $(0,\pm1,0)$
(c) $(\pm5,0)$ (d) $(0,\pm5)$

3. Equations of directrices is
(a) $5x\pm16=0$ (b) $5y\pm16=0$
(c) $5x\pm12=0$ (d) $5y\pm12=0$

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason is incorrect.**
(d) If **Assertion is incorrect** but **Reason is correct.**

1. **Assertion:** If the equation of a parabola is given to be $x^2 = 6y$, then length of its latus rectum is 6.
Reason: Length of the latus rectum is $2a$.

2. **Assertion:** If the equation of a parabola is of the form $y^2 = -12x$, then equation of its directrix is $x-3=0$.
Reason: Equation of the directrix is $x-a=0$

3. **Assertion :** If the equation of a parabola is of the form $x^2 = -16y$, then coordinates of the focus are $(0,-4)$.
Reason: Coordinates of the focus = $(0,-a)$

4. **Assertion:** If the equation of the ellipse is given to be $16x^2+25y^2=400$, then length of the major axis is 10.
Reason: Length of the major axis = $2b$

5. **Assertion:** Equation of an ellipse is of the form $3x^2+2y^2=6$, then its eccentricity 'e' is $\frac{1}{\sqrt{2}}$.
Reason: The eccentricity 'e' of the ellipse is given by,
$$e=\sqrt{1-\frac{a^2}{b^2}}$$

6. **Assertion:** If the equation of a circle is $(x+1)^2+(y-1)^2=4$, then its radius is 4.
Reason: Equation of a circle with radius r is given by, $(x-a)^2+(y-b)^2=r^2$

7. **Assertion:** If the equation of a hyperbola is $3x^2-6y^2=-18$, then length of its latus rectum is $4\sqrt{3}$.
Reason: Length of latus rectum of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is $\frac{2a^2}{b}$.

MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1. An equation of the parabola is given to be $4x^2+y=0$.

Column I	Column II
(A) Vertex	(p) $(0,-1/16)$
(B) Axis	(q) $y=-1/16$
(C) Focus	(r) $(0,0)$
(D) Directrix	(s) $16y+1=0$
	(t) $x=0$
	(u) $3^x-1=0$

HOTS HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Find the equation of the circle passing through the points (2, 3) and (–1, 1) and whose centre is on the line $x-3y-11=0$.

2. A man running a race-course notes that the sum of the distances from the two flag posts from him is always 10 metres and the distance between the flag posts is 8 metres. Find the equation of the path traced by the man.

3. The foci of an ellipse are $(\pm2,0)$ and its eccentricity is 1/2, find its equation.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. equidistant. 2. axis.
3. perpendicular 4. Focus
5. eccentricity 6. Major 7. X-axis.
8. $(\pm a, o)$ 9. $\sqrt{1-\frac{a^2}{b^2}}$ 10. 1
11. 2a 12. Vertices

TRUE / FALSE

1. True 2. True 3. False
4. True 5. False 6. True
7. False 8. True 9. False
10. True 11. True 12. False

MATCH THE FOLLOWING :

1. (A) → r ; (B) → s ; (C) → p ; (D) → q

Equation of the parabola is $x^2 = -9y, 4a = 9,\ a = \frac{9}{4}$

Focus $= \left(0, -\frac{9}{4}\right)$

Axis of parabola is Y-axis i.e., $x = 0$, Directrix is $y = \frac{9}{4}$

2. (A) → r ; (B) → p ; (C) → q ; (D) → s

We put the equation in standard form by dividing by 225,

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

This shows that a = 5 and b = 3.

Hence, $9 = 25(1 - e^2)$, so $e = \frac{4}{5}$.

Since the denominater of x^2 is larger, the major axis is along X-axis, minor axis along Y-axis, foci are (4, 0) and (–4, 0) and vertices are (5, 0) and (–5, 0).

VERY SHORT ANSWER QUESTIONS :

1. Focus (0, –3) and directrix $y = 3$

vertex is the mid point of (0, –3) and (0, 3).

i.e., vertex is (0, 0)

and $a = 3$, $\therefore 4a = 12$

$\therefore$ Equation of parabola, $x^2 = -12y$

2. $2x^2 = 25y$

3. $22x - 26y - 25 = (x + y)^2$

Hint : Let P(x, y) be the point on the parabola.

$\sqrt{(x-3)^2 + (y+4)^2} = \frac{|x-y+5|}{\sqrt{1+1}}$ and solve.

4. $\frac{8}{9}$

Hint - Writing the given equation as

$y = -\frac{2}{3}x + \frac{1}{3} = -\frac{2}{3}x - \frac{2/9}{-2/3},\quad \therefore a = \frac{2}{9}$

5. Taking vertex of the parabola reflector at origin, X-axis along the axis of parabola the equation of parabola is $y^2 = 4ax$. Given depth 5 cm, diameter 20 cm.

$\therefore$ (5, 10) point lies on parabola

$(10)^2 = 4a(5)$

$\therefore\quad a = 5$

$\therefore$ Focus is $(a, 0)$, i.e., (5, 0) which is mid-point of the given diameter.

6. Coordinates of foci are $(\pm ae, 0)$ $\left[\because e = \frac{1}{2}\right]$

$\Rightarrow a = 4$

$\therefore\quad b^2 = 16\left(1 - \frac{1}{4}\right) = 12$

Thus, the equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

7. Since, vertex is at (0, 0) and axis is along X-axis, so let the equation of the parabola be $y^2 = 4ax$

Since it passes through (4, 3), so

$9 = 16a \Rightarrow 4a = 9/4$

hence, the required equation will be $4y^2 = 9x$

8. Let P (x, y) be any point on the parabola having its focus at S (–3, 0) and directrix as the line $x + 5 = 0$.

$\Rightarrow\ SP^2 = PM^2$

$\Rightarrow\ (x+3)^2 + (y-0)^2 = \left|\frac{x + 0y + 5}{\sqrt{1+0}}\right|^2 \Rightarrow y^2 = 4x + 16$

This is the equation of the required parabola.

9. The coordinates of the focus are (a, 0) i.e. (2, 0) and the equation of the directrix is $x = -a$ i.e. $x = -2$

Length of the latus rectum = $4a = 8$.

10. Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{.............. (i)}$$

Since, the vertices of the ellipse are on Y-axis. So, the coordinates of the vertices are $(0,\pm b)$.

$\therefore\ b=10$.

Now, $a^2=b^2(1-e^2) \Rightarrow a^2=100\left(1-\frac{16}{25}\right)=36$

Substituting the values of a^2 and b^2 in (i), we obtain

$\frac{x^2}{36}+\frac{y^2}{100}=1$ as the equation of the required ellipse.

11. The equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$.

12. $\frac{x^2}{18}+\frac{y^2}{9}=1$

13. Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

We have, $e=\frac{2}{3}$, and $\frac{2b^2}{a}=5$.

Now, $\frac{2b^2}{a}=5$

$\Rightarrow a=\frac{9}{2}$ and $b^2=\frac{45}{4}$

$\therefore \frac{4x^2}{81}+\frac{4y^2}{45}=1$

14. Foci are $(\pm 4, 0)$, $\therefore c=4$ or $c^2=a^2+b^2$

$\therefore\ 16=a^2+b^2 \qquad \ldots(i)$

Latus Rectum $=\frac{2b^2}{a}=12 \qquad \ldots(ii)$

Eliminating b^2 from (i) & (ii), $16=a^2+6a$

or $a^2+6a-16=0$

or $(a+8)(a-2)=0\ (\because a\neq -8)$

$\therefore a=2$

$b^2=6a=6\times 2=12;\ a^2=4,\ b^2=12,\ \frac{x^2}{4}+\frac{y^2}{12}=1$.

15. $\therefore e=\sqrt{3}/2$

16. The equation of the circle,

$(x-2)^2+(y-(-3))^2=8^2$

[Using: $(x-h)^2+(y-k)^2=a^2$]

$\Rightarrow\ x^2+y^2-4x+6y+5=0$.

17. The equation of the required circle is

$x^2+y^2=6^2 \Rightarrow x^2+y^2=36$

18. Since, the vertices are on the Y-axis (with origin at the mid-point), the equation is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

As vertices are $(0,\pm 6)$, $a=6$, $b^2=a^2(e^2-1)$

$=36\left(\frac{25}{9}-1\right)=64$, so the required equation of the hyperbola is $\frac{y^2}{36}-\frac{x^2}{64}=1$ and the foci are $(0,\pm ae)=(0,\pm 10)$

SHORT ANSWER QUESTIONS :

1. Equation of circle is $2x^2+2y^2-x=0 \Rightarrow x^2+y^2-\frac{x}{2}=0$

$\Rightarrow\left(x^2-\frac{x}{2}\right)+y^2=0 \Rightarrow \left(x^2-\frac{x}{2}+\frac{1}{16}\right)+y^2=\frac{1}{16}$

$\Rightarrow\left(x-\frac{1}{4}\right)^2+y^2=\frac{1}{16}$

$\therefore$ Centre is $\left(\frac{1}{4},0\right)$ and radius is $\frac{1}{4}$

2. $\frac{x^2}{36}+\frac{y^2}{100}=1$ as the equation of the required ellipse.

3. Let P (x, y) be any point on the parabola having its focus is at S $(-3, 0)$ and directrix, $x+5=0$.

Then SP = PM, where PM is the length of the perpendicular from P on the directrix

$\Rightarrow\ SP^2=PM^2$

$\Rightarrow\ (x+3)^2+(y-0)^2=\left|\frac{x+0y+5}{\sqrt{1+0}}\right|^2 \Rightarrow y^2=4x+16$

This is the equation of the required parabola.

4. (i) The given parabola $x^2=6y$ is of the form $x^2=4ay$, where $4a=6$ i.e., $a=3/2$.

Clearly, the coordinates of the focus are $(0, a)$ $=(0,\ 3/2)$ and the equation of the directrix is $y=-a$ i.e. $y=-3/2$.

Length of the latus rectum $=4a=8$.

(ii) The given parabola $y^2=-12x$ is of the form $y^2=-4ax$, where $4a=12$ i.e., $a=3$.

Clearly, the coordinates of the focus are $(-a, 0)$ $=(-3, 0)$ and the equation of the directrix is $x=a$ i.e. $y=3$.

Length of the latus rectum $=4a=12$.

5. $\frac{x^2}{25}+\frac{y^2}{9}=1$, which is the equation of the required ellipse.

6. Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$.

7. Let the equation of the hyperbola be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad ...(i)$$

Length of the latus rectum = 8

$\Rightarrow a(e^2-1)=4$

$\Rightarrow a=5$

Putting $a=5$ in $b^2=4a$, we get $b^2=20$.

Hence, the equation of the required hyperbola is

$$\frac{x^2}{25}-\frac{y^2}{20}=1$$

8. Vertices are $(\pm a,0)=(\pm 4,0)$ and foci are $(\pm ae,0)=(\pm 5,0)$

9. The given parabola is of the form $y^2=4ax$. On comparing, we have $4a=12$ i.e., $a=3$.

We know that the focal distance of any point (x, y) on $y^2=4ax$ is $x+a$.

Let the given point on the parabola $y^2=12x$ be (x, y). Then its focal distance is $x+3$.

$\therefore x+3=4 \Rightarrow x=1.$

Hence, the abscissa of the given point is 1.

10. Since, the vertices of the required hyperbola lie on Y-axis. So, let its equation be

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \quad .... (i)$$

The coordinates of vertices of this hyperbola are $(0, \pm b)$. So, $b=7$

Now, $a^2=b^2(e^2-1) \Rightarrow a^2=49\left(\frac{16}{9}-1\right)$

$\Rightarrow a^2=49\times\frac{7}{9}=\frac{343}{9}$

Substituting the values of a^2 and b^2 in (i), we get

$\frac{9x^2}{343}-\frac{y^2}{49}=-1$ as the equation of the desired hyperbola.

LONG ANSWER QUESTIONS :

1. Let the required circle be $x^2+y^2+2gx+2fy+c=0$ ...(i)

Since, it passes through $(1, 0)$, $(-1, 0)$ and $(0, 1)$.

$\therefore 1+2g+c=0$ ...(ii), $1-2g+c=0$ ...(iii)

$1+2f+c=0$ ...(iv)

Subtracting equation (iii) from (ii), we get

$4g=0 \Rightarrow g=0.$

Putting $g=0$ in (ii), we get $c=-1$.

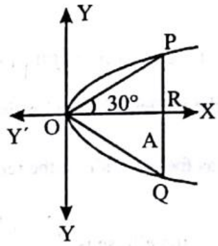
Now, putting $c=-1$ in (iv), we get $f=0$.

Substituting the values of g, f and c in (i), we get the equation of the required circle as $x^2+y^2=1$.

2. As shown in the figure ΔOPQ denotes the equilateral triangle with its equal sides of length l (say).

Here $OP=l$, so $OR=l\cos 30°$

$$=l\frac{\sqrt{3}}{2}$$



Also, $PR=l\sin 30°=\frac{1}{2}$.

Thus; $\left(\frac{l\sqrt{3}}{2},\frac{l}{2}\right)$ are the coordinates of the point P lying on the parabola $y^2=4ax$.

Therefore, $\frac{l^4}{4}=4a\left(\frac{l\sqrt{3}}{2}\right) \Rightarrow l=8\,a\sqrt{3}$.

Thus, $8\,a\sqrt{3}$ is the required length of the side of the equilateral triangle inscribed in the parabola $y^2=4ax$.

3. Major axis is X-axis

Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

As $(4, 3)$ and $(6, 2)$ lies on it, $\therefore \frac{16}{a^2}+\frac{9}{b^2}=1$ ...(i)

and $\frac{36}{a^2}+\frac{4}{b^2}=1$ ...(ii)

Subtracting $\frac{-20}{a^2}+\frac{5}{b^2}=0 \Rightarrow 5a^2=20b^2 \Rightarrow a^2=4b^2$

Putting the value of a^2 in equation. (i) $\frac{16}{4b^2}+\frac{9}{b^2}=1$,

$\therefore b^2=13$ and $a^2=4b^2=4\times 13=52$

$\therefore$ Equation of ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1$

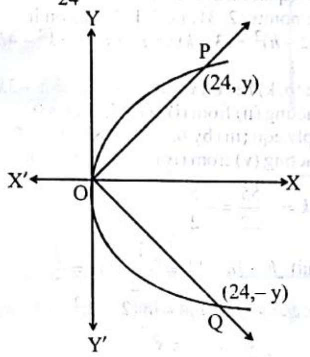
4. $3x^2+5y^2=32.$

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (b) 2. (b) 3. (c)

4. (b) Let P and Q be points on the parabola $y^2 = 6x$ and OP, OQ be the lines joining the vertex O to the points P and Q whose abscissa are 24.
$\Rightarrow y = \pm 12$.
Therefore, the coordinates of the points P and Q are (24, 12) and (24, – 12), respectively. Hence, the lines are $y = \pm\frac{12}{24}x \Rightarrow 2y = \pm x$.



5. (b) Hence, required equation of ellipse is $3x^2 + 5y^2 = 32$.

6. (b) Let P (x, y) be any point on the bar such that PA = a and PB = b, clearly from the figure.
$x = OL = b\cos\theta$ and $y = PL = a\sin\theta$

These give $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$. which is an ellipse.

7. (b) 8. (a)

9. (b) 10. (d)

11. (b) $e = \frac{1}{2}$. Directrix, $x = \frac{a}{e} = 4$
Equation of ellipse is,
$3x^2 + 4y^2 = 12$

12. (a) $e = \frac{3}{a} = \frac{3}{5}$

13. (c) The ellipse can be written as, $\frac{x^2}{25} + \frac{y^2}{16} = 1$
Here $a^2 = 25, b^2 = 16$, but $b^2 = a^2(1 - e^2)$
$\Rightarrow 16/25 = 1 - e^2$
$\Rightarrow e^2 = 1 - 16/25 = 9/25 \Rightarrow e = 3/5$
Foci of the ellipse are $(+ae, 0) = (\pm 3, 0)$, i.e., F_1 and F_2
$\therefore$ We have $PF_1 + PF_2 = 2a = 10$ for every point P on the ellipse.

14. (d) $e = \sqrt{1 - \frac{1}{7}} = \sqrt{\frac{6}{7}}$

15. (a) So, required points is $(2, \pm 4)$

16. The correct choice is (a), since the equation can be written as $(x-1)^2 + (y-1)^2 = 1$, which represents a circle touching both the axes with its centre (1, 1) and radius one unit.

17. The correct option is (a). The point of intersection of $3x + y - 14 = 0$ and $2x + 5y - 18 = 0$ are $x = 4, y = 2$, i.e., the point (4, 2).

Therefore, the radius is $= \sqrt{9+16} = 5$ and hence the equation of the circle is given by $(x-1)^2 + (y+2)^2 = 25$
$$x^2 + y^2 - 2x + 4y - 20 = 0$$

18. (c) 19. (c) 20. (a)

MORE THAN ONE CORRECT :

1. (b, c) Let the given points be A(0, 2) and B(0, –2). Let P(x, y) be a general point on the locus. By the given condition PA + PB = 6
$\Rightarrow \sqrt{x^2 + (y-2)^2} + \sqrt{x^2 + (y+2)^2} = 6$
$\Rightarrow 9x^2 + 5y^2 - 45 = 0 \Rightarrow -9x^2 - 5y^2 + 45 = 0$
$\Rightarrow 9(5 - x^2) = 5y^2$

2. (a,b)
$b = 5$ and $a = 4$
$\therefore$ Required equation of ellipse is $\frac{x^2}{(4)^2} + \frac{y^2}{(5)^2} = 1$
$\frac{x^2}{16} + \frac{y^2}{25} = 1$
$\frac{25x^2 + 16y^2}{400} = 1$
$25x^2 + 16y^2 = 400$

3. (a, b, d) 4. (a, b, c)

5. (b, c)
Equation of the parabola, $y^2 = 4ax$.
Since, it passes through (2,3), $a = 9/8$
$\therefore$ Equation of the parabola,
$y^2 = 4\left(\frac{9}{8}\right)x$.
$6y^2 - 27x = 0$

6. (b, d) 7. (a, c)

PASSAGE BASED QUESTIONS :

Passage-I

1. (c) The ellipse is $16x^2 + y^2 = 16$
Major axis is along Y-axis
$a^2 = 16$, $\therefore a = 4, b^2 = 1$, $\therefore b = 1$
$c = \sqrt{15}$
foci are $(0, \pm\sqrt{15})$

2. (a) Length of major axis $= 2a = 2 \times 4 = 8$

3. (b) Length of the latus rectum $= \frac{2b^2}{a} = \frac{2\times1}{4} = \frac{1}{2}$

Passage-II

1. (a) Comparing $y^2 = 8x$ with the standard equation $y^2 = 4ax$,

2. (d) Since, the given parabola lies along the X-axis, so equation of the axis, $y = 0$.

3. (c) Equation of the directrix,
$x+a=0$
$x=-2$

Passage-III

1. (a) Dividing both sides by 144,

$e=\sqrt{1+\frac{9}{16}}=\frac{5}{4}$

2. (d) Foci $(0,\pm be)\Rightarrow$ Foci $=(0,\pm 5)$

3. (b) Equations of directrics is, $y=\pm\frac{b}{e}\Rightarrow 5y\ \pm 16=0$

ASSERTION & REASON :

1. (c) $\therefore$ latus rectum $=4a=4\left(\frac{3}{2}\right)=6$

2. (a) $\therefore$ Equation of the directrix, $x-a=0$
$x-3=0$

3. (a) $\therefore$ Focus $=(0,-4)$

4. (c) We have, $\frac{x^2}{25}+\frac{y^2}{16}=1$
where $a^2=25$ and $b^2=16$
i.e., $a=5$ and $b=4$.
Clearly, a > b, therefore, the major and minor axes of the ellipse are along x and y axes respectively.
Hence, length of major axis $=2a=10$

5. (d) $a=\sqrt{2}$ and $b=\sqrt{3}$.
Clearly, $a<b$, so the major and minor axes of the given ellipse are along Y and X-axis respectively.
$\therefore$ Length of the major axis $=2b=2\sqrt{3}$,
Length of the minor axis $=2a=2\sqrt{2}$.
The coordinates of the vertices are $(0,\sqrt{3})$ and $(0,-\sqrt{3})$

$e=\frac{1}{\sqrt{3}}$

6. (d) $(x-(-1))^2+(y-1)^2=(2)^2$
radius $=2$

7. (a) Dividing both sides by 18,

$\frac{-x^2}{6}+\frac{y^2}{3}=1$

$a=\sqrt{6}$, $b=\sqrt{3}$

$\therefore$ latus rectum $=4\sqrt{3}$

MULTIPLE MATCHING QUESTIONS :

1. (A) → r ; (B) → t, u ; (C) → p ; (D) → q, s
(A) $4x^2=-y$.

$x^2=-\frac{1}{4}y$.

Vertex $=(0,0)$

(B) Since, the parabola lies along the Y-axis, so equation of the axis, $x=0$.
$3^x=1=3^0$
$x=0$

(C) Comparing $x^2=-\frac{1}{4}y$ with the standard equation of parabola $x^2=-4ay$,
$\therefore$ Focus $=(0,-1/16)$

(D) Equation of the directrix, $y=a$

HOTS SUBJECTIVE QUESTIONS :

1. Let the equation of the circle be $(x-h)^2+(y-k)^2=r^2$ ... (i)
As the points (2, 3) and (–1, 1) lies on it,
$\therefore\quad (2-h)^2+(3-k)^2=r^2\Rightarrow h^2+k^2-4h-6k+13=r^2$ ...(ii)
Centre (h, k) lies on $x-3y-11=0\Rightarrow h-3k-11=0$ ... (iii)
Subtracting (ii) from (i) $6h+4k-11=0$ ...(iv)
Multiply eqn (iii) by 6, $6h-18k-66=0$ ... (v)
Subtracting (v) from (iv), $22k+55=0$

$k=-\frac{55}{22}=-\frac{5}{2}$

from (iii) $h=3k+11=-\frac{15}{2}+11=\frac{7}{2}$

Put the value of h and k in $(2-h)^2+(3-k)^2=r^2$

$\left(2-\frac{7}{2}\right)^2+\left(3+\frac{5}{2}\right)^2=r^2$

$\Rightarrow\quad r^2=\frac{9}{4}+\frac{121}{4}=\frac{130}{4}=\frac{65}{2}$

$\therefore$ Equation of the circle $\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{65}{2}$

$\Rightarrow\quad x^2+y^2-7x+5y+\frac{49}{4}+\frac{25}{4}-\frac{65}{2}=0$

$\Rightarrow\quad x^2+y^2-7x+5y-14=0$.

2. Clearly, the path traced by the man is an ellipse having its focii at two flag posts. Let the equation of the ellipse be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2=a^2(1-e^2)$

It is given that the sum of the distances of the man from the two flag posts is 10 metres.
This means that the sum of the focal distances of a point on the ellipse is 10 m.
$\Rightarrow a=5$
It is also given that the distance between the flag posts is 8 metres.
$\Rightarrow ae=4$
Now, $b^2=a^2(1-e^2)\Rightarrow b=3$

Hence, the equation of the path is $\frac{x^2}{25}+\frac{y^2}{9}=1$

3. $\frac{x^2}{16}+\frac{y^2}{12}=1$.

chapter 19

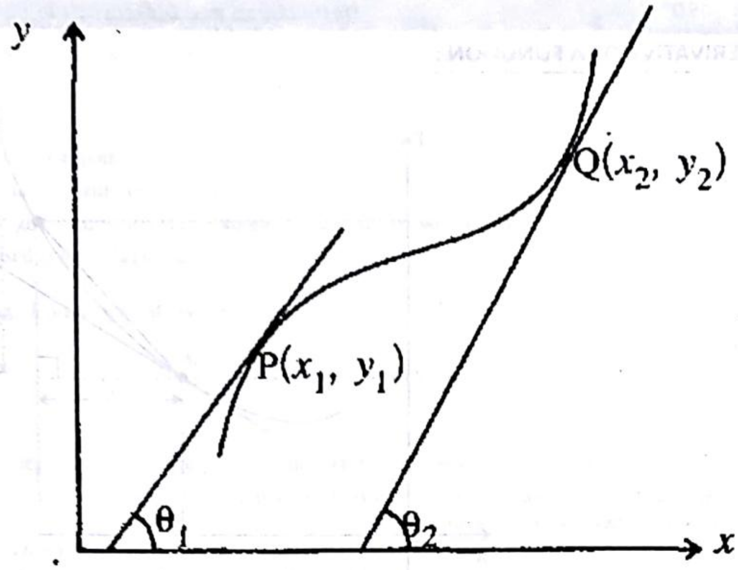


Fig. (A1)

DERIVATIVES AND DIFFERENTIABILITY

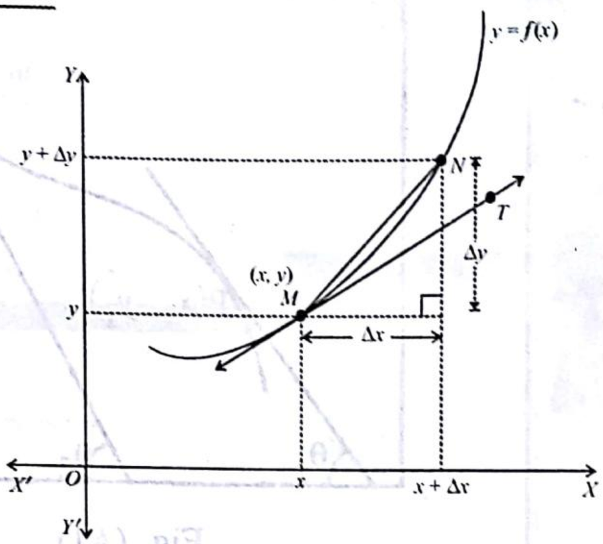
Introduction

At what rate is the economy growing? How fast is the number of subscribers to satellite radio growing? At what rate are the prices of LCD TVs decreasing ? At what rate the health care expenditure increasing?

The technique of differentiation will allow us to answer many questions like these that deal with rates of charge. Differentiation precise formulation of large number of physical concepts such as velocity at an instant, acceleration at an instant, curvature of a curve at a point etc.

Differentiation is widely used in Physics and Economics.

DERIVATIVE OF A FUNCTION :



Let $y = f(x)$ be a continuous curve. Consider a point $M(x, y)$ on the curve. Also suppose that MT is the tangent at point M to the curve.

Average rate of change of y with respect to x when x changes from x to $x + \Delta x$ and correspondingly y changes from y to $(y + \Delta y) = \frac{\Delta y}{\Delta x}$

Now $y = f(x)$

$\therefore \quad y + \Delta y = f(x + \Delta x)$

$\Rightarrow \quad (y + \Delta y) - y = f(x + \Delta x) - f(x)$

$\Rightarrow \quad \Delta y = f(x + \Delta x) - f(x)$

Hence, average rate of change in y with respect to x

$$= \frac{\Delta y}{\Delta x} \text{ or } \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

clearly $(x + \Delta x, y + \Delta y)$ will be a point on the curve. Let it be N.

Then $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$ = Slope of the chord MN

When $\Delta x \to 0$, then limiting value of average rate of change in y with respect to x becomes instantaneous rate of change in y with respect to x at point $M(x, y)$. Point N coincides with point M, chord MN coincides with the tangent MT.

Hence Instantaneous rate of change of y with respect to x.

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \text{ or } \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \text{slope of the tangent } MT.$$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ is represented by $\frac{dy}{dx}$ and $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ is represented by $f'(x)$

Now, $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ .... (i)

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ .... (ii)

$\frac{dy}{dx}$ (Formula (i)) is called derivative of y with respect to x.

$f'(x)$ (Formula (ii)) is called derivative of $f(x)$ with respect to x.

Formula (i) and (ii) are same and are called derivative by first principle or by Ab-into method or delta method.

Thus $\frac{dy}{dx}$ or $f'(x)$ is :

(a) Derivative of y or $f(x)$ with respect to x at point (x, y)

(b) Instantaneous rate of change of y or $f(x)$ with respect to x.

(c) Slope of the tangent at point (x, y) to the curve $y = f(x)$ or simply slope of the curve at (x, y)

Some times at the place of Δx, h is also used in formula (ii),. then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$ where h is a very small +ve quantity.

Derivative at $x = a$ is given by

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

Derivative of a function of a variable is simply the rate of change of the function with respect to the variable.

The derivative makes possible to study the character of change of a function. Derivative of some function $f(x)$ is positive means when the value of x increases then the value of $f(x)$ also increases and vice-versa. Derivative is $-$ve means when the value of x increases then the value of $f(x)$ decreases and vice-versa. Greater the absolute value of the derivative means more abruptly the value of $f(x)$ varies as x varies. Derivative is 0 (zero) means the value of $f(x)$ remains constant when x varies.

The derivative is widely applied in Geometry, Physics, Mechanics, Chemistry, Biology, Commerce and many other fields.

DERIVATIVE OF SOME STANDARD FUNCTIONS :

(a) $\frac{d(x^n)}{dx} = nx^{n-1}$

(b) $\frac{d(\text{constant})}{dx} = 0$

(c) $\frac{d(kx)}{dx} = k$, where k is a constant.

(d) $\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$

(e) $\frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}$

(f) $\frac{d\left(\frac{1}{x^2}\right)}{dx} = -\frac{2}{x^3}$

(g) $\frac{d\left(\frac{1}{x^2}\right)}{dx} = -\frac{3}{x^4}$

(h) $\frac{d(\sin x)}{dx} = \cos x$

(i) $\frac{d(\cos x)}{dx} = -\sin x$

(j) $\frac{d(\tan x)}{dx} = \sec^2 x$

(k) $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

(*l*) $\frac{d(\sec x)}{dx} = \sec x.\tan x$

(m) $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x.\cot x$

(n) $\frac{d(k.f(x))}{dx} = k.f'(x)$, where k is a constant and $f'(x)$ is the derivative of $f(x)$ with respect to x.

(o) $\frac{d[f(x)+g(x)]}{dx} = \frac{d(f(x))}{dx} + \frac{d(g(x))}{dx}$

(p) $\frac{d[f(x)-g(x)]}{dx} = \frac{d(f(x))}{dx} - \frac{d(g(x))}{dx}$

PRODUCT RULE OF DERIVATIVE :

(a) $\dfrac{d[f(x).g(x)]}{dx} = g(x).\dfrac{d[f(x)]}{dx} + f(x).\dfrac{d[g(x)]}{dx}$

(b) $\dfrac{d[f(x).g(x).r(x)]}{dx} = g(x).r(x).\dfrac{d[f(x)]}{dx} + f(x).r(x).\dfrac{d[g(x)]}{dx} + f(x).g(x).\dfrac{d[r(x)]}{dx}$

QUOTIENT RULE OF DERIVATIVE :

$$\frac{d\left[\dfrac{f(x)}{g(x)}\right]}{dx} = \frac{g(x).\dfrac{d}{dx}(f(x)) - f(x).\dfrac{d(g(x))}{dx}}{[g(x)]^2}$$

DIFFERENTIABILITY :

In day to day life we come across many examples where one variable depends on the other variable. For example, the price of certain commodity depends on its supply; weight of a human being depends on its height etc. In mathematics, we study how the value of a function varies with the values of the variable on which it depends. The limit, which measures the rate of change of one variable with other is know as derivative; and; it is one of the most important ideas in calculus. The concept of derivative is said to have been introduced by G.W. Leibnitz but it was first published by Sir Issac Newton. Therefore, Newton is considered as the father of calculus. Later on the calculus was presented in present modified form by Cauchy.

Derivative of a function $f(x)$ with respect to x at any point $x = a$ is denoted by $f'(a)$ and is defined as

$f'(a) = \lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$; provided that the limit exists finitely. Here 'h' is a very small +ve quantity .

A function is said to be differentiable at a point $x = a$ if it has a derivative at that point.

Similarly, the left hand derivative of a function $f(x)$ at a point $x = a$ is denoted by $Lf'(a)$ and is defined as

Hence, the function $f(x)$ is differentiable at $x = a$ if and only if $\lim\limits_{x\to a^-} \dfrac{f(x) - f(a)}{x - a} = \lim\limits_{x\to a^+} \dfrac{f(x) - f(a)}{x - a}$.

If the left hand limit $\lim\limits_{x\to a^-} \dfrac{f(x) - f(a)}{x - a} = \lim\limits_{h\to 0} \dfrac{f(a-h) - f(a)}{-h}$, exist finitely, it is called Left Hand Derivative (LHD) at $x = a$ and symbolically denoted by $Lf'(a)$ or $f'(a^-)$.

i.e. $Lf'(a) = \lim\limits_{h\to 0} \dfrac{f(a-h) - f(a)}{-h}$

If the right hand limit, $\lim\limits_{x\to a^+} \dfrac{f(x) - f(a)}{x - a} = \lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$, exists finitely, it is called Right Hand Derivative (RHD) at $x = a$ and symbolically denoted by $Rf'(a)$ or $f'(a^+)$.

i.e $Rf'(a) = \lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

The function $f(x)$ is differentiable at $x = a$ if both the right hand derivative, $Rf'(a)$ and the left hand derivative, $Lf'(a)$ exist finitely and are equal, that is, $Rf'(a) = Lf'(a)$

$f(x)$ is not differentiable if

i. either of the $Rf'(a)$ or $Lf'(a)$, do not exist; or

ii. both $Rf'(a)$ and $Lf'(a)$ exist but are not equal

ILLUSTRATION 19.1

Show that a function defined as $f(x) = x^2 + 2x + 7$ is differentiable at $x = 3$ and find $f' = 3$.

SOLUTION :

Since $f(x) = x^2 + 2x + 7$,

$$Rf'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h\to 0} \frac{(3+h)^2 + 2(3+h) + 7 - \{3^2 + 2(3) + 7\}}{h}$$

$$= \lim_{h\to 0} \frac{9 + h^2 + 6h + 6 + 2h + 7 - 9 - 6 - 7\}}{h} = \lim_{h\to 0} (h+8) = 8$$

Similarly, $$Lf'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{-h}$$

$$= \lim_{h\to 0} \frac{(3-h)^2 + 2(3-h) + 7 - \{3^2 + 2(3) + 7\}}{-h}$$

$$= \lim_{h\to 0} \frac{9 + h^2 - 6h + 6 - 2h + 7 - 9 - 6 - 7\}}{-h}$$

$$= \lim_{h\to 0} (8-h) = 8$$

Since $Rf'(3) = Lf'(3)$, $f(x)$ is differentiable at $x = 3$. Further, since $Rf'(3) = Lf'(3) = 8$, the value of $f'(3) = 8$.

ILLUSTRATION 19.2

Prove that the function f given by $f(x) = |x - 1|, x \in R$ is not differentiable at $x = 1$.

SOLUTION :

Sol. The given function may be written as, $f(x) = \begin{cases} x-1 & , \text{if } x \geq 1 \\ 1-x & , \text{if } x < 1 \end{cases}$

R.H.D. at x = 1 $= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{[(1+h)-1] - (1-1)}{h} = \lim_{h\to 0} \frac{h}{h} = 1$

L.H.D., at x = 1 $= \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{1-(1-h)-(1-1)}{-h} = \lim_{h\to 0} \frac{h}{-h} = -1$

R.H.D. $\neq$ L.H.D. $\Rightarrow$ f is not differentiable at x = 1.

DIFFERENTIABILITY IN AN INTERVAL :

A function $f(x)$ is said to be differentiable in an interval (a, b) if $f(x)$ is differentiable at every point of this interval (a, b).

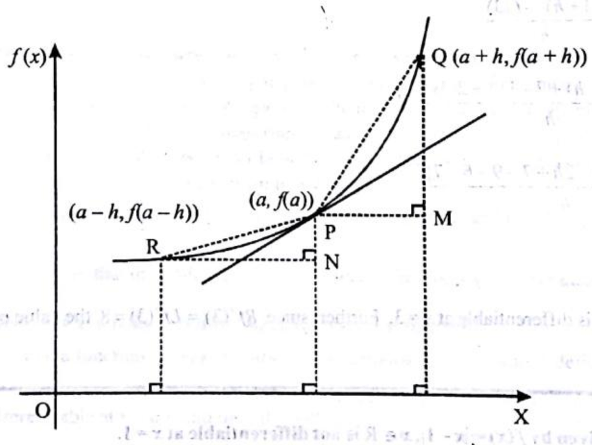
A function $f(x)$ is said to be differentiable in a closed interval $[a, b]$, if $f(x)$ is differentiable in (a, b), in addition $f(x)$ is differentiable at $x = a$ from right and differentiablie at $x = b$ from left.

SOME FACTS ABOUT DIFFERENTIABILITY:

1. A constant function is everywhere differentiable.
2. A polynomial function is everywhere differentiable.
3. Trigonometric are differentiable in their respective domains.
4. The sum, difference, product and quotient of two differentiable functions are respectively differentiable.
5. If f and g are two differentiable functions, then their composite function gof is also differentiable at every point in its domain.
6. Every differentiable function is continuous, however, the converse is not true, that is, every continuous function need not be differentiable.

GEOMETRICAL INTERPRETATION OF DIFFERENTIABILITY

Suppose $y = f(x)$ is a curve as shown in the figure and P be a point on the curve. The co-ordinates of P are $(a, f(a))$.
Let $Q\,(a+h,\ f(a+h))$ be a point on the curve on the right hand side of P and $R(a-h,\ f(a-h))$ be a point on the curve on the left hand side of P.



Slope of the chord $PQ = PQ = \dfrac{SQ}{SP} = \dfrac{f(a+h)-f(a)}{h}$

and, slope of the chord $PR = \dfrac{NP}{NR} = \dfrac{f(a-h)-f(a)}{-h}$

When the point Q tends to P, then the chord PQ becomes a tangent at P. When $h \to 0$, the point Q tends to P from the right hand side and the point R tends to P from the left hand side.

Hence, $\lim\limits_{Q\to P}$ (slope of chord PQ) = $\lim\limits_{R\to P}$ (slope of chord PR).

$$\Rightarrow \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

However, as per the definition, if $f(x)$ is differentiable at $x = a$, then

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

$\Rightarrow$ Slope of the tangent at P, which is limiting position of the chords drawn on the left side of P is the same as the slope of the tangent at P.

Hence $f(x)$ is differentiable at the point P, if and only if there exists a unique tangent at P.

MISCELLANEOUS SOLVED EXAMPLES

1. Show that $f(x)=|x|$ is not differentiable at $x=0$.

Sol. We have,

$$(\text{LHD at } x=0) = \lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}$$

$$\Rightarrow (\text{LHD at } x=0) = \lim_{h\to 0}\frac{f(0-h)-f(0)}{0-h-0}$$

$$\Rightarrow (\text{LHD at } x=0) = \lim_{h\to 0}\frac{|-h|-|0|}{-h} = \lim_{h\to 0}\frac{|-h|}{-h} = \lim_{h\to 0}\frac{h}{-h} = \lim_{h\to 0}(-1) = -1 \text{ and, } (\text{RHD at } x=0) = \lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}$$

$$\Rightarrow (\text{RHD at } x=0) = \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{|h|-|0|}{h} = \lim_{h\to 0}\frac{h}{h} = \lim_{h\to 0} 1 = 1$$

$\therefore$ (LHD at $x=0$) $\neq$ (RHD at $x=0$)

So, $f(x)$ is not differentiable at $x=0$.

2. If $f(x)$ is differentiable at $x=a$, find $\lim_{x\to a}\frac{x^2 f(a)-a^2 f(x)}{x-a}$

Sol. Since $f(x)$ is differentiable at $x=a$. Therefore,

$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists finitely.

Let $\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = f'(a)$ ...(i)

Now, $\lim_{x\to a}\frac{x^2 f(a)-a^2 f(x)}{x-a}$

$$= \lim_{x\to a}(x+a)f(a) - a^2\cdot\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$= 2a\cdot f(a) - a^2 f'(a)$$ [Using (i)]

3. Discuss the differentiability of $f(x)=x|x|$ at $x=0$.

Sol. We have,

$$f(x) = x|x| = \begin{cases} x^2, & x\ge 0 \\ -x^2, & x<0 \end{cases}$$

Now,

$$(\text{LHD at } x=0) = \lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}$$

$$\Rightarrow (\text{LHD at } x=0) = \lim_{x\to 0}(-x) = 0 \text{ and, } (\text{RHD at } x=0) = \lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}$$

$$\Rightarrow (\text{RHD at } x=0) = \lim_{x\to 0} x = 0$$

$\therefore$ (LHD at $x=0$) $=$ (RHD at $x=0$)

So, $f(x)$ is differentiable at $x=0$.

4. For the function f given by $f(x)=x^2-6x+8$, prove that $f'(5)-3f'(2)=f'(8)$.

Sol. Clearly, $f(x)$ being a polynomial function, is everywhere differentiable. The derivative of f at x is given by

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{\left\{(x+h)^2-6(x+h)+8\right\}-\left\{x^2-6x+8\right\}}{h}$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{2hx-6h+h^2}{h}$$

$$\Rightarrow \quad f'(x)=2x-6$$

$$\therefore \quad f'(5)-3f'(2)=(2\times5-6)-3(2\times2-6)=4+6=10$$

and, $\quad f'(8)=2\times8-6=10$

Hence, $\quad f'(5)-3f'(2)=f'(8)$

5. Find the derivative of cos x from first principle.

Sol. $f(x)=\cos x$

By first Principle, $f'(x)=\lim_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h}$

$$=\lim_{h\to 0}\frac{-2\sin\frac{x+(x+h)}{2}\sin\frac{x+h-x}{2}}{h}=\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$=\lim_{h\to 0}-\sin\left(x+\frac{h}{2}\right)\left(\frac{\sin\frac{h}{2}}{h/2}\right)=-\sin x \qquad \left[\because \text{As } h\to 0\left(\frac{\sin\frac{h}{2}}{h/2}\right)\to 1\right]$$

6. Find the derivative of 2tanx – 7 sec x

Sol. $\dfrac{d}{dx}\sec x=\sec x\tan x$

$$\frac{d}{dx}\tan x=\lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}=\lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}}{h}$$

$$=\lim_{h\to 0}\frac{\sin h}{h\cos x\cos(x+h)}=\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\frac{1}{\cos x\cos(x+h)}$$

$$=\frac{1}{\cos x\cos x}=\frac{1}{\cos^2 x}=\sec^2 x$$

$$\therefore \frac{d}{dx}(2\tan x-7\sec x)=2\sec^2 x-7\sec x\tan x$$

$$=\sec x(2\sec x-7\tan x)$$

7. Find the derivative of x^2-2 at $x=10$

Sol. Derivative of $f(x)$ at $x=a$ is given by $f'(a)=\lim\limits_{h\to0}\frac{f(a+h)-f(a)}{h}$.

$f(x)=x^2$ \$ 2 $\quad a=10$, $\therefore\ f(10+h)=(10+h)^2-2$

$f(10)=10^2$ \$ 2

$f(10+h)$ \$ $f(10)=[(10+h)+2]$ \$ $(10^2$ \$ $2)$

$=(10+h)^2$ \$ $10^2=h(20+h)$

$$\therefore\quad f(10)=\lim_{h\to0}\frac{f(10+h)-f(10)}{h}=\lim_{h\to0}\frac{h(20+h)}{h}=\lim_{h\to1}(20+h)=20$$

8. Find the derivative of the following functions from first principles $\cos\left(x-\frac{\pi}{8}\right)$.

Sol. Let $f(x)=\cos\left(x-\frac{\pi}{8}\right)$

$$\therefore f'(x)=\lim_{h\to1}\frac{\cos\left(x+h-\frac{\pi}{8}\right)-\cos\left(x-\frac{\pi}{8}\right)}{h}=\lim_{h\to0}\frac{-2\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$=\lim_{h\to0}-\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\left(\frac{\sin h/2}{h/2}\right)=-\sin\left(x-\frac{\pi}{8}\right)$$

9. Find the derivative of $(ax+b)(cx+d)^2$, where a, b, c, d are real constants.

Sol. Let $f(x)=(ax+b)(cx+d)^2$

To differentiate $(cx+d)^2$, put $cx+d=u$

$(cx+d)^2=u^2$

$$\therefore\quad \frac{d}{dx}(cx+d)^2=\frac{d}{dx}u^2$$

$$=2u\frac{du}{dx}=2(cx+d)\frac{d}{dx}(cx+d)$$

$$=2(cx+d)c=2c(cx+d)$$

Also $\frac{d}{dx}(uv)=u'v+uv'$

$$\therefore\quad f'(x)=\frac{d}{dx}(ax+b)(cx+d)^2=\left[\frac{d}{dx}(ax+b)\right](cx+d)^2+(ax+b)\frac{d}{dx}(cx+d)^2$$

$$=[a.(cx+d)^2+(ax+b).2c(cx+d)]$$

10. Find the derivative of $\frac{ax+b}{cx+d}$, where a, b, c, d are real constants.

Sol. Let $f(x)=\frac{ax+b}{cx+d}$. Now, $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$

$$\therefore\quad f'(x)=\frac{a.(cx+d)-(ax+b).c}{(cx+d)^2}=\frac{acx+ad-acx-bc}{(cx+d)^2}=\frac{ad-bc}{(cx+d)^2}$$

11. Find the derivative of $\frac{a}{x^4}-\frac{b}{x^2}+\cos x$, where a, b are real constants.

Sol. Let $f(x)=\frac{a}{x^4}-\frac{b}{x^2}+\cos x \Rightarrow f(x)=ax^{-4}-bx^{-2}+\cos x$

$\therefore \quad f'(x)=-4ax^{-5}-(-2)bx^{-3}-\sin x=-\frac{4a}{x^5}+\frac{2b}{x^3}-\sin x$

12. Find the derivative of cosec x cot x.

Sol. Let $f(x)=\operatorname{cosec} x \cot x$

$\therefore \quad f'(x)=\operatorname{cosec} x\frac{d}{dx}(\cot x)+\cot x\frac{d}{dx}(\operatorname{cosec} x)$ ...(i)

Now, $\frac{d}{dx}\operatorname{cosec} x=-\operatorname{cosec} x \cot x$

$$\frac{d}{dx}\cot x=\lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h}=\lim_{h\to 0}\frac{\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}}{h}=\lim_{h\to 0}-\left(\frac{\sin h}{h}\right)\frac{1}{\sin x\sin(x+h)}=\frac{1}{\sin^2 x}=-\operatorname{cosec}^2 x$$

Putting these values in (i)

$f'(x)=\operatorname{cosec} x.(-\operatorname{cosec}^2 x)+\cot x.(-\operatorname{cosec} x\cot x)$

$f'(x)=-\operatorname{cosec}^3 x-\operatorname{cosec} x\cot^2 x$

13. Find the derivative of $\frac{4x+5\sin x}{3x+7\cos x}$.

Sol. Let $f(x)=\frac{4x+5\sin x}{3x+7\cos x}$

$$\therefore \quad f'(x)=\frac{(4+5\cos x)(3x+7\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$$

$$=\frac{12x+28\cos x+15x\cos x+35\cos^2 x-(12x-28x\sin x+15\sin x-35\sin^2 x)}{(3x+7\cos x)^2}$$

$$=\frac{28(\cos x+x\sin x)+15(x\cos x-\sin x)+35}{(3x+7\cos x)^2}$$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. .... denotes the change in $f(x)$ at point 'a' with respect to x.
2. $y=\frac{dy}{dx}$ is referred to as .... of $f(x)$ with respect to x.
3. $\frac{d}{dx}[f(x)\cdot g(x)]=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$ is known as.............. rule.
4. Derivative of x^n is ....
5. Obtaining the derivative of a function from its basic definition is known as finding the derivative from ....
6. A function 'f' is said to be differentiable at a point 'c' in its domain $[a, b]$ only if left hand and right hand derivatives are finite and ....
7. $f(x)$ is differentiate at a point 'P', of there exists a unique .... at point 'P'.
8. Every polynomial function is .... at each $x \in R$.
9. If f is derivable in the open interval (a, b) and also at the end points a and b, then 'f' is said to be derivable in the ....
10. Derivative of $y=2x^5$ with respect to x is ....

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. The derivative of a real-valued function $f(x)$ at 'a' is defined by, $\lim\limits_{h\to 0}\frac{f(a+h)-f(a)}{h}$
2. Derivative of a constant function is always non-zero.
3. Derivative of $y=\cos x$ with respect to x is $\sin x$.
4. The process of finding derivative of a function is called differentiation.
5. Derivative of a function is defined, provided the limit exists.
6. A function is said to be differentiable in an interval (a, b), if it is differentiable at every point of (a, b).
7. Every constant function is differentiable at each $x \in R$.
8. Every continuous function is always differentiable.
9. If $y=2\sec x$, then $\frac{dy}{dx}$ is $2\sec x \cdot \tan x$.

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.*

1.

Column-I	Column-II
(A) $\sin x$	(p) $-4x^{-5}$
(B) x^{-4}	(q) $-2\sin x$
(C) $\frac{2}{\sec x}$	(r) $9x^2$
(D) $3x^3$	(s) $\cos x$
	(t) $9x^4$
	(u) $-\cos x$

2.

Column-I	Column-II
(A) $f(x)=x^2$	(p) not differentiable at $x=1$
(B) $f(x)=x^3-x^2+x+1$	(q) differentiable at $x=1$
(C) $f(x)=\begin{cases}x^2\sin\left(\frac{1}{x}\right) & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$	(r) not differentiable at $x=2$
(D) $f(x)=\begin{cases}x-1, & \text{if } x<2\\ 2x-3, & \text{if } x\geq 2\end{cases}$	(s) differentiable at $x=0$
	(t) differentiable at $x=2$

VSAQ Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Find the derivative of $99x$ at $x = 100$
2. Find the derivative of x at $x = 1$
3. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$ for some fixed real number.
4. For some constant a and b. Find the derivative of $(ax^2+b)^2$.
5. Find the derivative of the following functions from first principles.
   (i) $-x$ (ii) $\sin(x+1)$

DIRECTIONS : *Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers)*

6. $(x+a)$
7. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$
8. $4\sqrt{x}-\sqrt{2}$
9. Find the values of a and b so that the function
$$f(x)=\begin{cases} x^2+3x+a & , \text{if } x \le 1 \\ bx+2 & , \text{if } x > 1 \end{cases}$$
is differentiable at each $x \in R$.
10. If $f(x)=x^2+2x+7$, find $f'(3)$.
11. Find the derivative of $(x^2+1)\cos x$.

SAQ Short Answer Questions :

DIRECTIONS : *Give answer in 2-3 sentences.*

1. Find the derivative of $(-x)^{-1}$ from first principle.
2. Find the derivative of $\dfrac{ax+b}{px^2+qx+r}$
3. Find the derivative of $\dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x$
4. Find the derivative of $\dfrac{\cos x}{1+\sin x}$
5. Find the derivative of $\dfrac{px^2+qx+r}{ax+b}$
6. Find the derivative of $\dfrac{\sin x+\cos x}{\sin x-\cos x}$
7. Find the derivative of $\dfrac{\sec x-1}{\sec x+1}$
8. Find the derivative of $\dfrac{a+b\sin x}{c+d\cos x}$.
9. Find the derivative of $(x+\sec x)(x-\tan x)$.
10. Find the derivative of $\dfrac{x}{1+\tan x}$.
11. If $f(2)=4$ and $f'(2)=1$, then find $\lim\limits_{x\to 2}\dfrac{xf(2)-2f(x)}{x-2}$.

LAQ Long Answer Questions :

DIRECTIONS : *Give answer in four to five sentences.*

1. Find the derivatives of $(x-1)(x-2)$ from first principle.
2. Find the derivative of $f(x)=\tan(ax+b)$, by first principle.
3. Find the derivative of $f(x)=\sqrt{\sin x}$, by first principle.
4. If f is a real valued function defined by $f(x)=x^2+4x+3$, then find $f'(1)$ and $f'(3)$.
5. Prove that the function f given by $f(x)=|x-1|$, $x\in R$ is not differentiable at $x=1$.

2 EXERCISE

MCQ Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If $f(x)=x\sin x$, then $f'\left(\dfrac{\pi}{2}\right)$ is equal to
   (a) 0 (b) 1
   (c) 1 (d) $\dfrac{1}{2}$
2. Derivative of the function $f(x)=7x^{-3}$ is
   (a) $21x^{-4}$ (b) $-21x^{-4}$
   (c) $21x^4$ (d) $-21x^4$
3. If $y=3\cos x$, then $\dfrac{dy}{dx}$ at $x=\dfrac{\pi}{2}$ is
   (a) -3 (b) 3
   (c) 0 (d) -1

4. If $y = 2\sin x - 3x^4 + 8$, then $\frac{dy}{dx}$ is
(a) $2\sin x - 12x^3$ (b) $2\cos x - 12x^3$
(c) $2\cos x + 12x^3$ (d) $2\sin x + 12x^3$

5. Derivative of the function $f(x) = (x-1)(x-2)$ is
(a) $2x+3$ (b) $3x-2$
(c) $3x+2$ (d) $2x-3$

6. The set of the points where $f(x) = x|x|$ is twice differentiable, will be –
(a) R (b) R_0
(c) R^+ (d) R^-

7. If $f(x) = \sqrt{1-\sqrt{1-x^2}}$, then at $x = 0$,
(a) $f(x)$ is differentiable as well as continuous
(b) $f(x)$ is differentiable but not continuous
(c) $f(x)$ is continuous but not differentiable
(d) $f(x)$ is neither continuous nor differentiable

8. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals
(a) 3 (b) 4
(c) 5 (d) 6

9. Let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of all points where $f(x)$ is NOT differentiable is–
(a) $\{-1, 1\}$ (b) $\{-1, 0\}$
(c) $\{0, 1\}$ (d) $\{-1, 0, 1\}$

10. It is given that $f'(a)$ exists, then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ is equal to :
(a) $f(a) - af'(a)$ (b) $f'(a)$
(c) $-f'(a)$ (d) $f(a) + af'(a)$

11. If $f(x) = x[\sqrt{x} - \sqrt{x+1}]$, then :
(a) $f(x)$ is continuous but not differentiable at $x = 0$
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x)$ is differentiable at $x = 0$
(d) none of these

12. If $f(x) = \begin{cases} e^x + ax, & x < 0 \\ b(x-1)^2, & x \geq 0 \end{cases}$ is differentiable at $x = 0$, then (a, b) is
(a) $(-3, -1)$ (b) $(-3, 1)$
(c) $(3, 1)$ (d) $(3, -1)$

13. The value of the derivative of $|x-1| + |x-3|$ at $x = 2$ is :
(a) 2 (b) 1
(c) 0 (d) -2

14. Let $f(x+y) = f(x)f(y)\ \forall x, y \in R$, $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ equals :
(a) 4 (b) 1
(c) 1/2 (d) 6

More than One Correct :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following given statements is/are not correct?
(a) $\frac{d}{dx}(\text{cosec } x) = \text{cosec } x.\cot x.$
(b) $\frac{d}{dx}(\sec x) = \sec x.\tan x.$
(c) $\frac{d}{dx}(3\cot x) = -3\,\text{cosec}^2 x$
(d) $\frac{d}{dx}(2^\circ \tan x) = 2\sec^2 x$

2. Which of the following given statements is/are correct?
(a) If $Lf'(c) \neq Rf'(c)$, then $f(x)$ is not differentiable at $x = c$
(b) If a function is differentiable at a point, it is necessarily continuous at that point.
(c) If a function is differentiable at each $x \in R$ then it is said to be every where differentiable.
(d) $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$ where, c is a constant.

3. If $y = x^{-3/2}$, then $\frac{dy}{dx}$ is,
(a) $\frac{3}{2}x^{-5/2}$ (b) $\frac{9}{6}x^{-3/2}.x^4$
(c) $-\frac{3}{2}x^{-5/2}$ (d) $-\frac{9}{6}x^{3/2}.x^{-4}$

4. Derivative of the function $f(x) = x\sin x$ is
(a) $x\sin x + \cos x$
(b) $x\cos x + \sin x$
(c) $x\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)$
(d) $x\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)$

5. A function $f: R \to R$ satisfies $f(x) = f(2a-x)$. Suppose $f(x)$ is differentiable at $x = a$ then
(a) $f'(a) = 0$ (b) $f'(a^+) = f'(a^-)$
(c) $f'(a^+) = f'(a^-) = 0$ (d) None of these

6. Given $f(x) = \max\{1+x, 2, 1-x\}$ then
(a) $f(x)$ is continuous $\forall x$ except $x = \pm 1$
(b) $f(x)$ is differentiable $\forall x$ except $x = \pm 1$
(c) $f(x)$ is continuous $\forall x \in R$
(d) $f(x)$ is differentiable $\forall x \in R$

PBQ Passage Based Questions :

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

$$f(x)=\begin{cases} x^2+3x+a & \text{, for } x\le 1 \\ bx+2 & \text{, for } x>1 \end{cases}$$

is everywhere differentiable.

1. Value of constant 'a' is
   (a) 3 (b) 1/3
   (c) 5 (d) None of these
2. Value of constant 'b' is
   (a) 5 (b) 3
   (c) 1/5 (d) None of these
3. $f'(1)=?$
   (a) 3 (b) 5
   (c) 0 (d) None of these

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** Derivative of $\dfrac{x^n-a^n}{x-a}$ for some constant n is $\dfrac{(n-1)x^n - nax^{n-1}+a^n}{(x-a)^2}$
   Reason : $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{u'v-uv'}{v^2}$
   where u and v are two distinct functions.
2. **Assertion :** Derivative of $3\cot x+5\operatorname{cosec} x$ is $-\operatorname{cosec} x\,(3\operatorname{cosec} x+5\cot x)$
   Reason : $f'(a)=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$
3. **Assertion :** Derivative of $(px+q)\left(\dfrac{r}{x}+s\right)$ is $ps+\dfrac{qr}{x^2}$.
   Reason : $\dfrac{d}{dx}(uv)=u'v+uv'$
   where u and v are two distinct functions.
4. **Assertion :** Let $f(x)=x^2+7x+4$ be a polynomial function, then $f'(2)=11$.
   Reason : A polynomial function is differentiable everywhere.
5. **Assertion :** $f(x)=\begin{cases} x^2 & \text{, for } x\le c \\ ax+b & \text{, for } x>c \end{cases}$
   If $f(x)$ is differentiable at $x=c$, then $a=2c$ and $b=-c^2$.
   Reason : A continuous function is differentiable everywhere.

MMQ Multiple Matching Questions :

DIRECTIONS : *Following question has four statements (A, B, C and D) given in Column I and four statements (p, q, r and s) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

Column-I	Column-II
(A) $f'(x)=x^2-2$, $f'(x)$ at $x=10=?$	(p) $\dfrac{\cos a}{\cos^2 x}$
(B) $f(x)=\cos\left(x-\dfrac{\pi}{8}\right)$, $f'(x)=?$	(q) 20
(C) $f(x)=\dfrac{\sin(x+a)}{\cos x}$, $f(x)=?$	(r) $\dfrac{(1+\tan^2 x)}{\sec a}$
(D) $f(x)=(-x)^{-1}$, $f'(x)=?$	(s) $-\sin\left(x-\dfrac{\pi}{8}\right)$
	(t) $\dfrac{1}{x^2}$
	(u) $\sin\left(\dfrac{7\pi}{8}+x\right)$

HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Find the derivative of $f(x)=2x^2+3x-5$ at $x=-1$. Also, prove that $f'(0)+3f'(-1)=0$.
2. Differentiate the following functions with respect to x from first principles : $\dfrac{2x+3}{3x+2}$
3. Differentiate the following functions w.r.t. x : $\dfrac{\sin x+\cos x}{\sin x-\cos x}$

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. $f'(a)$
2. derivative
3. Product
4. nx^{n-1}
5. first principle
6. equal
7. tangent
8. Differentiable
9. [a, b]
10. $10x^4$

TRUE / FALSE

1. True
2. False
3. False
4. True
5. True
6. True
7. True
8. False
9. True

MATCH THE FOLLOWING :

1. (A) $\to$ s, (B) $\to$ p, (c) $\to$ q, (D) $\to$ r
2. (A) $\to$ q, (B) $\to$ p, (c) $\to$ s, (D) $\to$ r

VERY SHORT ANSWER QUESTIONS :

1. Derivative of $f(x)$ at $x = 100$ is

$$\lim_{h\to 0} = \frac{f(100+h)-f(100)}{h}$$

Now, $f(x) = 99x$

$f(100+h) = 99(100+h)$

$f(100) = 99 \times 100$

$\therefore \quad f(100+h) - f(100) = 99(100+h) - 99 \times 100$

$= 99[100 + h - 100] = 99 \times h$

$$\therefore \quad f(100) = \lim_{h\to 0}\frac{99h}{h} = 99$$

2. Derivative $f(x) = x$ at $x = 1$

$$f(1) = \lim_{h\to 0}\frac{f(1+h)-f(x)}{h} = \lim_{h\to 0}\frac{(1+h)-1}{h} = \lim_{h\to 0}\frac{h}{h} = 1$$

3. Let $f(x) = x^n + ax^{n-1} + a^2x^{n-2} + \ldots\ldots + a^{n-1}x + a^n$

Now, $\frac{d}{dx}x^n = nx^{n-1}$, $\frac{d}{dx}x^{n-1} = (n-1)x^{n-1}.etc$ and

$\frac{d}{dx}[ag(x)] = ag'(x)$, $\frac{d}{dx}a^n = 0$

$$f'(x) = nx^{n-1} + (n-1)ax^{n-2} + (n-2)a^2x^{n-3} + \ldots\ldots + a^{n-1}$$

4. $f(x) = (ax^2+b)^2 = a^2x^4 + 2abx^2 + b^2$

Now, $\frac{d}{dx}\left(x^4\right) = 4x^3$ and $\frac{d}{dx}\left(x^2\right) = 2x$, $\frac{d}{dx}\left(b^2\right) = 0$

$\therefore \quad f'(x) = a^2.4x^3 + 2ab.2x + 0 = 4a^2x^3 + 4abx$

5. (i) $f'(x) = -1$

(ii) $f'(x) = \cos(x+1)$

6. $\frac{d}{dx}x^n = nx^{n-1}$

$$\therefore \quad \frac{d}{dx}(x+a) = \frac{d}{dx}x + \frac{d}{dx}a = 1 + 0 = 1$$

7. Let $f(x) = \dfrac{1+\frac{1}{x}}{1-\frac{1}{x}} = \dfrac{x+1}{x-1}$

$$\therefore \quad f'(x) = \frac{1.(x-1)-(x+1).1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

8. Let $f(x) = 4\sqrt{x} + \sqrt{2} = 4x^{1/2} + \sqrt{2}$

$$\therefore \quad f'(x) = 4\left(\frac{1}{2}\right)x^{-1/2} + 0 = 2x^{-1/2}$$

$$f'(x) = \frac{2}{\sqrt{x}}$$

9. $a = 3, b = 5$

Hint : Use (LHD at $x = 1$) = (RHD at $x = 1$)

and $\lim\limits_{x\to 1^-} f(x) = \lim f(x) = \lim\limits_{x\to 1^+} f(x)$

10. We know that a polynomial function is everywhere differentiable. Therefore, $f(x)$ is differentiable at $x = 3$.

$$\therefore \quad f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$

$$\Rightarrow f'(3) = \lim_{h\to 0}\frac{\left\{(3+h)^2 + 2(3+h) + 7\right\} - \{9+6+7\}}{h}$$

$$\Rightarrow f'(3) = \lim_{h\to 0}\frac{8h+h^2}{h} = \lim_{h\to 0}(8+h) = 8$$

11. Let $f(x) = (x^2+1)\cos x$,

$\therefore f'(x) = 2x\cos x - (x^2+1)\sin x$

SHORT ANSWER QUESTIONS :

1. Let $f(x)=(-x)^{-1}=\frac{-1}{x}$

$\therefore\ f'(x)=\lim_{x\to 0}\frac{-\frac{1}{x+h}-\left(-\frac{1}{x}\right)}{h}=\lim_{h\to 0}\frac{-\frac{1}{x+h}+\frac{1}{x}}{h}$

$=\lim_{h\to 0}\frac{[x+h]-x}{h(x+h)x}=\lim_{h\to 0}\frac{h}{hx(x+h}$

$=\lim_{h\to 0}\frac{1}{x(x+h}=\frac{1}{x^2}$

2. Let $f(x)=\frac{ax+b}{px^2+qx+r}$

$f'(x)=\frac{a.(px^2+qx+r)-(ax+b)(2px+q)}{(px^2+qx+r)^2}$

$=\frac{-apx^2+2bpx+bq-ar}{(px^2+qx+r)^2}$

3. Let $f(x)=\frac{a}{x^4}-\frac{b}{x^2}+\cos x$

$\therefore\ f'(x)=-4ax^{-5}-(-2)bx^{-3}-\sin x=-\frac{4a}{x^5}+\frac{2b}{x^3}-\sin x$

4. Let $f(x)=\frac{\cos x}{1+\sin x}$

$\therefore\ f'(x)=\frac{-\sin x(1+\sin x)-\cos x.\cos x}{(1+\sin x)^2}$

$=\frac{-(\sin x+1)}{(1+\sin x)^2}=\frac{-1}{1+\sin x}$

5. Let $f(x)=\frac{px^2+qx+r}{ax+b}$

$f'(x)=\frac{(2px+q)(ax+b)-(px^2+qx+r)a}{(ax+b)^2}$

$=\frac{(2apx^2+2bpx+aqx+bq)-(apx^2+aqx+ar)}{(ax+b)^2}$

$=\frac{apx^2+2bpx+bq-ar}{(ax+b)^2}$

6. Let $f(x)=\frac{\sin x+\cos x}{\sin x-\cos x}$

$\therefore\ f'(x)=\frac{(\cos x-\sin x)(\sin x-\cos x)-(\sin x+\cos x)(\sin x+\cos x)}{(\sin x-\cos x)^2}$

$=\frac{-[(\sin x-\cos x)^2+(\sin x+\cos x)^2]}{(\sin x-\cos x)^2}$

$=\frac{-2}{(\sin x-\cos x)^2}$

7. Let $f(x)=\frac{\sec x-1}{\sec x+1}$

$\therefore\ f'(x)=\frac{\sec x\tan x(\sec x+1)-\sec x\tan x(\sec x-1)}{(\sec x+1)^2}$

$=\frac{2\sec x\tan x}{(\sec x+1)^2}$

8. Let $f(x)=\frac{a+b\sin x}{c+d\cos x}$,

$\therefore\ f'(x)=\frac{b\cos x(c+d\cos x)+d\sin x(a+b\sin x)}{(c+d\cos x)^2}$

$=\frac{bc\cos x+ad\sin x+bd(\cos^2 x+\sin^2 x)}{(c+d\cos x)^2}$

$=\frac{bc\cos x+ad\sin x+bd}{(c+d\cos x)^2}$

9. Let $f(x)=(x+\sec x)(x-\tan x)$

$\therefore\ f'(x)=(1+\sec x\tan x)(x-\tan x)+(x+\sec x)(1-\sec^2 x)$

$=(x+\sec x)(1-\sec^2 x)+(x-\tan x)(1+\sec x\tan x)$

10. Let $f(x)=\frac{x}{1+\tan x}$

$\therefore f'(x)=\frac{1(1+\tan x)-x.(\sec^2 x)}{(1+\tan x)^2}=\frac{1+\tan x-x\sec^2 x}{(1+\tan x)^2}$

11. We have,

$\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$

$=\lim_{x\to 2}\frac{xf(2)-2f(2)+2f(2)-2f(x)}{x-2}$

$=\lim_{x\to 2}\frac{(x-2)f(2)}{x-2}-2\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$

$=f(2)-2f'(2)\quad\left[\because f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}\right]$

$=4-2\times 1=2\quad[\because f(2)=4\text{ and }f'(2)=1]$

LONG ANSWER QUESTIONS :

1. Let $f(x)=(x-1)(x-2)$

$\therefore \quad f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$\Rightarrow \quad f'(x)=\lim\limits_{h\to 0}\dfrac{(x+h-1)(x+h-2)-(x-1)(x-2)}{h}$

$\Rightarrow \quad f'(x)=\lim\limits_{h\to 0}\dfrac{\{(x-1)+h\}\{(x-2)+h\}-(x-1)(x-2)}{h}$

$\Rightarrow \quad f(x)=\lim\limits_{h\to 0}\dfrac{(x-1)(x-2)+h(x-1)+h(x-2)+h^2-(x-1)(x-2)}{h}$

$\Rightarrow \quad f'(x)=\lim\limits_{h\to 0}\dfrac{h(x-1)+h(x-2)+h^2}{h}$

$=\lim\limits_{h\to 0}\{(x-1)+(x-2)\}+h=2x-3$

2. We have $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\tan(a(x+h)+b)-\tan(ax+b)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\sin(ax+ah+b)\cos(ax+b)-\sin(ax+b)\cos(ax+ah+b)}{h\cos(ax+b)\cos(ax+ah+b)}$

$=\lim\limits_{h\to 0}\dfrac{a}{\cos(ax+b)\cos(ax+ah+b)}$

$\lim\limits_{ah\to 0}\dfrac{\sin ah}{ah}$ [as $h\to 0,\ ah\to 0$]

$=\dfrac{a}{\cos^2(ax+b)}=a\sec^2(ax+b)$

3. By definition,

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\sqrt{\sin(x+h)}-\sqrt{\sin x}}{h}$

$=\lim\limits_{h\to 0}\dfrac{\left(\sqrt{\sin(x+h)}-\sqrt{\sin x}\right)+\left(\sqrt{\sin(x+h)}+\sqrt{\sin x}\right)}{h\left(\sqrt{\sin(x+h)}+\sqrt{\sin x}\right)}$

$=\lim\limits_{h\to 0}\dfrac{2\cos\left(\dfrac{2x+h}{2}\right)\sin\dfrac{h}{2}}{2\cdot\dfrac{h}{2}\left(\sqrt{\sin(x+h)}+\sqrt{\sin x}\right)}$

$=\dfrac{\cos x}{2\sqrt{\sin x}}=\dfrac{\cos x}{2\sqrt{\sin x}}\times\dfrac{\sqrt{\sin x}}{\sqrt{\sin x}}=\dfrac{1}{2}\cdot\dfrac{\cos}{\sin x}\sqrt{\sin x}$

$=\dfrac{1}{2}\cot x\sqrt{\sin x}$

4. We have,

$f(x)=x^2+4x+3$

$\therefore \quad f'(1)=\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$

$\Rightarrow \quad f'(1)=\lim\limits_{h\to 0}\dfrac{\left\{(1+h)^2+4(1+h)+3\right\}-\left\{1^2+4\times1+3\right\}}{h}$

$\Rightarrow \quad f'(1)=\lim\limits_{h\to 0}h+6=6$

and $f'(3)=\lim\limits_{h\to 0}\dfrac{f(3+h)-f(3)}{h}$

$\Rightarrow \quad f'(3)=\lim\limits_{h\to 0}\dfrac{\left\{(3+h)^2+4(3+h)+3\right\}-\left\{3^2+4\times3+3\right\}}{h}$

$\Rightarrow \quad f'(3)=\lim\limits_{h\to 0}h+10$

$\Rightarrow \quad f'(3)=10$

5. The given function may be written as ,

$f(x)=\begin{cases}x-1 & \text{, if } x\ge 1\\ 1-x & \text{, if } x<1\end{cases}$

R.H.D. at $x=1=\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$

$=\lim\limits_{h\to 0}\dfrac{[(1+h)-1]-(1-1)}{h}=\lim\limits_{h\to 0}\dfrac{h}{h}=1$

L.H.D. at $x=1=\lim\limits_{h\to 0}\dfrac{f(1-h)-f(1)}{-h}$

$=\lim\limits_{h\to 0}\dfrac{1-(1-h)-(1-1)}{-h}=\lim\limits_{h\to 0}\dfrac{h}{-h}=-1$

R.H.D. $\neq$ L.H.D. $\Rightarrow$ f is not differentiable at $x=1$.

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (b) As $f'(x)=x\cos x+\sin x$

So, $f'\left(\dfrac{\pi}{2}\right)=\dfrac{\pi}{2}\cos\dfrac{\pi}{2}+\sin\dfrac{\pi}{2}=1$

2. (b)

$f(x)=7(-3)x^{-3-1}=-21x^{-4}$.

3. (a) $\dfrac{dy}{dx}=-3\sin x$

$\left.\dfrac{dy}{dx}\right|_{x=\frac{\pi}{2}}=-3\sin\dfrac{\pi}{2}=-3$

4. (b)

5. (d) Applying product rule,

$$f'(x) = (x-1)\frac{d}{dx}(x-2) + (x-2)\frac{d}{dx}(x-1)$$

$= x-1+x-2 = 2x-3$

6. (b) $f(x) = \begin{cases} x^2 & , x \geq 0 \\ -x^2, & x < 0 \end{cases}$

$\Rightarrow f'(x) = 2x$, when $x > 0$ and $f'(x) = -2x$, when $x < 0$.
Also $f'(0+0) = 0$, $f'(0-0) = 0 \Rightarrow f'(0) = 0$

$$\therefore f'(x) = \begin{cases} 2x & , x > 0 \\ 0 & , x = 0 \\ -2x & , x < 0 \end{cases}$$

$$\Rightarrow f''(x) = \begin{cases} 2 & , x > 0 \\ -2, & x < 0 \end{cases}$$

Also $f''(0+0) = 2, f''(0-0) = -2 \Rightarrow f''(0)$ does not exist.
Hence $f(x)$ is twice differentiable in R_0

7. (c)

8. (c) $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

As function is differentiable, so it is continuous as it is given that $\lim_{h\to 0} \frac{f(1+h)}{h} = 5$ and hence $f(1) = 0$.

Hence $f'(1) = \lim_{h\to 0} \frac{f(1+h)}{h} = 5$

9. (d) $f(x) = \max.\{x, x^3\}$

$$= \begin{cases} x\ ; & x < -1 \\ x^3; & -1 \leq x \leq 0 \\ x\ ; & 0 \leq x \leq 1 \\ x^3; & x \geq 1 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 1\ ; & x < -1 \\ 3x^2; & -1 \leq x \leq 0 \\ 1\ ; & 0 \leq x \leq 1 \\ 3x^2; & x \geq 1 \end{cases}$$

Clearly f is not differentiable at -1, 0 and 1.

10. (a)

11. (c) A function $f(x)$ is said to be differentiable at $x = a$ if it's derivative $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists and left hand derivative is equal to right hand derivative.

Given : $f(x) = x[\sqrt{x} - \sqrt{x+1}]$

Since, given function is algebraic and every algebraic function is continuous.

$\therefore$ f (x) is continuous at $x = 0$.

Now, check for differentiability at $x = 0$

L.H.D. $Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0}$

Replacing 'x' by $(0-h)$ and 'a' by 0

$$= \lim_{h\to 0} \frac{-h[\sqrt{-h} - \sqrt{-h+1}]}{-h} = -1$$

R.H.D., $Rf'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$

Replacing x by $0+h$.

$$Rf'(0) = \lim_{h\to 0} \frac{h[\sqrt{h} - \sqrt{h+1}]}{h} = -1$$

Since $Lf'(0) = Rf'(0)$

$\therefore$ The function is differentiable at $x = 0$

12. (b)

13. (c) Given : $f(x) = |x-1| + |x-3|$

At $x = 2$, $|x-1| = x-1$

and $|x-3| = -x+3 \Rightarrow f(x) = x-1-x+3 = 2$

which is a constant function $\Rightarrow f'(2) = 0$

14. (d)

MORE THAN ONE CORRECT :

1. (a, d) 2. (a, b, c, d)

3. (c, d) $\frac{dy}{dx} = \frac{-3}{2}x^{(-3/2)-1} = \frac{-3}{2}x^{-5/2} = \frac{-3\times 3}{3\times 2}x^{(3/2)-4}$

$$= \frac{-9}{6}x^{3/2} \cdot x^{-4}$$

4. (b, c)

$$f'(x) = x\frac{d}{dx}(\sin x) + \sin x\frac{dx}{dx}$$

$= x\cos x + \sin x$

$$= x\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)$$

5. (a), (c) 6. (b, c)

PASSAGE BASED QUESTIONS :

1. (a) 2. (a) 3. (b)

ASSERTION & REASON :

1. (a) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$$\therefore \frac{d}{dx}\left(\frac{x^n - a^n}{x-a}\right) = \frac{\left[\frac{d}{dx}(x^n - a^n)\right](x-a) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$

$$= \frac{nx^{n-1}(x-a) - (x^n - a^n).1}{(x-a)^2} = \frac{nx^n - n.x^{n-1}a - x^n + a^n}{(x-a)^2}$$

$$= \frac{(n-1)x^n - nax^{n-1} + a^n}{(x-a)^2}$$

2. (a) Now, $\frac{d}{dx}(\cot x) = \lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h}$

$$= \lim_{h\to 0}\frac{\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}}{h}$$

$$= \lim_{h\to 0} -\left(\frac{\sin h}{h}\right)\cdot\frac{1}{\sin x.\sin(x+h)}$$

$$= -\frac{1}{\sin x \sin x}\lim_{h\to 0}\frac{\sin h}{h} = 1 = -\operatorname{cosec}^2 x$$

Also, $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$$\frac{d}{dx}(3\cot x + 5\operatorname{cosec} x) = -3\operatorname{cosec}^2 x - 5\operatorname{cosec} x \cot x$$

$= -\operatorname{cosec} x\,(3\operatorname{cosec} x + 5\cot x)$

3. (d) Let $f(x) = (px+q)\left(\frac{r}{x}+s\right)$

We have, $(uv) = u'v + uv'$

$$\therefore \quad f'(x) = \left[\frac{d}{dx}(px+q)\right]\left(\frac{r}{x}+s\right) + (px+q)\frac{d}{dx}\left(\frac{r}{x}+s\right)$$

$$= p\left(\frac{r}{x}+s\right) + (px+q)\left(\frac{-r}{x^2}\right)$$

$$= \frac{pr}{x} + ps - \frac{pr}{x} - \frac{qr}{x^2} = ps - \frac{qr}{x^2}$$

4. (a) We know that a polynomial function is everywhere differentiable. Therefore, $f(x)$ is everywhere differentiable. The derivative of f at x is given by,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\{(x+h)^2 + 7(x+h) + 4\} - \{x^2 + 7x + 4\}}{h}$$

$$\Rightarrow f'(2) = 2\times 2 + 7 = 11$$

5. (c) It is given that $f(x)$ is differentiable at $x = c$ and every differentiable function is continuous. So, $f(x)$ is continuous at $x = c$.

$$\therefore \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

$$\Rightarrow \lim_{x\to c} x^2 = \lim_{x\to c}(ax+b) = c^2 \quad \text{[Using def. of } f(x)]$$

$$\Rightarrow c^2 = ac + b \quad \text{...(i)}$$

Now, $f(x)$ is differentiable at $x = c$.

$\Rightarrow$ (LHD at $x = c$) = (RHD at $x = c$)

$$\Rightarrow \lim_{x\to c^-}\frac{f(x)-f(c)}{x-c} = \lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}$$

$$\Rightarrow \lim_{x\to c}\frac{x^2-c^2}{x-c} = \lim_{x\to c}\frac{(ax+b)-c^2}{x-c}$$

[Using def. of $f(x)$]

$$\Rightarrow \lim_{x\to c}(x+c) = \lim_{x\to c} a \quad \text{...(ii)}$$

From (i) and (ii), we get

$c^2 = 2c^2 + b \Rightarrow b = -c$

Hence, $a = 2c$ and $b = -c^2$.

MULTIPLE MATCHING QUESTIONS :

1. (A) $\to$ q; (B) $\to$ s, u; (c) $\to$ p, r; (D) $\to$ t

(A) $f'(x) = 2x$

$f'(10) = 2(10) = 20$

(B) $f'(x) = -\sin\left(x - \frac{\pi}{8}\right)$

$$\sin\left(\pi + x - \frac{\pi}{8}\right) = \sin\left(\frac{7\pi}{8} + x\right)$$

(C) $f'(x) = \frac{\cos x\cdot\cos(x+a) + \sin x\cdot\sin(x+a)}{\cos^2 x}$

$$= \frac{\cos(x+a-x)}{\cos^2 x} = \frac{\cos a}{\cos^2 x} = \sec^2 x\cdot\cos a$$

$$= \frac{(1+\tan^2 x)}{\sec a}$$

(D) $f'(x) = -(-1)x^{-1-1} = 1/x^2$

HOTS SUBJECTIVE QUESTIONS :

1. Let us first find the derivatives of $f(x)$ at $x = 0$ and $x = -1$

We have,

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$\Rightarrow f'(0) = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$\Rightarrow f'(0) = \lim_{h\to 0}\frac{(2h^2+3h-5) - \{2\times(0)^2 + 3\times(0) - 5\}}{h}$$

$$\Rightarrow f'(0) = \lim_{h\to 0}(2h+3) = 2\times 0 + 3 = 3$$

and, $f'(-1) = \lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h}$

$$\Rightarrow f'(-1) = \lim_{h\to 0}\frac{2h^2-h}{h} = \lim_{h\to 0}(2h-1) = 2\times 0 - 1 = -1$$

$$\therefore \quad f'(0) + 3f'(-1) = 3 + 3\times(-1) = 3 - 3 = 0$$

2. Let $f(x)=\dfrac{2x+3}{3x+2}$. Then, $f(x+h)=\dfrac{2(x+h)+3}{3(x+h)+2}$

$$\therefore \quad \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \quad \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{\dfrac{2(x+h)+3}{3(x+h)+2}-\dfrac{2x+3}{3x+2}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x))$$

$$=\lim_{h\to 0}\frac{(2x+3+2h)(3x+2)-(2x+3)(3x+2+3h)}{h(3x+2)(3x+2+3h)}$$

$$\Rightarrow \quad \frac{d}{dx}(f(x))$$

$$=\lim_{h\to 0}\frac{(2x+3)(3x+2)+2h(3x+2)-(2x+3)(3x+2)-3h(2x+3)}{h(3x+2)(3x+2+3h)}$$

$$\Rightarrow \quad \frac{d}{dx}(f(x))=\lim_{h\to 0}\frac{-5}{(3x+2)(3x+2+3h)}=-\frac{5}{(3x+2)^2}$$

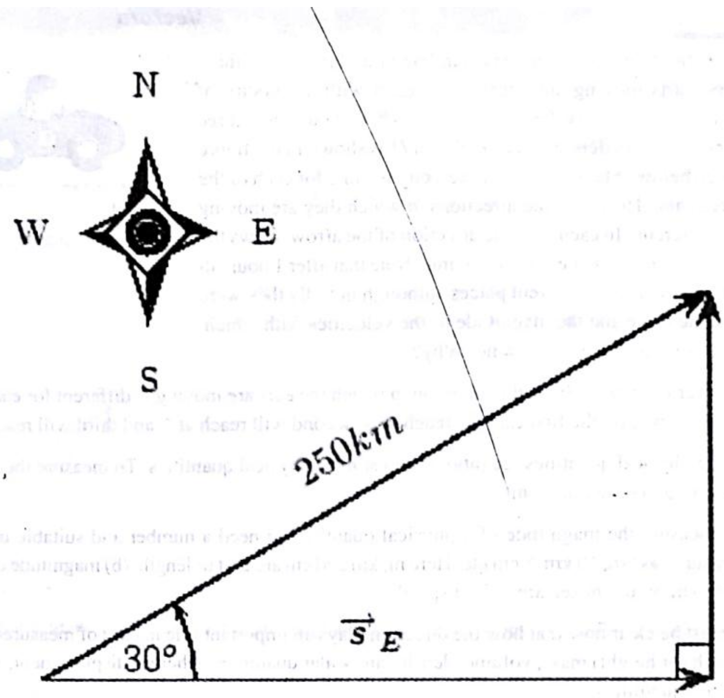
3. Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$$

$$=\frac{(\sin x-\cos x)\dfrac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\dfrac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$$=\frac{-\left[(\sin x-\cos x)^2+(\sin x+\cos x)^2\right]}{(\sin x-\cos x)^2}$$

$$=\frac{-2\left[\sin^2x+\cos^2x\right]}{\sin^2x+\cos^2x-2\sin x\cos x}=\frac{-2}{1-\sin 2x}$$



VECTORS

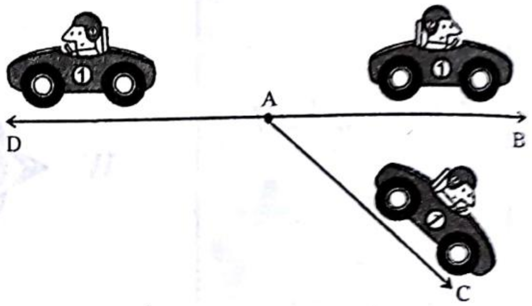
Introduction

There are two types of quantities. One type of quantities can be measured. For example : Length, mass, time, volume, speed, velocity, displacement acceleration, force etc. Other type of quantities can not be measured. For example: Happiness, sorrow etc. Those quantities which can be measured are called physical quantities.

The physical quantities can be divided into two categories – one which have only magnitude and other which have both magnitude and direction. The quantities which have only magnitude and are not related to any fixed direction are called scalar quantities or scalars. However, the quantities which have both magnitude and direction are known as vector quantities or simply vectors.

In this chapter, you will study clear difference between scalar and vector quantities. Representation of vector quantities, product of a scalar and vector quantity, sum and difference of two vector quantities.

Consider there are three cars standing at a place A. All three cars starts moving simultaneously, each with a velocity of 50 km. per hour in different directions. After 1 hour, these three cars reach at different places B, C and D as shown in the figure given below. Magnitude of the velocity is same for each of the three cars. However, the directions in which they are moving are different. In each case the direction of the arrow shows the direction in which the care is moving. Note that after 1 hour, all the cars will be at different places, although initially they were at same place and the magnitude of the velocities with which they are moving are also same. Why?



Answer is simple. Since the direction in which the cars are moving is different for each of them. As a consequence they will reach at different places; the first car will reach at B, second will reach at C and third will reach at D as shown in the figure.

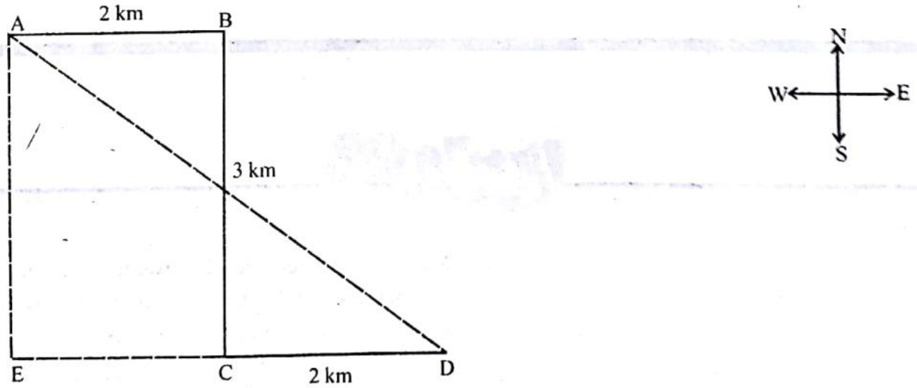
Some physical quantities are ratio of two similar physical quantities. To measure the magnitude of such physical quantities, we need only a number not any unit.

To measure the magnitude of a physical quantity, we need a number and suitable unit. For example : (a) magnitude of length are measured as 5m, 10 km, 8 cm etc. Here m, km and cm are unit of length. (b) magnitude of speed are measured as 40 km/hr, 25 m/sec. etc. Here km/hr and m/sec are unit of speed.

It must be clear now that how the direction plays an important role in case of measurement of some physical quantities. Temperature, length (or height) mass, volume, density are scalar quantities, whereas displacement, velocity, acceleration, force and momentum are vector quantities.

The following examples clearly illustrate the difference between the vectors and scalars:

Example 1: Suppose Ram walks 2 Kilometer towards East, then he walks 3 Kilometer towards south and from there he walks 2 Kilometer towards East. We draw a figure showing the movement of Ram. First he walks 2 Km from A to B, then he walks 3 Km from B to C and finally he walks 2 Km from C to D.



Total distance travelled by Ram is sum of AB, BC and CD, i.e. $2+3+2=7$ Kilometer. However, if we have to find the displacement, we have to find the change between the final and initial positions of Ram. Therefore, the displacement would be equal to AD, shown by dotted lines in the direction shown by the arrow. To find the distance AD, extend DC and drop perpendicular from A to DC to form the right triangle AED. Now using Pythagoras Theorem, we can find out the value of AD. In triangle AED.

$$AD^2 = AE^2 + ED^2$$

$$= 3^2 + 4^2 = 25$$

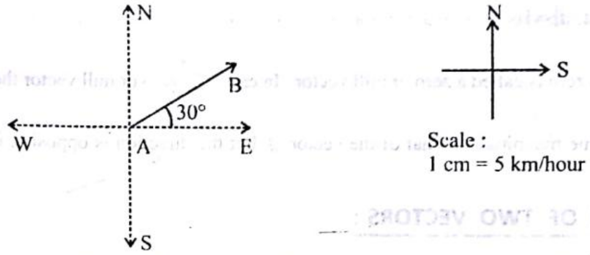
Hence, $AD = 5$ Km.

We can see that the total distance travelled by Ram is 7 Km., whereas the displacement is 5 Km. in the direction shown by arrow AD. Note that distance is a scalar quantity and it has no effect on it whether any direction is chosen. However, direction has to play an important role while calculating the displacement. Next example clarify this aspect.

Example 2: Suppose a person drives a car 10 Km. towards the west direction and returns from there to his initial position. Then the total distance travelled by him is 20 Km., whereas the displacement is zero. This is because displacement is a vector quantity and sense of direction is important in its expression, whereas the distance is a scalar quantity and only magnitude is required while expressing it.

GRAPHICAL REPRESENTATION OF VECTORS :

Vectors are represented graphically by arrows. The length of the arrow is proportional to the magnitude of the vector according to some suitable scale choosen. Hence length of the arrow represents the magnitude of the vector and its arrow specifies the direction of the vector.



In the above figure arrow AB of length 3cm represent velocity : 15 km/hour from east to north 30°. In the figure, A is called Tail (or initial point) and B is called Head (or terminal point).

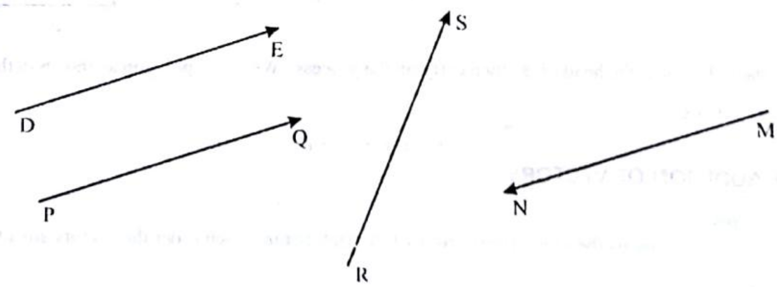
SYMBOLIC REPRESENTATION OF VECTORS :

A vector is denoted using two capital letters and an arrow above it. There is no space between the two letters. The first letter is the tail and the second letter is the head of the vector. The arrow above the two letters is from tail to head neither the length of arrow not represent the magnitude of the vector nor the arrow specifies the direction of the arrow. The arrow above the two letters only indicates that the physical quantity involved a direction. The vector represented graphically is the above topic. Graphical representation of a vector is represented symbolically as $\overrightarrow{AB}$. $|\overrightarrow{AB}|$ represent the magnitude of the vector $\overrightarrow{AB}$. Magnitude of a vector also represented by any single small letter. Let $|\overrightarrow{AB}| = \vec{a}$, then vector $\overrightarrow{AB} = \vec{a}$. Hence the vector represented graphically in the above topic can be represented by either $\overrightarrow{AB}$ or $\vec{a}$. $\overrightarrow{AB}$ and $\vec{a}$ are read as vector AB and vector a respectively. Sometimes we denoted the vectors by writing them in bold letters as **AB** or **a**.

EQUALITY OF VECTORS :

Two vectors $\vec{a}$ and $\vec{b}$ are equal if they are of same length and they are parallel to each other pointed in the same direction.

In the following figure, $\overrightarrow{DE}$ and $\overrightarrow{PQ}$ are two equal vectors. Note, that the magnitude and the directions of both the vectors $\overrightarrow{DE}$ and $\overrightarrow{PQ}$ are same. However, the vectors $\overrightarrow{DE}$ and $\overrightarrow{MN}$ are not same. Note that though the vectors $\overrightarrow{DE}$ and $\overrightarrow{MN}$ are same in magnitude and are parallel to each other, but their directions are opposite to each other. Hence $\overrightarrow{DE}$ and $\overrightarrow{MN}$ are not equal. Vector $\overrightarrow{DE}$ and $\overrightarrow{RS}$ are not equal because neither their magnitude nor their directions are same.



TYPES OF VECTORS :

Like and Unlike vectors:

Two vectors are said to be like, if they have same direction and unlike if they have opposite directions. For example, in figure given in the above topic : Equality of vectors, and $\overrightarrow{DE}$ and $\overrightarrow{PQ}$ are like vectors; whereas $\overrightarrow{DE}$ and $\overrightarrow{MN}$ are unlike vectors.

Collinear vectors:

Two or more vectors are said to be collinear if their directions are parallel, whether same or opposite, irrespective of their magnitudes. Both Like and Unlike vectors are collinear.

Unit Vector:

A vector whose magnitude is 1 or unity is called a unit vector. The unit vector in the direction of a vector $\vec{a}$ is denoted by $\hat{a}$ and is read as 'a cap'. It is clear that $|\hat{a}| = 1$.

Zero vector or Null vector:

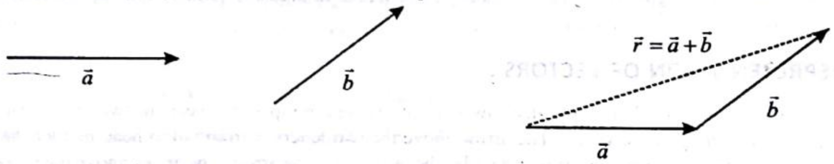
A vector whose magnitude is zero is called a zero or null vector. In case of a zero or null vector the head and tail coincide.

Negative of a vector:

The vector which has the same magnitude as that of the vector $\vec{a}$ but the direction is opposite, is called the negative of $\vec{a}$ and is denoted by $-\vec{a}$.

GRAPHICAL ADDITION OF TWO VECTORS :

Suppose we have to add the vector $\vec{b}$ to vector $\vec{a}$. We show the process of addition of two vectors in the below given figure.

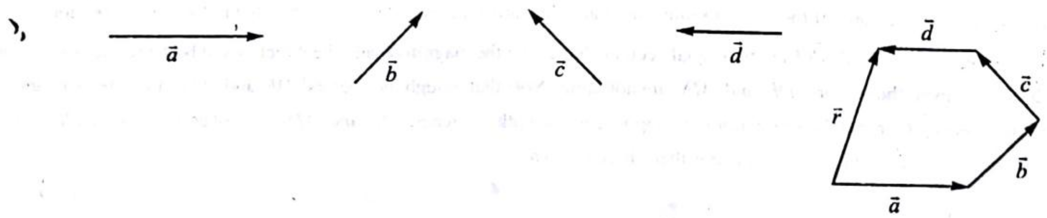


To find out $\vec{a} + \vec{b}$, first draw a line segment parallel to vector $\vec{a}$ of a length equal to that of $\vec{a}$ and put an arrow mark at one end as in $\vec{a}$. Draw a line segment parallel to $\vec{b}$ of length equal that of $\vec{b}$ and put an arrow mark as in vector $\vec{b}$. Now join the tail of new $\vec{a}$ to the head of new $\vec{b}$, as shown by dotted lines in the figure and put an arrow mark at the head of $\vec{b}$. This will denote the resultant vector $\vec{a} + \vec{b}$. We denote the resultant by $\vec{r}$. Thus addition of two vectors is again a vector and we write it as

$$\vec{r} = \vec{a} + \vec{b}$$

GRAPHICAL ADDITION OF MORE THAN TWO VECTORS :

The same process is followed for addition if more than two vectors are involved.



First draw $\vec{a}$, place the tail of $\vec{b}$ at the head of $\vec{a}$, then carry on the process. We find a polygon as shown in the figure. The resultant vector is shown by $\vec{r}$. Thus

$$\vec{r} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$$

PROPERTIES OF ADDITION OF VECTORS :

(i) **Commutative property :**

Addition of two vectors remains the same, irrespective of the fact that in which order the vectors are added.

i.e. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(ii) **Associative property :**
Addition of three vectors remain the same, irrespective of the fact that in which order the vectors are added.

i.e. $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$

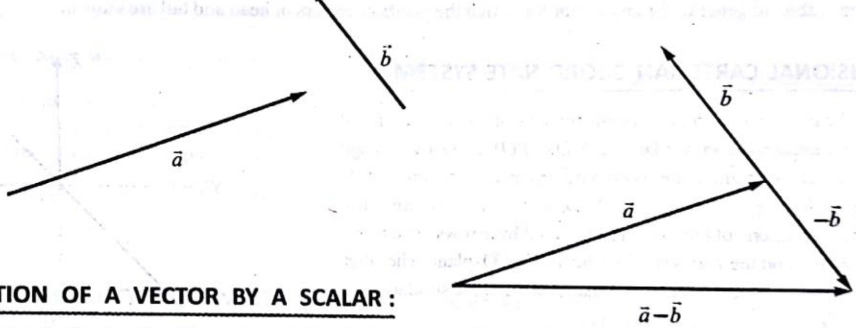
NOTE : Two vectors are called co-initial if their initial points are same.

GRAPHICAL SUBTRACTION OF VECTORS :

If $\vec{a}$ and $\vec{b}$ are two vectors and we have to subtract $\vec{b}$ from $\vec{a}$. We do the subtraction as follows :

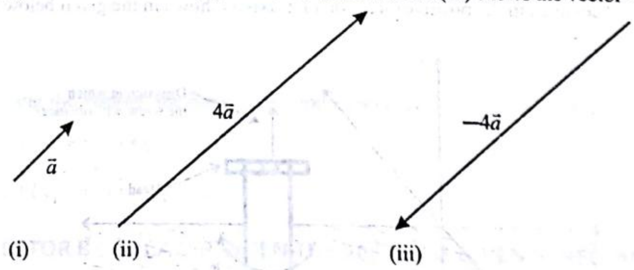
$\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$

Thus to subtract $\vec{b}$ from $\vec{a}$, we add $(-\vec{b})$ to $\vec{a}$ where $(-\vec{b})$ denotes the vector $\vec{b}$ taken in reverse direction. Figure no. ____ illustrates the subtraction of two vectors.



MULTIPLICATION OF A VECTOR BY A SCALAR :

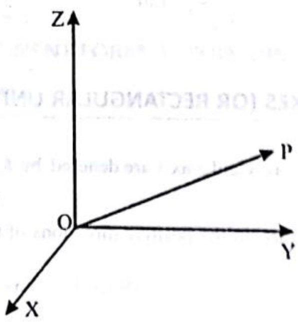
Let $\vec{a}$ be a vector and k be a scalar. The product of k and $\vec{a}$ represented by $k\vec{a}$ is a vector whose magnitude is equal to $|k|$ times the magnitude of original vector. Its direction will remain the same, if $k > 0$. However, the direction will become opposite, if $k < 0$. For example, in the figure below (i) shows the vector $\vec{a}$ (ii) shows the vector $4\vec{a}$ and (iii) shows the vector $-4\vec{a}$.



If $k=0$, then $k\vec{a}$ is the zero vector and represented by $\vec{O}$.

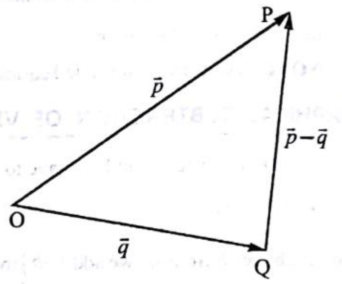
POSITION VECTOR OF A POINT :

If O is the origin and P is any point in the space, then $\overrightarrow{OP}$ is called the position vector of point P.



POINT TO BE REMEMBER :

If $\overline{PQ}$ is a vector with P as head and Q as tail of the vector. Suppose $\bar{P}$ that is the position vector of the point P and $\bar{q}$ is the position vector of the point Q. Then the vector $\overline{PQ}$ can be expressed in terms of the position vectors of its head and tail as follows :



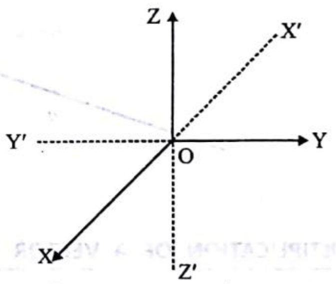
$\overline{PQ}$ = (Position vector of head Q) ş (Position vector of tail P)

= $\bar{p}$ ş $\bar{q}$

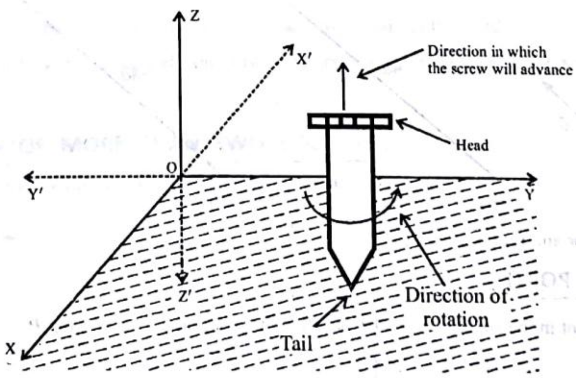
The formula is applicable, in general, for any vector for which the position vectors of head and tail are known.

THREE DIMENSIONAL CARTESIAN COORDINATE SYSTEM :

Three dimensional cartesian coordinate system consists of an arbitrary point O in space and three mutually perpendicular lines X'OX, $Y'OY$, and $Z'OZ$ through O. The point O is called the origin of the coordinate system and the lines $X'OX$, $Y'OY$, $Z'OZ$ are called respectively the X-axis, the Y-axis, and the Z-axis. The positive directions of the axes are indicated by arrows. The plane determined by the X-axis and the Y-axis is called the XOY or XY-plane. The other two planes YOZ (or YZ-plane) and XOZ (or XZ-plane) are defined similarly. These three planes are called the coordinate planes.



The positive direction of X, Y and Z-axis are such that if we place a screw perpendicular to the XY-plane so that tail of the screw lies in the XY-plane in between the positive direction of X and Y-axis and rotate the screw from positive direction of X-axis to the positive direction of Y-axis, the screw will advance in the positive direction of Z-axis as shown in the given below figure.

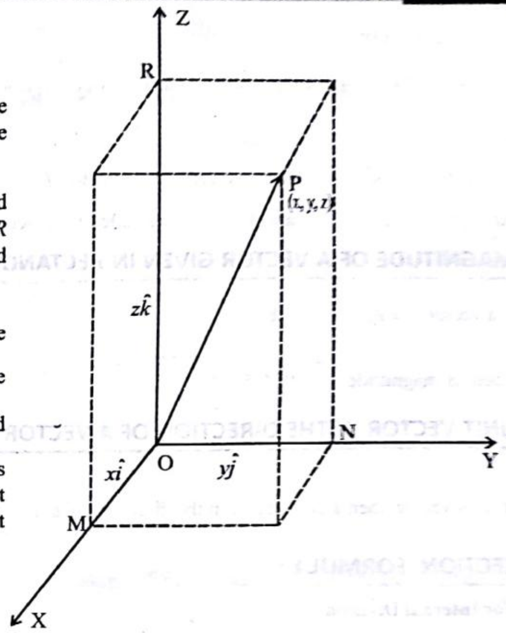


UNIT VECTORS IN THE DIRECTION OF AXES (OR RECTANGULAR UNIT VECTORS) :

If unit vectors in the directions of positive x-axis, y-axis and z-axis are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively, i.e. $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

Hence $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually perpendicular vectors, in the positive directions of the x-axis, y-axis and z-axis respectively such that

RECTANGULAR COMPONENT VECTORS OF A VECTOR :



Let $\overrightarrow{OP}$ be a vector such that the coordinates of the point P are (x, y, z). Since coordinate of P is (x, y, z). Hence, the perpendicular distance of the point $P(x, y, z)$ from YZ, ZX and XY-planes are x, y and z respectively. The distances x, y and z are called X, Y and Z-components of vector $\overrightarrow{OP}$.

In other words, if we draw the perpendicular from point $P(x, y, z)$ to X, Y and Z-axis and the perpendicular intersects the X, Y and Z-axis at M, N and R respectively then the length of OM, ON and OR from the origin are x, y and z respectively and are called X, Y and Z components of vector $\overrightarrow{OP}$.

When we multiply x, y and z-components by unit vector $\hat{i}$ (in the positive direction of X-axis), $\hat{j}$ (in the positive direction of Y-axis) and $\hat{k}$ (in the positive direction of z-axis). We get $x\hat{i}, y\hat{j}$ and $z\hat{k}$ respectively and are called X, Y and Z-components vectors of vector $\overrightarrow{OP}$. X, Y and Z-components vectors of any vectors are called rectangular component vectors of that vector. If we add three rectangular component vectors of any vector, we get that vector. Hence $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$ or, position vector of point P(x, y, z) $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$.

ADDITION AND SUBTRACTION OF VECTORS WHEN VECTORS ARE GIVEN IN RECTANGULAR COMPONENTS FORM :

Suppose $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are two vectors which are given in the rectangular components form as follow :

$$\overrightarrow{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\overrightarrow{OQ} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

Then, which adding or subtracting the vectors, the corresponding components are added or subtracted. Thus

$$\overrightarrow{OP} + \overrightarrow{OQ} = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k}$$

$$\overrightarrow{OP} - \overrightarrow{OQ} = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j} + (z_1 - z_2)\hat{k}$$

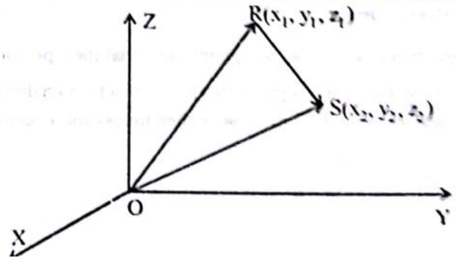
MULTIPLICATION OF A VECTOR BY A SCALAR WHEN THE VECTOR IS GIVEN IN RECTANGULAR COMPONENT FORM :

In case of multiplication of the vector by a scalar, every component is multiplied by the scalar. Therefore,

$$\overrightarrow{OP} = cx_1\hat{i} + cy_1\hat{j} + cz_1\hat{k}$$

where c is a constant or a scalar.

VECTOR IN THEIR RECTANGULAR COMPONENT FORM, IF COORDINATES OF HEAD AND TAIL ARE GIVEN :



If R(x_1, y_1, z_1) and S(x_1, y_1, z_1) are two points in space.
Then position vector of point R.

$$\overrightarrow{OR} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

And position vector of point S,

$$\overrightarrow{OS} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

Now $\overrightarrow{RS}$ = (Position vector of head S) – (Position vector of tail R)

$= \overrightarrow{OS} - \overrightarrow{OR}$

$= \left(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}\right) - \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right) = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Here $(x_2 - x_1)\hat{i}, (y_2 - y_1)\hat{j}$ and $(z_2 - z_1)\hat{k}$ are called x, y and z component vectors respectively of vector $\overrightarrow{RS}$.

And $(x_2 - x_1), (y_2 - y_1)$ and $(z_2 - z_1)$ are called x, y, z-component of vector $\overrightarrow{RS}$.

MAGNITUDE OF A VECTOR GIVEN IN RECTANGULAR COMPONENT FORM :

If a vector $\vec{r} = r_1\hat{i} + r_2\hat{j} + r_3\hat{k}$

Then its magnitude, $|\vec{r}|$ or $r = \left|\sqrt{r_1^2 + r_2^2 + r_3^3}\right|$

UNIT VECTOR IN THE DIRECTION OF A VECTOR :

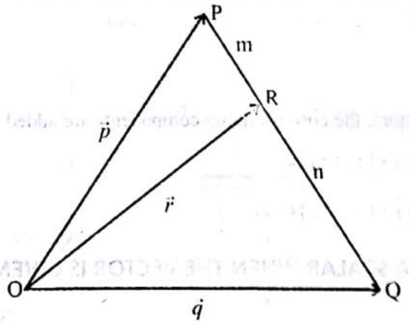
If $\vec{a}$ is vector, then a unit vector in the direction of $\vec{a}$ is $\frac{\vec{a}}{|\vec{a}|}$

SECTION FORMULA :

For Internal Division :

Suppose that P and Q be two points such that their position vectors are $\vec{P}$ and $\vec{q}$ respectively, and suppose that R be a point which divides PQ internally in the ratio m : n, then the position vectors of R is given by

$$\overrightarrow{OR} = \vec{r} = \frac{m\vec{q} + n\vec{p}}{m + n}.$$



Remark : if $m = n$, then R will be mid point of PQ. Setting m = n in the above formula, we get the formula to find out the position vector of mid point R of PQ.

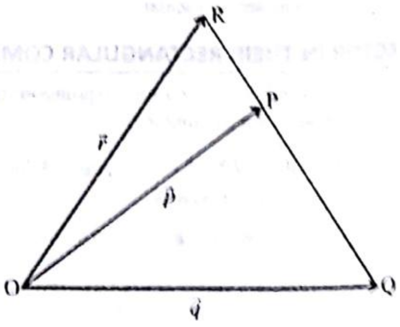
$$\overrightarrow{OR} = \vec{r} = \frac{\vec{q} + \vec{p}}{2}$$

This is called mid-point formula.

For External Division :

Suppose that P and Q be two points such that their position vectors are $\vec{P}$ and $\vec{q}$ respectively and suppose that R be a point which divides PQ externally in the ratio $m : n$ i.e. $PR : RQ = m : n$ then the position vector of R is given by

$$\overrightarrow{OR} = \vec{r} = \frac{m\vec{q} - n\vec{p}}{m - n}$$



MISCELLANEOUS SOLVED EXAMPLES

1. Compute the magnitude of the following vectors:

$\vec{a} = \hat{i}+\hat{j}+\hat{k},\ \vec{b} = 2\hat{i}-7\hat{j}-3\hat{k},\ \vec{c} = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$

Sol. (i) $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $|\vec{a}| = \sqrt{1^2+1^2+1^2}$ $\left[\because |x\hat{i}+y\hat{j}+z\hat{k}| = \sqrt{x^2+y^2+z^2}\right]$

$\therefore\ |\vec{a}| = \sqrt{3}$

(ii) $\vec{b} = 2\hat{i}-7\hat{j}-3\hat{k}$ $\therefore\ |\vec{b}| = \sqrt{2^2+(-7)^2+3^2} = \sqrt{4+49+9} = \sqrt{62}$

(iii) $\vec{c} = \frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}-\frac{1}{\sqrt{3}}\hat{k}$, $|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{-1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \sqrt{\frac{3}{3}} = 1$

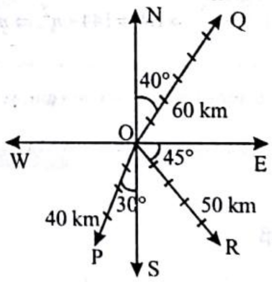
2. Represent graphically (i) a displacement of 40 km, 30° west of south, (ii) 60 km, 40° east of north (iii) 50 km south-east.

Sol. (i) The vector $\overline{OP}$ represents the required displacement vector.

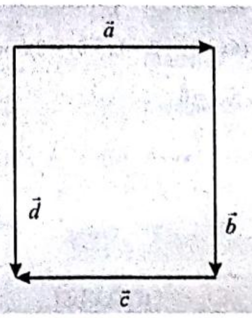
(ii) The vector $\overline{OQ}$ represents the required vector.

(iii) $\overrightarrow{OR}$ represents the required vector.

Scale :
1 smaller division = 10 km.



3. In fig. (a square), identify the following vectors :
(i) Coinitial
(ii) Equal
(iii) Collinear but not equal



Sol. Clearly,

(i) $\vec{a}, \vec{d}$ are co-initial vectors

(ii) $\vec{d}$ and $\vec{b}$ are equal vectors

(iii) $\vec{a}$ and $\vec{c}$ are collinear but not equal vectors.

4. If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B respectively, then find the position vector of points of trisection of AB.

Sol. Let P and Q be points of trisection of AB. Then, $AP = PQ = QB = \lambda$ (say).
Since P divides AB in the ratio $\lambda : 2\lambda$ i.e. $1 : 2$.

$\therefore$ Position vector of $P = \dfrac{1\cdot\vec{b} + 2\cdot\vec{a}}{1+2} = \dfrac{\vec{b}+2\vec{a}}{3}$

$\therefore$ Position vector of $Q = \dfrac{\dfrac{\vec{b}+2\vec{a}}{3}+\vec{b}}{2} = \dfrac{4\vec{b}+2\vec{a}}{6} = \dfrac{\vec{a}+2\vec{b}}{3}$

5. If the position vector $\vec{a}$ of point $(12, n)$ is such that $|\vec{a}| = 13$, find the value of n.

Sol. The position vector of the point $(12, n)$ is $12\hat{i} + n\hat{j}$

$\therefore$ $|\vec{a}| = 12\hat{i} + n\hat{j}$

$|\vec{a}| = \sqrt{12^2 + n^2}$

Now, $|\vec{a}| = 13$

$\Rightarrow$ $13 = \sqrt{12^2 + n^2} \Rightarrow 169 = 144 + n^2 \Rightarrow n^2 = 25 \Rightarrow n = \pm 5$

6. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors and vectors $\vec{\alpha} = (x+4y)\vec{a} + (2x+y+1)\vec{b}$ and $\vec{\beta} = (-2x+y+2)\vec{a} + (2x-3y-1)\vec{b}$ are connected by the relation $3\vec{\alpha} = 2\vec{\beta}$, find x, y.

Sol. We have,

$3\vec{\alpha} = 2\vec{\beta}$

$\Rightarrow$ $3\{(x+4y)\vec{a} + (2x+y+1)\vec{b}\} = 2\{(-2x+y+2)\vec{a} + (2x-3y-1)\vec{b}\}$

$\Rightarrow$ $(3x+12y+4x-2y-4)\vec{a} + (6x+3y+3-4x+6y+2)\vec{b} = \vec{0}$

$\Rightarrow$ $(7x+10y-4)\vec{a} + (2x+9y+5)\vec{b} = \vec{0}$

$\Rightarrow$ $7x+10y-4 = 0$ and $2x+9y+5 = 0$

Solving the above equations,

$\Rightarrow$ $x = 2,\ y = -1.$

1 EXERCISE

Fill in the Blanks :

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

1. Quantities having only magnitude, but no fixed direction are known as _________.
2. Vectors quantities have _______ magnitude and direction.
3. Displacement, velocity, acceleration etc. are examples of __________ quantities.
4. Measure of length of a vector $\vec{a}$, denoted by $|\vec{a}|$ is known as ________ of the vector.
5. A vector whose modulus is unity is called a ________.
6. Vector $\overline{AB}$ having A and B as its initial and terminal points, respectively, is called the ________ of the point A with respect to B.
7. A vector whose initial and terminal points coincide is called a _____ vector.
8. A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called ____ of the given vector.
9. Two or more vectors are said to be _____ if they are parallel to the same line.
10. Three vectors are ______, if one of them can be expressed as a linear combination of the other two.

True / False :

DIRECTIONS : *Read the following statements and write your answer as true or false.*

1. Mass, volume, temperature etc. are examples of scalar quantities.
2. The point A from where the vector $\overline{AB}$ starts is called its terminal point.
3. For every vector $\vec{a}$, $\vec{a}+\vec{O}=\vec{a}$ where $\vec{O}$ is the null vector.
4. For any two vectors $\vec{a}$ and $\vec{b}$, $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
5. If a is vector, then $\vec{a}=|\vec{a}|.\frac{1}{\vec{a}}$
6. For any vector $\vec{a}, O\vec{a}=\vec{O}$
7. If a point P in a plane has coordinates (x, y), then $\overline{OT}=x\hat{i}+y\hat{j}$
8. If two vectors $\vec{a}$ and $\vec{b}$ are represented by two sides of a triangle, then their sum is given by triangle law of addition of vectors.
9. Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively, and let C be a point dividing AB internally in the ratio $m:n$. then the position vector of C is given by,
$$\overline{OC}=\frac{m\vec{b}-n\vec{a}}{m-n}$$
10. If a point P in a plane has coordinates (x, y), then $|\overline{OP}|=\sqrt{x^2+y^2}$

Match the Following :

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D....) in column I have to be matched with statements (p, q, r, s....) in column II.*

1.

Column-I	Column-II
(A) Reciprocal vector	(p) $-\vec{a}$
(B) Negative vector	(q) $\vec{a}^{-1}$
(C) Triangle law of addition	(r) $O\vec{a}$
(D) Null vector	(s) $\vec{a}+\vec{b}$
	(t) $\vec{a}-\vec{b}$
	(u) $O+\vec{a}$

2.

Column-I	Column-II
(A) Unit vector	(p) $|\vec{a}|$ or $|\vec{b}|$
(B) Modulus	(q) $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
(C) Commutative law	(r) $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$
(D) Associative law	(s) $\frac{\vec{a}}{|\vec{a}|}$ or $\frac{\vec{b}}{|\vec{b}|}$

Very Short Answer Questions :

DIRECTIONS : *Give answer in one word or one sentence.*

1. Represent graphically a displacement of 40km, 30° east of north.
2. Classify the following as scalar and vector quantities
(i) time period (ii) distance (iii) force (iv) velocity (v) work.
3. In a square, identify the following vectors (i) Co-initial (ii) Equal (iii) collinear but not equal

4. Find the scalar and vector components of the vector with initial point (2,1) and terminal point (–5, 7).

5. Find the unit vector in the direction of the vector $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$.

6. Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

7. Find the scalar components and magnitude of the vector joining the points P (x_1, y_1, z_1) and $Q\ (x_2, y_2, z_2)$.

8. Find the value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

9. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

10. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, prove that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, provided $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

11. If $A\equiv(2\hat{i}+3\hat{j}), B\equiv(p\hat{i}+9\hat{j})$ and $C\equiv(\hat{i}-\hat{j})$ are collinear, then find the value of p.

12. If $\overline{PO}+\overline{OQ}=\overline{QO}+\overline{OR}$, show that the points P, Q, R are collinear.

13. A, B, P, Q and R are five points in a plane. Show that the sum of the vectors $\overrightarrow{AP}, \overrightarrow{AQ}, \overrightarrow{AR}, \overrightarrow{PB}, \overrightarrow{QB}$ and $\overrightarrow{RB}$ is $3\,\overrightarrow{AB}$.

14. The position vectors of points A, B, C, D are $\vec{a},\vec{b},2\vec{a}+3\vec{b}$ and $\vec{a}-2\vec{b}$ respectively. Show that $\overrightarrow{DB}=3\vec{b}-\vec{a}$ and $\overrightarrow{AC}=\vec{a}+3\vec{b}$

15. Find the position vectors of the points which divide the join of the points $2\vec{a}-3\vec{b}$ and $3\vec{a}-2\vec{b}$ internally and externally in the ratio 2 : 3.

16. Find the values of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.

17. Let O be the origin and let $P(-4, 3)$ be a point in the xy-plane. Express $\overline{OP}$ in terms of vector $\hat{i}$ and $\hat{j}$. Also, find $|\overline{OP}|$.

18. If the position vector $\vec{a}$ of the point (5, n) is such that $|\vec{a}|=13$, find the value of n.

19. Find a unit vector parallel to the vector $-3\hat{i}+4\hat{j}$.

20. Find the magnitude of the vector $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$

SAQ *Short Answer Questions :*

DIRECTIONS : *Give answer in two to three sentences.*

1. Find the unit vector in the direction of vector $\overline{PQ}$. Where P and Q are the points (1, 2, 3) and (4,5,6) respectively.

2. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ.

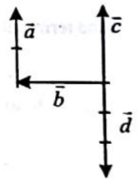
3. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non zero vectors, no two of which are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$, and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ then evaluate $\vec{a}+2\vec{b}+6\vec{c}$.

4. Find the position vector of a point R which divides the line joining the points whose positive vector are $P(\hat{i}+2\hat{j}-\hat{k})$ and $Q(-\hat{i}+\hat{j}+\hat{k})$ in the ratio 2 : 1 externally.

5. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

6. Classify the following measures as scalars and vectors
   (i) 10 kg (ii) 10 meters north-west
   (iii) 10 Newton (iv) 30 km/hr
   (v) 50 m/ sec towards north
   (vi) 10^{-19} cloumb.

7. In fig. which of the vectors are :



   (i) Collinear (ii) Equal (iii) Co-initial

8. If $\vec{a},\vec{b},\vec{c}$ be the vectors represented by the sides of a triangle, taken in order, then prove that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$.

9. If D is the mid-point of the side BC of a triangle ABC, prove that $\overline{AB}+\overline{AC}=2\overline{AD}$.

LAQ *Long Answer Questions :*

DIRECTIONS : *Give answer in four to five sentences.*

1. Show that the points A, B and C with position vector $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$, respectively form the vectices of a right angled triangle.

2. Show that the points A (1, 2, 7), B (2, 6, 3) and C(3, 10, –1) are collinear.

3. Show that the points A (1,– 2,– 8), B (5,0,–2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

4. Find the coordinates of the tip of the position vector which is equivalent to $\overline{AB}$, where the coordinates of A and B are (3, 1) and (5, 0) respectively.

2 EXERCISE

Multiple Choice Questions :

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. If C is the middle point of AB and P is any point outside AB, then

(a) $\overline{PA} + \overline{PB} = \overline{PC}$ (b) $\overline{PA} + \overline{PB} = 2\overline{PC}$
(c) $\overline{PA} + \overline{PB} + \overline{PC} = \bar{0}$ (d) $\overline{PA} + \overline{PB} + 2\overline{PC} = \bar{0}$

2. $ABCD$ is a parallelogram whose diagonals meet at P. If O is a fixed point, then $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD}$ equals

(a) $\overline{OP}$ (b) $2\overline{OP}$
(c) $3\overline{OP}$ (d) $4\overline{OP}$

3. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then $ABCD$ is a

(a) rectangle (b) square
(c) rhombus (d) None of these

4. A unit vector parallel to the sum of the vectors $\vec{a} = 2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$ is

(a) $\dfrac{3\hat{i}+6\hat{j}+2\hat{k}}{5}$ (b) $\dfrac{-3\hat{i}+6\hat{j}-2\hat{k}}{7}$
(c) $\dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$ (d) none of these

5. In a triangle ABC three forces of magnitudes $3\overrightarrow{AB}$, $2\overrightarrow{AC}$ and $6\overrightarrow{CB}$ are acting along the sides AB, AC and CB respectively. If the resultant meets AC at D, then the ratio $DC : AD$ will be equal to :

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

6. In triangle ABC, which of the following is not true?

(a) $\overline{AB} + \overline{BC} + \overline{CA} = \bar{0}$ (b) $\overline{AB} + \overline{BC} - \overline{AC} = \bar{0}$
(c) $\overline{AB} + \overline{BC} - \overline{CA} = \bar{0}$ (d) $\overline{AB} - \overline{CB} + \overline{CA} = \bar{0}$

7. If $\vec{a}$ and $\vec{b}$ are two collinear vectors then which of the following is incorrect?

(a) $\vec{b} = \lambda\vec{a}$ for some scalar
(b) $\vec{a} = \pm\vec{b}$
(c) the respective components of $\vec{a}$ and $\vec{b}$ are proportional
(d) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitude.

8. If $\bar{a}$ is a non-zero vector of magnitude a and λ a non-zero scalar, then $\lambda\ \bar{a}$ is a unit vector if

(a) $\lambda = 1$ (b) $\lambda = -1$
(c) $a = |\lambda|$ (d) $a = \dfrac{1}{|\lambda|}$

9. Let $\vec{A} = (x+4y)\ \vec{a} + (2x+y+1)\ \vec{b}$ and $\vec{B} = (y-2x+2)\ \vec{a} + (2x-3y-1)\ \vec{b}$ where $\vec{a}$ and $\vec{b}$ are non collinear vectors, if $3\ \vec{A} = 2\ \vec{B}$; then

(a) $x=1, y=2$ (b) $x=2, y=1$
(c) $x=2, y=-1$ (d) $x=-1, y=2$

10. The vector $\vec{a} = \alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i}+\hat{j}$ and $\vec{c} = \hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(a) $\alpha=2, \beta=2$ (b) $\alpha=1, \beta=2$
(c) $\alpha=2, \beta=1$ (d) $\alpha=1, \beta=1$

11. If the position vectors of the vertices A, B, C of a triangle ABC are $7\hat{j}+10\hat{k}, -\hat{i}+6\hat{j}+6\hat{k}$ and $-4\hat{i}+9\hat{j}+6\hat{k}$ respectively, the triangle is :

(a) equilateral
(b) isosceles
(c) scalene
(d) right angled and isosceles also

12. Forces $3\overline{OA}, 5\overline{OB}$ act along OA and OB. If their resultant passes through C on AB, then

(a) C is a mid-point of AB
(b) C divides AB in the ratio 2 : 1
(c) $3AC = 5CB$
(d) $2AC = 3CB$

13. If points $A(60\hat{i}+3\hat{j}), B(40\hat{i}+8\hat{j})$ and $C(a\hat{i}+52\hat{j})$ are collinear, then a is equal to

(a) 40 (b) −40
(c) 20 (d) −20

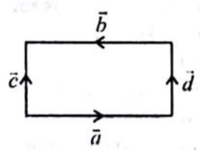
14. The vector $\cos\alpha\cos\beta\hat{i}+\cos\alpha\sin\beta\hat{j}+\sin\alpha\hat{k}$ is a

(a) null vector (b) unit vector
(c) constant vector (d) none of these

MTOC More than One Correct

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d). Out of which ONE OR MORE may be correct.*

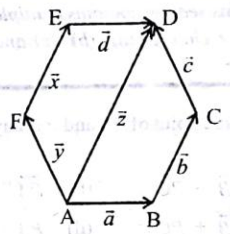
1. Which of the given below physical quantities are vectors?
   (a) displacement (b) acceleration
   (c) time period (d) velocity
2. Among the following given options, identify the scalar quantities
   (a) distance (b) work
   (c) force (d) momentum
3. Which among the below given options is/are correct?
   (a) Scalar quantities have both magnitude as well as direction.
   (b) Two vectors are always coplanar.
   (c) A vector whose modulus is unity is called a unit vector.
   (d) A vector whose initial and terminal points are cooincident is called a null vector.
4. Which among the below given options is/are correct?
   (a) $m(-\vec{a}) = -(m\vec{a})$
   (b) $m(\vec{a}+\vec{b}) = m\vec{a}+m\vec{b}$
   (c) $(m+n)\vec{a} = m\vec{a}+n\vec{a}$
   (d) $(-m)(-\vec{a}) = -m\vec{a}$
   where $\vec{a}$ is a vector and m is a scalar
5. Which among the following given options is/are not false?
   (a) $\vec{a}$ and $-2\vec{a}$ are collinear.
   (b) If t is a scalar, then $\vec{a}$ and $t\vec{a}$ are collinear
   (c) $|-2\vec{a}| = -2|\vec{a}|$
   (d) Magnitude of $-3\vec{a}$ is thrice that of $\vec{a}$.
6. Select the statements which are false?
   (a) Two collinear vectors are always equal in magnitude.
   (b) Two vectors having same magnitude are collinear
   (c) Zero vector is unique
   (d) Two collinear vectors having the same magnitude are equal
7. In the given figure, which vectors are collinear?
   (a) $\vec{c}$ and $\vec{d}$
   (b) $\vec{a}$ and $\vec{b}$
   (c) $\vec{a}$ and $\vec{c}$
   (d) $\vec{a}$ and $\vec{d}$



PBQ Passage Based Questions :

DIRECTIONS : *Study the given passage(s) and answer the questions given below each passage.*

Passage- I



In the above figure, which vectors are

1. Collinear
   (a) $\vec{a}$ and $\vec{d}$ (b) $\vec{b}, \vec{x}$ and $\vec{z}$
   (c) $\vec{c}, \vec{y}$ and $\vec{z}$ (d) $\vec{x}$ and $\vec{y}$
2. Equal
   (a) $\vec{z}$ and $\vec{d}$ (b) $\vec{c}$ and $\vec{y}$
   (c) $\vec{b}$ and $\vec{x}$ (d) $\vec{a}$ and $\vec{y}$
3. Collinear but not equal
   (a) $\vec{b}$ and $\vec{z}$ (b) $\vec{a}$ and $\vec{d}$
   (c) $\vec{c}$ and $\vec{y}$ (d) $\vec{x}$ and $\vec{z}$

Passage-II

Let $\vec{a} = 2\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{b} = 4\hat{i}+6\hat{j}+12\hat{k}$ are two given vectors with components along the three coordinate axes.

1. Modulus of $\vec{a}+\vec{b}$ is
   (a) 21 (b) 20
   (c) 19 (d) none of these
2. Unit vector along the $\vec{b}$ is
   (a) $\frac{1}{7}(4\hat{i}+6\hat{j}+12\hat{k})$ (b) $\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})$
   (c) $\frac{1}{14}(2\hat{i}+3\hat{j}+6\hat{k})$ (d) none of these
3. Relation among $\vec{a}$ and $\vec{b}$ is
   (a) collinear (b) equal
   (c) reciprocal (d) negative

A&R Assertion & Reason :

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If **Assertion** is **incorrect** but **Reason** is **correct**.

1. **Assertion :** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, then $|\vec{a}| = \sqrt{3}$

   Reason : Modulus = $\sqrt{x^2 + y^2 + z^2}$

   where x, y and z are coordinates along the X, Y and Z axes respectively.

2. **Assertion :** If $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$, then a vector having magnitude of 8 units along $\vec{a}$ is $\frac{8}{\sqrt{30}}\left(5\hat{i} - \hat{j} + 2\hat{k}\right)$

   Reason : Vector having modulus m along a given vector 'a' is given by, $m \times \hat{a}$

3. **Assertion :** Position vector of a point R, which divides the line joining the points $P\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $Q\left(-\hat{i} + \hat{j} + \hat{k}\right)$ in the ratio 2 : 1 internally is $\frac{-1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$.

   Reason :

   $m : n$

   $P(\vec{a})$ $R(\vec{r})$ $Q(\vec{b})$

   $$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

4. **Assertion :** 40 watts is a vector quantity.

   Reason : Energy is a scalar quantity.

5. **Assertion :** If $\vec{a}$ and $\vec{b}$ are given by two sides of a triangle, then $\vec{a} + \vec{b}$ is given by the third side.

   Reason : Two vectors can be added up by triangle law of addition.

MMQ Multiple Matching Questions :

DIRECTIONS : *Each question has statements (A, B, C, D....) given in Column I and statements (p, q, r, s....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.*

1.

Column-I		Column-II	
(A)	vectors	(p)	40°
(B)	scalars	(q)	0
(C)	$\lvert\vec{0}\rvert$	(r)	20 m/s^2
(D)	$\lvert\hat{i}\rvert$	(s)	10^{-19} columb
		(t)	2m North-West
		(u)	1

HOTS HOTS Subjective Questions :

DIRECTIONS : *Answer the following questions.*

1. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{i} - 5\hat{k}$ and $(3\hat{i} - 4\hat{j} - 4\hat{k})$ from the vertices of a right angled triangle.

2. If $\vec{a}, \vec{b}$ are the position vectors of the points $(1, -1), (-2, m)$, then find the value of m for which $\vec{a}$ and $\vec{b}$ are collinear.

3. Using, vectors, show that the points $A(-2, 1), B(-5, -1)$ and $C(1, 3)$ are collinear.

4. If the points with position vectors $60\hat{i} + 3\hat{j}, 40\hat{i} - 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ are collinear, find the value of a.

SOLUTIONS

Brief Explanations of Selected Questions

Exercise 1

FILL IN THE BLANKS :

1. scalars 2. both 3. vector
4. modulus 5. unit vector 6. position vector
7. Null 8. negative 9. collinear
10. coplanar

TRUE / FALSE :

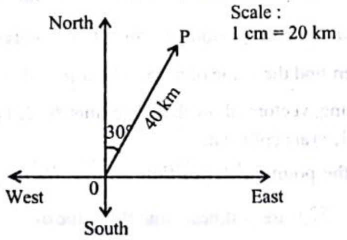
1. True 2. False 3. True
4. True 5. False 6. True
7. True 8. True 9. False
10. True

MATCH THE FOLLOWING :

1. (A) → q, (B) → p, (C) → s, (D) → r
2. (A) → s, (B) → p, (C) → q, (D) → r

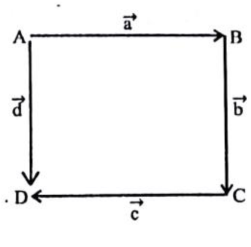
VERY SHORT ANSWER QUESTIONS :

1. A line segment of 2 cm is drawn on the right of OY making an angle of 30° with it. Vector $\overrightarrow{OP}$ represents displacement of 40 km, 30° east of north.



2. Scalar Quantity : (i) time period (ii) distance (v) work.
Vector Quantity : (iii) force (iv) velocity

3.



(i) Coinitial vectors are $\vec{a}, \vec{d}$
(ii) Equal Vectors are $\vec{b}$ and $\vec{d}$
(iii) Collinear but not equal vectors are $\vec{a}$ and $\vec{c}$.

4. The vector components are $-7\hat{i}$ and $6\hat{j}$ and scalar components are – 7 and 6.

5. Unit vector in the direction of vector $\vec{a}$

$$= \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k}) = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

6. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k})$

vector $\vec{a}$ and $\vec{b}$ have the same direction

∴ they are collinear.

7. Scalar components of vector $\overline{PQ}$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

8. $x = \pm \frac{1}{\sqrt{3}}$

9. Let vector $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$,

$$\vec{c} = \vec{a} + \vec{b} = (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j}$$

$$|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

Unit vector $\vec{c} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{j}) = \frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j}$

∴ Vector parallel to $\vec{c}$, of magnitude 5units

$$= \frac{15}{\sqrt{10}}\hat{i} + \frac{5}{\sqrt{10}}\hat{j} = \frac{3\sqrt{10}}{2}\hat{i} + \frac{\sqrt{10}}{2}\hat{j}$$

10. **Hint** – Given $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(i)

and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ ...(ii)

Now equ (i) – equ (ii) and solve it.

11. $\overline{AB} = (p-2)\hat{i} + 6\hat{j}$, $\overline{AC} = \hat{i} - 4\hat{j}$

Now A,B,C are collinear ⇔ $\overline{AB} \parallel \overline{AC}$

$$\Leftrightarrow \frac{p-2}{-1} = \frac{6}{-4} \Rightarrow p = -7/2$$

12. We have,

$\overrightarrow{PO}+\overrightarrow{OQ}=\overrightarrow{QO}+\overrightarrow{OR}$

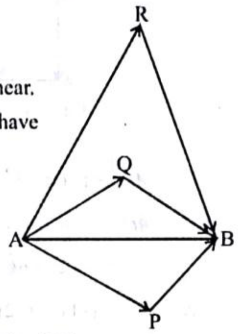
$\Rightarrow \overrightarrow{PQ}=\overrightarrow{QR}$ [By triangle law]

Thus, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are either parallel or collinear. But Q is a point common to them.

So, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are collinear.

Hence, points P, Q, R are collinear.

13. In Δs, APB, AQB and ARB we have



$\overrightarrow{AP}+\overrightarrow{PB}=\overrightarrow{AB}$

$\overrightarrow{AQ}+\overrightarrow{QB}=\overrightarrow{AB}$

and $\overrightarrow{AR}+\overrightarrow{RB}=\overrightarrow{AB}$

Adding all these we get

$\overrightarrow{AP}+\overrightarrow{PB}+\overrightarrow{AQ}+\overrightarrow{QB}+\overrightarrow{AR}+\overrightarrow{RB}=3\overrightarrow{AB}$

Hence, the sum of the given vectors is $3\overrightarrow{AB}$.

14. We have,

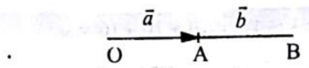
$\overrightarrow{DB}$ = Position vector of B – Position vector of D

$\Rightarrow \overrightarrow{DB}=\vec{b}-(\vec{a}-2\vec{b})=3\vec{b}-\vec{a}$

and $\overrightarrow{AC}$ = Position vector C – Position vector of A

$\Rightarrow \overrightarrow{AC}=(2\vec{a}+3\vec{b})-\vec{a}=\vec{a}+3\vec{b}$

15. Let A and B be the given points with position vectors $2\vec{a}-3\vec{b}$ and $3\vec{a}-2\vec{b}$ respectively. Let P and Q be the points dividing AB in the ratio 2 : 3 internaly and externally respectively. Then,



Position vector of

$P=\dfrac{3(2\vec{a}-3\vec{b})+2(3\vec{a}-2\vec{b})}{3+2}=\dfrac{12\vec{a}}{5}-\dfrac{13\vec{b}}{5}$

Position vector of $Q=\dfrac{3(2\vec{a}-3\vec{b})-2(3\vec{a}-2\vec{b})}{3-2}=-5\vec{b}$

16. We know that

$a_1\hat{i}+b_1\hat{j}=a_2\hat{i}+b_2\hat{j} \Leftrightarrow a_1=a_2$ and $b_1=b_2$

$\therefore\ 2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j} \Rightarrow x=2$ and $y=3$.

17. The position vector of point P is $-4\hat{i}+3\hat{j}$.

$\therefore\ \overrightarrow{OP}=-4\hat{i}+3\hat{j} \Rightarrow |\overrightarrow{OP}|=\sqrt{(-4)^2+3^2}=5$

18. $n=\pm 12$

19. Unit vector parallel to

$\vec{a}=\hat{a}=\dfrac{\vec{a}}{|\vec{a}|}=\dfrac{1}{5}(-3\hat{i}+4\hat{j})=-\dfrac{3}{5}\hat{i}+\dfrac{4}{5}\hat{j}$

20. We have

$|\vec{a}|=\sqrt{3^2+(-2)^2+6^2}=\sqrt{49}=7$

SHORT ANSWER QUESTIONS :

1. Unit vector in the direction of $\overrightarrow{PQ}$

$=\dfrac{1}{|\overrightarrow{PQ}|}\overrightarrow{PQ}=\dfrac{1}{3\sqrt{3}}(3\hat{i}+3\hat{j}+3\hat{k})=\dfrac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

$=\dfrac{1}{\sqrt{3}}\hat{i}+\dfrac{1}{\sqrt{3}}\hat{j}+\dfrac{1}{\sqrt{3}}\hat{k}$

2. Point R is $3\vec{a}+5\vec{b}$

3. Let $\bar{a}+2\bar{b}=x\bar{c}$ and $\bar{b}+3\bar{c}=y\bar{a}$.

Then $\bar{a}+2\bar{b}+6\bar{c}=(x+6)\bar{c}$ and also,

$\bar{a}+2\bar{b}+6\bar{c}=(1+2y)\bar{a}$.

So $(x+6)\ \vec{c}=(1+2y)\ \vec{a}$.

Since $\vec{a}$ and $\vec{c}$ are non-zero and non collinear, We have $x+6=0$ and $1+2y=0$, i.e., $x=-6$ and $y=-1/2$. In either case, we have $a+2b+6c=0$.

4. The point 'R' which divides PQ externally in the ratio $m:n$ is given by

$\vec{R'}=\dfrac{m\vec{b}-n\vec{a}}{m-n}=\dfrac{2(-\hat{i}+\hat{j}+\hat{k})-1\cdot(\hat{i}+2\hat{j}-\hat{k})}{2-1}$

$=\dfrac{(-2-1)\,\hat{i}+(2-2)\,\hat{j}+(2+1)\,\hat{k}}{1}$

$=-3\hat{i}+0\cdot\hat{j}+3\hat{k}=-3\hat{i}+3\hat{k}$

5. Unit vector parallel to $2\vec{a}-\vec{b}+3\vec{c}$

$=\dfrac{3}{\sqrt{22}}\hat{i}-\dfrac{3}{\sqrt{22}}\hat{j}+\dfrac{2}{\sqrt{22}}\hat{k}$

6. (i) Mass-scalar (ii) Directed distance-vector

(iii) Force-vector (iv) Speed-scalar

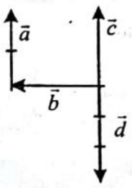
(v) Velocity-vector (vi) Electric charge-vector

7. Clearly,

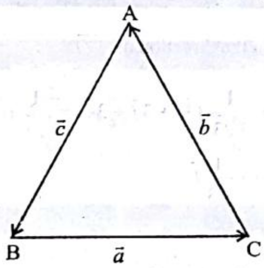
(i) $\vec{a}, \vec{c}$ and $\vec{d}$ are collinear vectors.

(ii) $\vec{a}$ and $\vec{c}$ are equal vectors each of magnitude 2 units.

(iii) $\vec{b}, \vec{c}$ and $\vec{d}$ are co-initial vectors.



8. Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}, \overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$.



$\vec{a} + \vec{b} + \vec{c} = \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}$

$\Rightarrow \quad \vec{a} + \vec{b} + \vec{c} = \overrightarrow{BA} + \overrightarrow{AB} \quad \left[\because \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{BA}\right]$

$\Rightarrow \quad \vec{a} + \vec{b} + \vec{c} = \overrightarrow{BB}$ [By triangle law]

$\Rightarrow \quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$ [By def. of null vector]

Hence, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

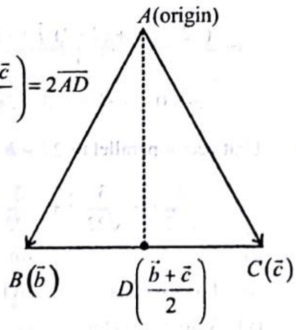
9. Let A be the origin and let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively. Then, the position vector of the mid-point of BC is $\dfrac{\vec{b}+\vec{c}}{2}$.

$\therefore$ Position vector of $D = \dfrac{\vec{b}+\vec{c}}{2}$

Now,

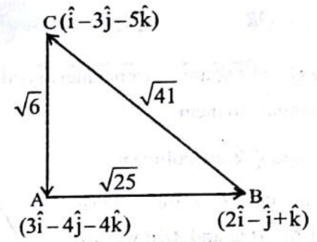
$\overrightarrow{AB} + \overrightarrow{AC} = \vec{b} + \vec{c}$

$\Rightarrow \quad \overrightarrow{AB} + \overrightarrow{AC} = 2\left(\dfrac{\vec{b}+\vec{c}}{2}\right) = 2\overrightarrow{AD}$



LONG ANSWER QUESTIONS :

1. $\overrightarrow{AB} = \vec{b} - \vec{a} = (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k}) = -\hat{i} + 3\hat{j} + 5\hat{k}$



$\therefore \quad |\overrightarrow{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

$\overrightarrow{BC} = \vec{c} - \vec{b} = (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} - + \hat{k})$

$= \hat{i} - 2\hat{j} - 6\hat{k}$

$\therefore \quad |\overrightarrow{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$

$\overrightarrow{CA} = \vec{a} - \vec{c} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k}) = 2\hat{i} - \hat{j} + \hat{k}$

$\therefore \quad |\overrightarrow{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$

Now $|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = |\overrightarrow{BC}|^2$

Hence ΔABC is a right angled triangle

2. The position vectors of points A, B, C are $\hat{i} + 2\hat{j} + 7\hat{k}$, $2\hat{i} + 6\hat{j} + 3\hat{k}$, $3\hat{i} + 10\hat{j} - \hat{k}$ respectively.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (2\hat{i} + 6\hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} + 7\hat{k})$

$= \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (3\hat{i} + 10\hat{j} - \hat{k}) - (2\hat{i} + 6\hat{j} + 3\hat{k})$

$= \hat{i} + 4\hat{j} - 4\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OB} = (3\hat{i} + 10\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 7\hat{k})$

$= 2\hat{i} + 8\hat{j} - 8\hat{k} = 2(\hat{i} + 4\hat{j} - 4\hat{k})$

$|\overrightarrow{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$,

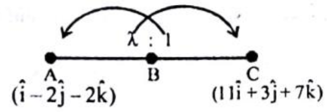
$|\overrightarrow{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$

$|\overrightarrow{AC}| = \sqrt{2^2 + 8^2 + (-8)^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$

Thus $|\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|$, Hence A, B, C are collinear.

3. The position vectors of A and C are $\hat{i} - 2\hat{j} - 8\hat{k}$ and $11\hat{i} + 3\hat{j} + 7\hat{k}$ respectively.

let the point B lying on AC divides it in the ratio $\lambda : 1$



$\therefore$ position vector of point B is

$$\frac{\lambda\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+1\cdot\left(\hat{i}-2\hat{j}-8\hat{k}\right)}{\lambda+1}$$

$$=\left(\frac{11\lambda+1}{\lambda+1}\right)\hat{i}+\left(\frac{3\lambda-2}{\lambda+1}\right)\hat{j}+\left(\frac{7\lambda-8}{\lambda+1}\right)\hat{k}$$

$=5\hat{i}+0\hat{j}-2\hat{k}$ (Given)

$$\Rightarrow \left(\frac{3\lambda-2}{\lambda+1}\right)=0, \quad \therefore \lambda=\frac{2}{3}$$

Now, $\dfrac{11\lambda+1}{\lambda+1}=\dfrac{11\times\frac{2}{3}+1}{\frac{2}{3}+1}=\dfrac{22+3}{2+3}=\dfrac{25}{5}=5$

$$\frac{7\lambda-8}{\lambda+1}=\frac{7\times\frac{2}{3}-8}{\frac{2}{3}+1}=\frac{14-24}{2+3}=\frac{-10}{5}=-2$$

$\Rightarrow$ A, B, C are collinear and B divides it in the ratio 2 : 3.

4. Let O be the origin and let $P(x, y)$ be the required point. Then, P is the tip of the position vector $\overrightarrow{OP}$ of point P.

We have,

$$\overrightarrow{OP}=x\hat{i}+y\hat{j}$$

and $\overrightarrow{AB}$ = Position vector of B – position vector of A

$\Rightarrow$ $\overrightarrow{AB}=\left(5\hat{i}+0\hat{j}\right)-\left(3\hat{i}+\hat{j}\right)=2\hat{i}-\hat{j}$

$\therefore$ $\overrightarrow{OP}=\overrightarrow{AB}$

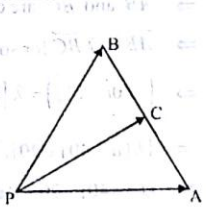
$\Rightarrow$ $x\hat{i}+y\hat{j}=2\hat{i}-\hat{j} \Leftrightarrow x=2$ and $y=-1$

Hence, the coordinates of the required point are (2, –1).

Exercise 2

MULTIPLE CHOICE QUESTIONS :

1. (b) $\because \dfrac{AC}{CB}=\dfrac{1}{1}$



$\Rightarrow$ $\overline{AC}=\overline{CB}$

$\Rightarrow$ $\overline{AP}+\overline{PC}=\overline{CP}+\overline{PB}$

$\Rightarrow$ $\overline{PA}+\overline{PB}=2\overline{PC}$

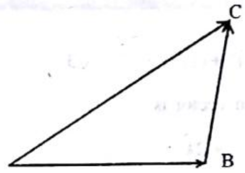
2. (d) Since, P bisects both the diagonal AC and BD, so

$\therefore$ $\overline{OA}+\overline{OC}=2\overline{OP}$ and $\overline{OB}+\overline{OD}=2\overline{OP}$

$\Rightarrow$ $\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}=4\overline{OP}$

3. (d) 4. (c) 5. (b)

6. (a) By triangle law of vector addition $\overline{AB}+\overline{BC}=\overline{AC}$



or $\overline{AB}+\overline{BC}=-\overline{CA}$ or $\overline{AB}+\overline{BC}+\overline{CA}=\bar{0}$

7. (d) Option (d) is incorrect since both the vectors $\bar{a}$ and $\bar{b}$, being collinear, are not necessarily in the same direction. They may have opposite directions. Their magnitude may be different.

8. (d) $\bar{a}$ is a non-zero vector of magnitude a $\Rightarrow |\bar{a}|=a$

Now, $\lambda\,\bar{a}$ is a unit vector if $|\lambda\,\bar{a}|=1$

or $|\lambda||\bar{a}|=1$ or $|\lambda|a=1 \Rightarrow a=\dfrac{1}{|\lambda|}$

9. (c) $3\vec{A}=3(x+4y)\,a+3(2x+y+1)\,b$

$2\vec{B}=2(y-2x+2)\,a+2(2x-3y-1)\,b$

$\therefore$ $3\vec{A}=2\vec{B} \Rightarrow 3(x+4y)=2(y-2x+2)$

$\Rightarrow$ $3(2x+y+1)=2(2x-3y-1)$

$\Rightarrow$ $7x+10y=4$ and $2x+9y=-5$

$\Rightarrow$ $x=2, y=-1$

10. (d) 11. (d) 12. (c)

13. (b) 14. (b)

MORE THAN ONE CORRECT :

1. (a, b, d) 2. (a, b) 3. (c, d)
4. (a, b, c) 5. (a, b, d) 6. (a, b, c, d)
7. (a, b)

PASSAGE BASED QUESTIONS :

PASSAGE-1

1. (a, b) 2. (b, c) 3. (a, d)

PASSAGE-2

1. (a) $\bar{a}+\bar{b}=(2+4)\hat{i}+(3+6)\hat{j}+(6+12)\hat{k}$

$=6\hat{i}+9\hat{j}+18\hat{k}$

$|\bar{a}+\bar{b}|=\sqrt{(6)^2+(9)^2+(18)^2}=\sqrt{441}=21$

2. (b) $|\bar{b}|=\sqrt{(4)^2+(6)^2+(12)^2}=14$

$$\hat{b}=\frac{\bar{b}}{|\bar{b}|}=\frac{1}{14}\left(4\hat{i}+6\hat{j}+12\hat{k}\right)=\frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}$$

3. (a) Collinear

$\bar{b}=2\left(2\hat{i}+3\hat{j}+6\hat{k}\right)=2\bar{a}$

ASSERTION & REASON :

1. (a) $|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

2. (a) The given vector is

$\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30}$

$\therefore$ Unit vector in the direction of vector

$\hat{a} = \frac{1}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

$\therefore$ Vector of magnitude 8 in the direction of vector $\vec{a}$

$= 8 \times$ unit vector $\hat{a} = 8 \cdot \frac{1}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

$= \frac{8}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k}) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

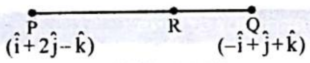
3. (c) The point R which divides the line joining the point P ($\vec{a}$) and Q ($\vec{b}$) in the ratio m : n,

$\vec{R} = \frac{m\vec{b} + n\vec{a}}{m+n}$

$\Rightarrow m = 2, n = 1, \vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$

$\therefore$ $\vec{R}$ is $\frac{2 \times (-\hat{i} + \hat{j} + \hat{k}) + 1 \times (\hat{i} + 2\hat{j} - \hat{k})}{2+1}$

$\vec{R} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$



4. (d)

5. (a)

MULTIPLE MATCHING QUESTIONS :

(1) (A) $\to$ r, t; (B) $\to$ p, s; (C) $\to$ q; (D) $\to$ u

HOTS SUBJECTIVE QUESTIONS :

1. The position vectors of the points A, B and C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{i} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively.

$\overline{AB} = \overline{OB} - \overline{OA} = (\hat{i} - 3\hat{i} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$:

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$|\overline{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$\therefore$ $|\overline{AB}|^2 = 41$

$\overline{BC} = \overline{OC} - \overline{OB} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= 2\hat{i} - \hat{j} + \hat{k}$

$|\overline{BC}|^2 = BC^2 = 2^2 + (-1)^2 + 12 = 4 + 1 + 1 = 6$

$|\overline{AC}| = \overline{OC} - \overline{OA} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= \hat{i} - 3\hat{j} - 5\hat{k}$

$|\overline{AC}|^2 = AC^2 = 1^2 + (-3)^2 + (-5)^2 = 1 + 9 + 25 = 35$,

$BC^2 + AC^2 = AB^2 \Rightarrow$ Triangle ABC is a right angled triangle.

2. We have,

$\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = -2\hat{i} + m\hat{j}$

Since $\vec{a}$ and $\vec{b}$ are collinear

$\therefore$ $\vec{a} = \lambda\vec{b}$, for some scalar λ

$\Rightarrow \hat{i} - \hat{j} = (-2\lambda)\hat{i} + (m\lambda)\hat{j}$

$\Rightarrow 1 = -2\lambda$ and $-1 = m\lambda$

$\Rightarrow \lambda = -\frac{1}{2}$ and $\lambda = -\frac{1}{m} \Rightarrow -\frac{1}{2} = -\frac{1}{m} \Rightarrow m = 2$

3. We have,

$\overline{AB}$ = Position vector of B – Position vector of A

$\Rightarrow \overline{AB} = (-5\hat{i} - \hat{j}) - (-2\hat{i} + \hat{j}) = -3\hat{i} - 2\hat{j}$

and, $\overline{BC}$ = Position vector of C – Position vector of B

$\Rightarrow \overline{BC} = (\hat{i} + 3\hat{j}) - (-5\hat{i} - \hat{j})$

$\Rightarrow \overline{BC} = 6\hat{i} + 4\hat{j}$

Clearly, $\overline{BC} = -2\,\overline{AB}$

Therefore, $\overline{AB}$ and $\overline{BC}$ are parallel vectors.

But, B is a common pont of $\overline{AB}$ and $\overline{BC}$.

Hence, the points A, B, C are collinear.

4. Let the points be A, B and C respectively.

Then, A, B, C are collinear

$\Rightarrow \overline{AB}$ and $\overline{BC}$ are collinear

$\Rightarrow \overline{AB} = \lambda\overline{BC}$ for some scalar λ

$\Rightarrow \left(-20\hat{i} - 11\hat{j}\right) = \lambda\{(a-40)\hat{i} - 44\hat{j}\}$

$\Rightarrow \{\lambda(a-40) + 20\}\hat{i} - (44\lambda - 11)\hat{j} = \vec{0}$

$\Rightarrow \lambda(a-40) + 20 = 0$ and $44\lambda - 11 = 0$

[$\because \hat{i}, \hat{j}$ are non-collinear]

$\Rightarrow \lambda = \frac{1}{4}$ and $\lambda(a-40) + 20 = 0$

$\Rightarrow \frac{1}{4}(a-40) + 20 = 0 \Rightarrow a = -40$

Hence, the given points will be collinear, if $a = -40$.